Solutions
Assignment 6
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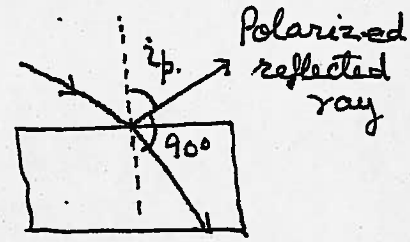
Q.1

Find the angle of incidence for which the light reflected from water of refractive index 1.333 is plane polarized. : Ans: 53°

Solution: when the angle of incidence is such that the angle between the reflected and refracted rays is 90°, the reflected ray is completely polarized. We call the angle of incidence polarizing angle, i_p. If n is the index of refraction of the reflecting material



$$n = \tan i_p \quad \text{(Brewster law)}$$

Given, $n = 1.333$

$$\therefore \; i_p = \tan^{-1} 1.333 = 53^\circ$$

Q.2

Light reflected from a flint-glass surface is plane-polarized when the angle of incidence is 59°. What is the index of refraction of the glass? Ans: (1.66)

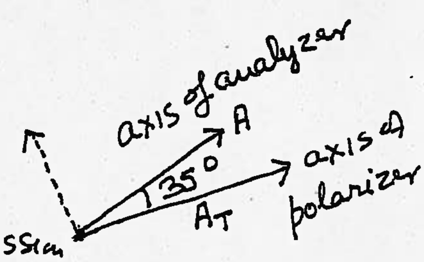
Solution: using Brewster Law.

$$n = \tan i_p = \tan 59^\circ = 1.66$$

Q.3

Two Nocol prisms have their planes of vibration parallel. One of the prisms is then turned so that its plane of vibration makes an angle of 35° with that of the other. What fraction of the amplitude incident on the second Nicol prism is transmitted? What percentage of light incident on the second Nicol prism is transmitted? Ans: (0.819, 67.%)

Solution: The linearly polarized light transmitted by the polarizer may be resolved into two components, one parallel and the other perpendicular to the transmission



direction of the analyzer. Only the parallel component of the amplitude $A\cos\theta$ will be transmitted by the analyzer. Since

Since the intensity is proportional to the square of the amplitude

$$\boxed{I = A^2 \cos^2\theta = I_{max} \cos^2\theta} \rightarrow \text{Malus' Law}$$

$I_{max} \rightarrow$ max amount of light transmitted, and I is the amount transmitted at angle θ.

In the problem $\theta = 35°$. The fraction of amplitude incident on second Nicol prism transmitted is

$$A_T = A \cos 35°$$

$$\therefore \frac{A_T}{A} = \cos 35° = 0.819$$

% age of light incident on the second Nicol from Malus' Law.

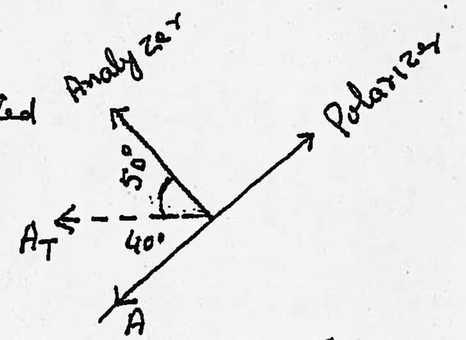
$$\frac{I}{I_0} = \cos^2 35° = 0.67 \text{ or } 67\%$$

Q4 Two polaroids are crossed. If the analyzer is now rotated through an angle of 50°, what percentage of the plane polarized light from the polarizer is transmitted, assuming the Polaroid is perfect transmitter of one polarization and a perfect absorber of the other? What percentage of the ordinary light incident on the polarizer is transmitted through the analyzer? Ans: (58.7%, 29.3 %)

Solution

Fraction of polarized light transmitted

$$\frac{I}{I_{max}} = \left(\frac{A_T}{A}\right)^2 = (\cos^2 40°)^2 = 0.587 \text{ or } 58.7\%$$



