University of Ottawa

Department of Mathematics and Statistics

MAT 2322C-Winter 2018-Midterm 2 – March 15, 17:30-18:50

Professor: Dr.Dr. Alessandro Selvitella

Surname ____________________ First Name ____________________ Student # __________

- Time: 80 min. Total points: 25.
- Only the following calculators are allowed during Faculty of Science examinations: Texas Instruments TI-30 and TI-34, Casio FX-260 and Casio FX-300 (scientific and non-programmable calculators).
- Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite.
- **Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.**

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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Question	1 - 4	5	6	7	8	Total
Maximum	8	4	4	4	5	25
Grade						

Put the answers of multiple choice questions into the following table.

Question	1	2	3	4
Answer				

Part I: Multiple Choice Questions, 2 points each, only answers are marked.

1. Which of the following integrals computes $\iiint_E y\, dV$ in cylindrical coordinates, where E is the solid that lies between cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, above the xy-plane and below the plane $z = x + 4$?

- (a) $\int_0^{2\pi} \int_1^2 \int_0^{r\cos\theta+4} r^2 \sin\theta dz dr d\theta$
- (b) $\int_0^{2\pi} \int_1^2 \int_0^{r\cos\theta+4} r \sin\theta dz dr d\theta$
- (c) $\int_0^{2\pi} \int_1^2 \int_0^{r\sin\theta} (r\cos\theta + 4) dz dr d\theta$
- (d) $\int_0^{2\pi} \int_1^2 \int_0^{r} r^2 \sin\theta dz dr d\theta$
- (e) $\int_0^{2\pi} \int_1^2 \int_0^{r} r \sin\theta dz dr d\theta$

Solution: (a).
$y = r\sin\theta$, $dV = r dr d\theta dz$, $0 \le z \le x + 4$ becomes $0 \le z \le r\cos\theta + 4$.

2. Let R be the ellipse given by $\frac{x^2}{9} + \frac{y^2}{4} = 1$. By using the transformation $x = 3r\cos\theta,\ y = 2r\sin\theta$, the Jacobian $J(r, \theta) =$

(a) $2r$ (b) $6r$ (c) $12r$ (d) $2r\sin(2\theta)$ (e) $3r\sin(2\theta)$

Solution: (b).
$J(r, \theta) = abr = 3(2)r = 6r$

3. Let the curve C be $\vec{r}(t) = (t^2, -t^2, t)$. Find the unit tangent vector at $t = 1$.

(a) $(\frac{2}{9}, -\frac{2}{9}, \frac{1}{9})$ (b) $(-\frac{2}{9}, -\frac{2}{9}, \frac{1}{9})$ (c) $(-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3})$ (d) $(\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3})$ (e) $(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3})$

Solution: (e).
$\vec{r}\,'(t) = (2t, -2t, 1)$, $\vec{r}\,'(1) = (2, -2, 1)$, $|\vec{r}\,'(1)| = 3$. Thus

$$\vec{T}(1) = \frac{\vec{r}\,'(1)}{|\vec{r}\,'(1)|} = (\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}).$$

4. The length of the curve determined by equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

- (a) $\int_0^4 \sqrt{1 + 4t}dt$
- (b) $2\int_0^4 \sqrt{1 + t^2}dt$
- (c) $\int_0^4 \sqrt{1 + 2t^2}dt$
- (d) $\int_0^4 \sqrt{1 + 4t^2}dt$
- (e) $2\pi \int_0^4 \sqrt{1 + 4t^2}dt$

Solution: (d).
$L = \int_0^4 \sqrt{(2t)^2 + 1^2}dt = \int_0^4 \sqrt{4t^2 + 1}dt$

Part II: Four Long Answer Questions, you have to show your work clearly.

5. (4 points) Consider the parallelepiped given by

$$P := \{(x, y, z) \in \mathbb{R}^3 | 1 \leq x \leq 3, 0 \leq y \leq 2, 4 \leq z \leq 5\}.$$

Compute $\iiint_P z \, dV$.

Solution: (2 points domain, 2 points correct integration)

$$\iiint_P z \, dV = \int_1^3 dx \int_0^2 dy \int_4^5 z dz = 2 * 2 * \left(\frac{5^2}{2} - \frac{4^2}{2}\right) = 18.$$

6. (4 points) Compute the curvature of the curve

$$\vec{r}(t) = (\cos t^3, \sin t^3, t)$$

at $t = 0$.

Solution:

$$r'(t) = (-3t^2 \sin t^3, 3t^2 \cos t^3, 1)$$

(1.5 points) and

$$r''(t) = (-6t \sin t^3 + 9t^4 \cos t^3, 6t \cos t^3 - 9t^4 \sin t^3, 0).$$

(1.5 points) Therefore: $r'(0) = (0, 0, 1)$ and $r''(0) = (0, 0, 0)$. This implies: $\kappa(0) = 0$. (1 point)

7. (4 points) Evaluate the triple integral $\iiint\limits_E 2x\ dV$, where E is the solid in the first octant, enclosed by the four planes: $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + z = 6$.

Solution: The limits of z are: $0 \leq z \leq 6 - 2x - 3y$. The region D in the xy-plane is the triangle with vertices at (0,0), (3,0) (0,2). The triple integral is evaluated by the iterated integral:

$$\iiint\limits_E 2xdV = \iint_D \int_0^{6-2x-3y} (2x)dzdA$$

$$= \int_0^3 \int_0^{-\frac{2}{3}x+2} [2x(6-2x-3y)]dydx = \int_0^3 [\frac{4}{3}x^3 - 8x^2 + 12x]dx = 9.$$

8. (5 points) Find the integral $\iiint_E \sqrt{x^2+y^2+z^2}dV$, where E is region between the sphere $x^2+y^2+z^2=4$, and the sphere $x^2+y^2+z^2=9$.

Solution: We use spherical coordinates

$$x=\rho\sin\phi\cos\theta, y=\rho\sin\phi\sin\theta, z=\rho\cos\phi, dV=\rho^2\sin\phi d\rho d\phi d\theta.$$

It is easy to see that $2\le\rho\le 3, 0\le\theta\le 2\pi, 0\le\phi\le\pi$. Thus

$$\iiint_E \sqrt{x^2+y^2+z^2}dV = \int_0^{2\pi}\int_0^{\pi}\int_2^3 \sqrt{\rho^2}\rho^2\sin\phi d\rho d\phi d\theta$$

$$= \left(\frac{1}{4}\rho^4\right)\Big|_2^3 (-\cos\phi)\ |_0^{\pi}\ (\theta)\ |_0^{2\pi} = 65\pi.$$

Page for rough work!