MATH 1402B **Term test #1 – solution** **29 January 2012**

Maximum score: 35 marks

****

Non-programmable calculators are permitted. This test is closed book.

Supply your answers on this sheet, but TA's have extra paper if you need it.

PLEASE PRINT ____________ First name Last name ______ Student number

Please show your work where appropriate!

1. [4 x 2 = 8 marks] Find the domain of the following functions, given the rule:

a. $y = f(x) = \dfrac{12x-4}{\sqrt{3x-6}}$ $\Rightarrow$ $3x-6>0 \Rightarrow x>2$

$\therefore$ dom $f = (2,\infty)$

b. $y = f(x) = e^x \cdot \ln(2x-5)$ $\Rightarrow$ $2x-5>0 \Rightarrow x > 5/2$

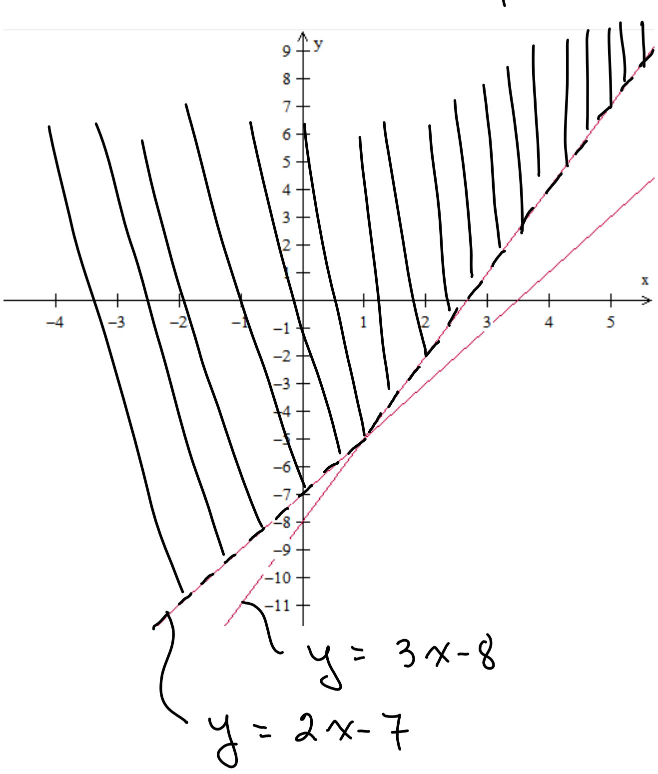
$\therefore$ dom $f = (5/2, \infty)$

c. $y = f(x) = 2x^3 - 5x^2 + 3x + 2$ $\Rightarrow$ dom $f = \mathbb{R}$

d. $z = f(x,y) = \dfrac{\ln(y-3x+8)}{\sqrt{7-y-2x}}$ $\Rightarrow$ $y-3x+8>0 \Rightarrow y > 3x-8$

AND $7-y-2x>0 \Rightarrow y > 2x-7$

No need to plot the region, but I cannot resist:



NB- the dashed border is not included in the domain.

2. [3+1] Let: $z = f(x,y) = 12x^2 - 3x - 3y + 34$.

a. Determine the rule $y = g(x)$ of the function obtained when considering an iso–z section (i.e. constant z) where z= z_0=10.

$10 = 12x^2 - 3x - 3y + 34 \Rightarrow 3y = 12x^2 - 3x + 24$

$\boxed{y = g(x) = 4x^2 - x + 8}$

****

b. Determine $f(-2,2)$ $f(-2,2) = 12(-2)^2 - 3(-2) - 3(2) + 34$
$= 48 + 6 - 6 + 34 = 82$

3. [5 x 2=10] $z = f(x,y) = 5x^3 - x^2y + 3y^2 + 2y - x$ Determine the following: $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}$

$\frac{\partial z}{\partial x} = \boxed{15x^2 - 2xy - 1}$; $\frac{\partial z}{\partial y} = \boxed{-x^2 + 6y + 2}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial x}\left(15x^2 - 2xy - 1\right) = \boxed{30x - 2y}$

$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial y}\left(-x^2 + 6y + 2\right) = \boxed{6}$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(-x^2 - 6y + 2\right) = \boxed{-2x}$

4. [2] Again, considering the same case, $z = f(x,y) = 5x^3 - x^2y + 3y^2 + 2y - x$, show that Young's Theorem holds.

Need to compute $\frac{\partial^2 z}{\partial y \partial x}$: $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) =$

$\frac{\partial}{\partial y}\left(15x - 2xy - 1\right) = \boxed{-2x} = \frac{\partial^2 z}{\partial x \partial y}$

5. [8] Let $Q = f(K,L) = 10K^{\frac{2}{5}}L^{\frac{4}{5}}$ be the rule for a production function, where ***K*** corresponds to ***input capital***, ***L*** corresponds to ***input labour*** and ***Q*** corresponds to ***output production***. Determine the following quantities:

a. [2] The rule $K = g(L)$ that represents an isoquant for which $Q = Q_0 = 20$.

$f(K,L) = 20 = 10K^{2/5}L^{4/5} \Rightarrow K^{2/5} = \frac{2}{L^{4/5}} \Rightarrow K = \frac{2^{5/2}}{(L^{4/5})^{5/2}} = \frac{\sqrt{32}}{L^2} = \frac{4\sqrt{2}}{L^2}$

b. [3] Again, when $Q = Q_0 = 20$, determine the ***marginal rate of substitution*** **(MRS)**, i.e. $\left|\frac{dK}{dL}\right|$, for $L = 2$. $\Rightarrow Q_0 = 20 \Rightarrow K = \frac{4\sqrt{2}}{L^2} = 4\sqrt{2}L^{-2}$

$\frac{dK}{dL} = -8\sqrt{2}L^{-3} \Rightarrow \text{MRS} = \left|\frac{dK}{dL}\right| = \frac{8\sqrt{2}}{L^3} \xRightarrow{L=2} \boxed{\text{MRS} = \sqrt{2}}$

c. [3] The ***marginal product of capital*** **(MPK – i.e.** $\frac{\partial Q}{\partial K}$**)**, when K=3 and L=9

$\frac{\partial Q}{\partial K} = 10\left(\frac{2}{5}\right)K^{-3/5}L^{4/5} = 4\sqrt[5]{\frac{L^4}{K^3}}$; $(K,L) = (3,9) \Rightarrow \frac{\partial Q}{\partial K} = 4\sqrt[5]{\frac{9^4}{3^3}}$
$= 4 \cdot 3 = \boxed{12}$

6. [3] Let $z = f(x,y) = 2x^2 - 5y^2 + 12x + 10y - 10$. Determine the point where there is a minimum, maximum or inflection point.

$\left.\begin{array}{l}\frac{\partial z}{\partial x} = 4x + 12 \\ \frac{\partial z}{\partial y} = -10y + 10\end{array}\right\}$ $\frac{\partial z}{\partial x} = 0 \Rightarrow x = -3$ $\therefore (-3, 1)$ is a critical point.
$\frac{\partial z}{\partial y} = 0 \Rightarrow y = 1$

$\left.\frac{\partial^2 z}{\partial x^2} = 4 \; ; \; \frac{\partial^2 z}{\partial y^2} = -10\right\} \Rightarrow$ opposite sign $\Big\} \Rightarrow$ <u>SADDLE!</u>