% age of ordinary light transmitted $= \frac{58.7}{2} = 29.4\%$

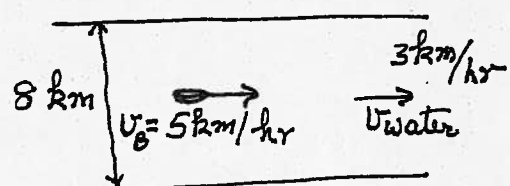
Q.5 A river 8 km wide has a current with a speed of 3 km/hr. How much longer will it take a boat with a speed of 5 km/hr to go upstream 8 km and return than go directly across the river and return to the same point? Ans: (2.26 hr)

Solution: Time taken to go east 8 km

$$= \frac{8}{(5+3)} = 1 \text{ hr.}$$

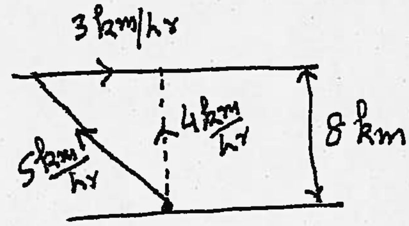
Time taken to come down 8 km

$$= \frac{8}{(5-3)} = 4 \text{ hr.}$$



$t_1 \rightarrow$ Total time $= 1 + 4 = 5$ hr.

Time taken to go directly across and Come back is



$$t_2 = 2 \times \frac{8}{4} = 4\,hr$$

$\therefore$ time difference $= t_1 - t_2 = 5 - 4 = 1\,hr$

Q.6 Find the mass of an electron traveling at 0.6 times the speed of light. How many times as great as the rest mass is this value? Ans: (1.14×10^{-30}kg, 1.25)

Solution: Rest mass of electron $\rightarrow m_0 = 9.11 \times 10^{-31}$ km.

speed of electron $= 0.6c$

$\therefore$ Relativistic electron mass $\rightarrow m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \dfrac{9.11 \times 10^{-31}}{\sqrt{1 - (0.6)^2}} = 1.14 \times 10^{30}$ kg

$$\therefore \frac{m}{m_0} = \frac{1}{\sqrt{1 - .36}} = 1.25$$

Q7 Find the work, which must be done on an electron to increase its speed from 0.5c to 0.9c where c is the speed of light. Ans: (9.33×10^{-20} J)

Solution: Kinetic energy of electron at a speed 0.5c

$$K_1 = m_0 c^2 \left\{ \frac{1}{\sqrt{1 - (0.5)^2}} - 1 \right\} = 0.154\, m_0 c^2$$

Kinetic energy of electron at speed 0.9c

$$K_2 = m_0 c^2 \left\{ \frac{1}{\sqrt{1 - (.9)^2}} - 1 \right\} = 1.294\, m_0 c^2$$

$\therefore$ work which must be done $\rightarrow W = K_2 - K_1 = 1.294 m_0 c^2 - 0.154 m_0 c^2$

$$= (m_0 c^2)(1.140)$$

Rest mass of electron $\rightarrow m_0 = 0.511$ MeV/c^2

$\therefore$ Work Done $= (0.511 \times 1.14) = 0.583$ MeV

1 MeV $= 1.6 \times 10^{-13}$ J

work done in Joules $= 0.583 \times 1.6 \times 10^{-13}$

$$= 9.33 \times 10^{-14}\,J$$

Q.8 How many electron volts of energy must an electron gain to bring its mass to (a) 1.05 m_0, (b) 2 m_0? In each case what is the speed of electron?
Ans: {(a) 25500, 9.2x10^7 m/s, (b) 511000, 2.6x 10^8}

Solution: mass of electron (relativistic) → m

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1-\beta^2}} \quad \text{where } \beta = \frac{v}{c} \quad (1)$$

(a) Energy gained → $(1.05 m_0 - m_0)c^2 = 0.05\, m_0 c^2$

$$m_0 = 0.511 \ \frac{MeV}{c^2}$$

$$\therefore \text{Energy gained} = 0.05 \times 0.511 \text{ MeV}$$
$$= 25550 \text{ eV}.$$

(b) Energy gained → $(2m_0 - m_0)c^2 = m_0 c^2 = 0.511$ MeV
$$= 511000 \text{ eV}.$$

(a) Speed of electron is given by.

From (1)
$$\frac{1.05 m_0}{m_0} = \frac{1}{\sqrt{1-\beta^2}} \quad \text{or} \quad (1-\beta^2) = \frac{1}{(1.05)^2}$$

$$\beta = \sqrt{0.093} = 0.305$$

$$\therefore v = 0.305 \times (3\times10^8) = 9.2\times10^7 \text{ m/s}$$

(b) Speed of electron is given by

$$\frac{2 m_0}{m_0} = \frac{1}{\sqrt{1-\beta^2}}$$

$$(1-\beta^2) = \frac{1}{4} = 0.25$$

$$\beta^2 = 0.75 \quad \text{or} \quad \beta = \frac{v}{c} = 0.866$$

$$\therefore v = 0.866 \times (3\times10^8)$$
$$= \underline{2.6\times10^8 \text{ m/s}}$$

Q 9.

A proton of rest mass 938 MeV (1.67×10^{-27} kg) is given a kinetic energy of 9.38×10^{9} eV in a proton synchrotron. Find the mass of proton at this speed. What must be the radius of the orbit if the vertical magnetic field is 1.5 webers/m^2?

Ans: (1.84×10^{-26} kg, 23m)

<u>Solution:</u> Relativistic mass m_p of proton is

$$m_p c^2 = m_0 c^2 + K \rightarrow \text{kinetic energy.}$$

$$K = 938 \text{ Mev} = 938 \times (1.6 \times 10^{-13}) = 15.0 \times 10^{-10} \text{ J}$$

$$\therefore \; m_p c^2 = (1.67 \times 10^{-27}) c^2 + 15 \times 10^{-10}$$

$$m_p = (1.67 \times 10^{-27}) + \frac{15 \times 10^{-10}}{(3 \times 10^8)^2} = 1.84 \times 10^{-26} \text{ kg.}$$

From Newton's Second Law $\frac{m_p v^2}{R} = q v B$ or $R = \frac{(1.84 \times 10^{-26}) v}{qB}$

$$R = \frac{(1.84 \times 10^{-26}) v}{(1.6 \times 10^{-19}) \times 1.5} \qquad (1)$$

To find v, we use

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad \text{or} \quad 1.84 \times 10^{-26} = \frac{1.67 \times 10^{-27}}{\sqrt{1 - \beta^2}}$$

$$\therefore \; 338.56 (1 - \beta^2) = 2.789$$

$$\text{or } \beta = 0.992 \quad \text{or} \quad v = 0.996 \times 3 \times 10^8$$

$$= 2.988 \times 10^8 \text{ m/s}$$

Putting the value of v in (1)

$$R = \frac{(1.84 \times 10^{-26})(2.988 \times 10^8)}{(1.6 \times 10^{-19})(1.5)}$$

$$= 23 \text{ m.}$$

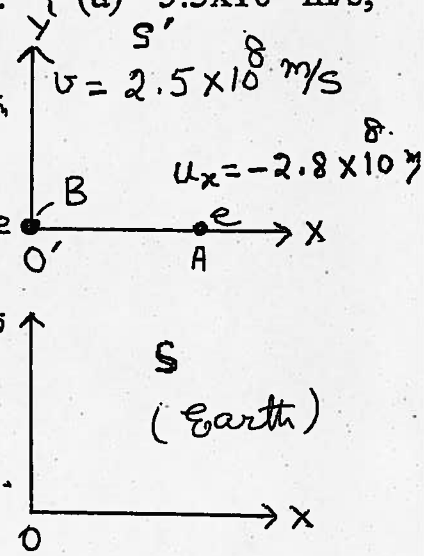
Q 10 An arrow passes an observer with a speed 0.6 times that of light If the rest length of the arrow is 0.75 m, find its apparent length as it passes the observer.
Ans: (0.6 m)

Solution: $v = 0.6c$

Rest length of arrow means that it would be the length of the arrow if the observer was at rest with respect to the arrow. That would be the proper length of the arrow. The observer on earth will see the contracted length, given by

$$l' = l_0\sqrt{1-\frac{v^2}{c^2}} = 0.75 \times \sqrt{1-(0.6)^2} = 0.6\,m.$$

Q.11 An electron moving to the right with a speed of 2.5×10^8 m/s passes an electron moving to the left with a speed of 2.8×10^8 m/s. Find the speed of one electron relative to the other as predicted by (a) the Newtonian-Galilean transformation and (b) the relativistic transformation. { (a) 5.3×10^8 m/s, 2.98×10^8 m/s }



Solution: According to Galilean transformation the relative speed of electron B with respect to A is:

$$U_r = 2.8 \times 10^5 + 2.5 \times 10^8 = 5.3 \times 10^8\ m/s$$

B. Take the moving frame on electron B so that it is at rest in the S' frame. Then the speed u_x of electron A is given by $u_x = -2.8 \times 10^8$ m/s. Using the relativistic addition of velocities transformation we get:

$$u_x' = \frac{u_x - v}{\left(1-\frac{u_x v}{c^2}\right)} = \frac{2.5 \times 10^8 - (-2.8 \times 10^8)}{1 - \frac{(2.5 \times 10^8)(-2.8 \times 10^8)}{c^2}} = \frac{5.3 \times 10^8}{1 + 0.778}$$

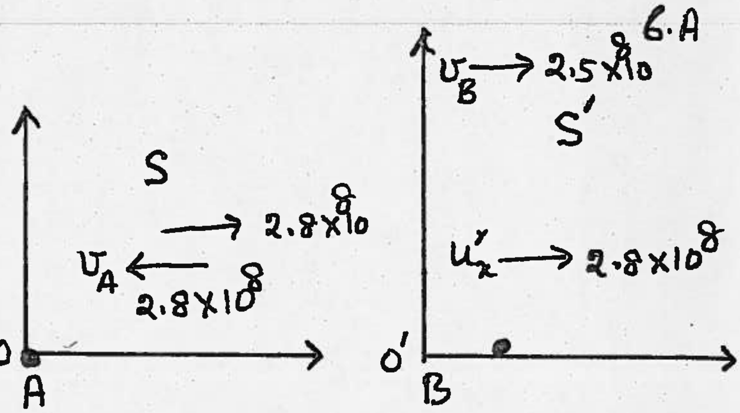
$$= 2.98 \times 10^8\ m/s$$

Q11

Alternative Approach

6.A

In the earth's frame S, two electrons A and B are moving with uniform speeds $v_A = -2.8 \times 10^8$ m/s and $v_B = 2.5 \times 10^8$ m/s.



Let two moving frames be attached to the two electrons namely O, O'. Superimpose on both electrons a speed equal and opposite to the speed of A i.e $(+2.8 \times 10^8)$, thus bringing the A to rest with its frame and the electron in S' moving with a speed $u'_x = +2.8 \times 10^8$. Now we can use relativistic addition ~~trans~~formation of velocity:

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

Here $u'_x = 2.8 \times 10^8$ $\quad v = v_B = 2.5 \times 10^8$

Relative speed of B with respect to the electron A is given by

$$u_{BA} = \frac{(2.8 \times 10^8) + (2.5 \times 10^8)}{1 + \frac{(2.8 \times 10^8)(2.5 \times 10^8)}{c^2}}$$

$$= \frac{5.3 \times 10^8}{1 + 0.778} = 2.98 \times 10^8 \text{ m/s.}$$

Q.12 Although the speed of light is enormous, particles such as electrons, protons, and neutrons encountered in atomic and nuclear physics frequently have speeds comparable that of light. If a particle is moving at speed of exactly0.6c, by what percent do its momentum and kinetic energy differ from the Newtonian values? Ans: (1.25, 1.39)

Solution:

Relativistic momentum $\rightarrow p = m u_x = \dfrac{m_0 u_x}{\sqrt{1-\frac{v^2}{c^2}}}$

Newtonian value of $p = m_0 u_x$

$$\therefore \frac{\text{Rel. momentum}}{\text{Newtonian momentum}} = \frac{(m u_x)}{(m_0 u_x)} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$v = 0.6c$

$$\therefore \frac{p_R}{p_N} = \frac{1}{\sqrt{1-(0.6)^2}} = 1.25$$

- Relativistic kinetic energy $K_R = m_0 c^2 \left\{ \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right\}$

$$= \frac{2}{2} \cdot \frac{v^2 c^2}{v^2} m_0 \left\{ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right\}$$

$$= \frac{2 v^2}{2 \beta^2} m_0 \left\{ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right\} \qquad \left[\beta = \frac{v}{c}\right]$$

- Newtonian kinetic energy $= \frac{1}{2} m_0 v^2$

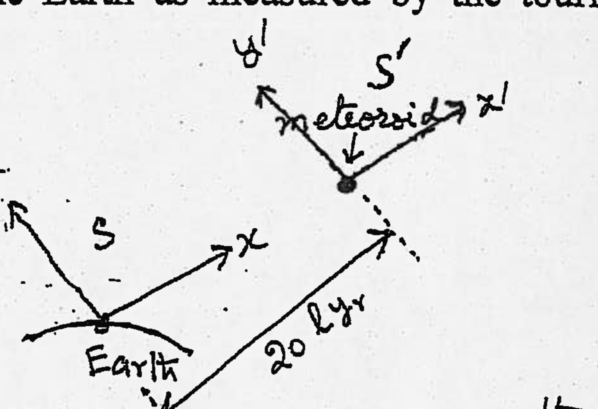
$$\therefore \frac{K_R}{K_N} = \frac{2}{\beta^2} \left\{ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right\}$$

$v = 0.6c$

$$\therefore \frac{K_R}{K_N} = \frac{2}{(0.6)^2} \left\{ \frac{1}{\sqrt{1-(0.6)^2}} - 1 \right\}$$

$$= \frac{2}{0.36} \left\{ 1.25 - 1 \right\} = \underline{1.39}$$

13 An astronomer on Earth observes a meteoroid in the southern sky approaching the earth at a speed of 0.8c. At the time of its discovery the meteoroid is 20.0 light years from the Earth. Calculate (a) the time interval required for the meteoroid to reach the Earth as measured by the Earth-bound observer, (b) the time interval as measured by a tourist on the meteoroid, and (c) the distance to the Earth as measured by the tourist. Ans: 25.0 yr., 15.0 yr, 12.0 ly



Solution:

(a) $u_x = 0.8c$

distance $d = 20\,lyr$

$\therefore \Delta t = \frac{20\,lyr}{0.8c} = 25\,yr$

(b) For a tourist on meteorite the observer and the earth are approaching him/her at a speed 0.8c. So for the tourist 25 yr is improper time.

$$\therefore \quad \Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{or} \quad 25 = \frac{\Delta t' = \Delta t_p}{\left[1-\frac{v^2}{c^2}\right]^{1/2}} \quad \text{or} \quad \Delta t' = 25\sqrt{1-(0.8)^2} = 15\,yr.$$

Alternative approach

The tourist is approaching the earth with a speed 0.8C and the distance measured on earth will contract for the tourist.

$$\therefore \quad d' = d_0\left(1-v^2/c^2\right)^{1/2}$$

$$= 20\left\{1-(0.8)^2\right\}^{1/2} = \boxed{12\,lyr} \quad (A)$$

$\therefore$ Time measured by the tourist $= \frac{12\,lyr}{0.8C} = 15\,yr$

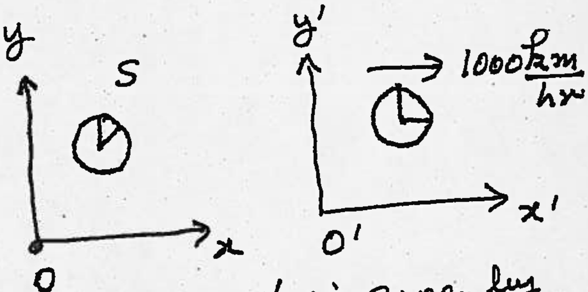
(C) The distance for the tourist will appear to contract as given in eq (A) in 13(b) i.e $d' = 20\left(1-\frac{v^2}{c^2}\right)^{1/2} = 12\,l.yr.$

14 An atomic clock moves at 1000 km/hr for 1.0 hr as measured by an identical clock on the Earth. How many nanoseconds slow will the moving clock be compared with the Earth clock, at the end of 1.00 hr. interval? Ans: (1.54 ns)

Solution:

The time interval measured by the clock on the earth is the proper time. In the problem

$\therefore \Delta t_p = 1\,hr.$



$\therefore$ Time interval on the moving clock $\Delta t'$ is given by

$$\Delta t' = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$v = 1000\ km/hr = \frac{1000\times 10^3}{3600} = \frac{10^4}{36}$$

$$\Delta t' = \left\{1 - \frac{v^2}{c^2}\right\}^{-1/2}(3600)$$

$$\Delta t' = \left\{1 + \frac{1}{2}\frac{v^2}{c^2} \cdots\right\}(3600)$$

$$\beta = \frac{v}{c} = \left(\frac{10^3\times 10^3}{36\times 10^2}\right)\frac{1}{(3\times 10^8)} = \frac{10^{-4}}{108}$$

$$\therefore \Delta t' - \Delta t_p \approx \frac{1}{2}\frac{v^2}{c^2}(3600)$$

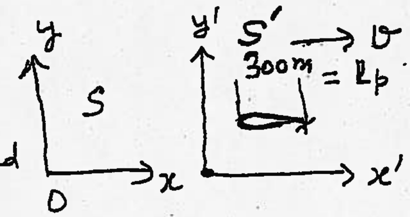
$$= \frac{1}{2}\left(10^{-4}/108\right)^2 (3.6\times 10^3)$$

$$= 1.54\times 10^{-9} = 1.54\ n.s$$

15 A spacecraft with a proper length of 300 m takes 0.750 μs to pass an Earth observer. Determine the speed of the spacecraft as measured by the Earth observer. Ans: (0.80 c)

Solution: Let S' be on the Spacecraft and its speed $v = \beta c$ ($\beta = v/c$)

The observer in S will see the Contracted length, $l = l_p(1-\beta^2)^{1/2} = 300(1-\beta^2)^{1/2}$.



$$\therefore \frac{300(1-\beta^2)^{1/2}}{\beta c} = 0.75\times 10^{-6} \text{ or } 300\left(\frac{1}{\beta^2}-1\right)^{1/2} = 0.75\times 10^{-6} c$$

$$\therefore \left(\frac{1}{\beta^2}-1\right) = \left[\frac{0.75\times 10^{-6} c}{300}\right]^2 = 0.563 \text{ or } \beta^2 = \frac{1}{1+0.563}$$

$$v = \beta c = 0.80 c$$

Q16 A friend passes by you in a spacecraft traveling at a high speed. He tells you that his craft is 20.0 m long and that the identically constructed craft you are sitting in is 19.0 m long. According to your observations, (a) how long is your space craft, (b) how long is your friend's craft, and (c) what is the speed of your friend's craft?

Ans:(20 m, 19 m, 0.312c)

Solution: (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is the proper length of 20m.

(b) Length L' of your friend's spacecraft, which is moving with respect to you, is measured as Contracted length which is 19m.

(c) $$L' = L\sqrt{1-\frac{v^2}{c^2}}$$

Here $L' = 19m$, $L = 20m$ we find v?

$$\therefore 19 = 20\sqrt{1-\frac{v^2}{c^2}} \quad \text{or} \quad \sqrt{1-\frac{v^2}{c^2}} = \frac{19.0}{20.0} = 0.95$$

$$\therefore v = c\sqrt{1-(0.95)^2} = 0.312C$$

Q.17 A physicist drives through a stop light. When he is pulled over, he tells the police officer that the Doppler shift made the red light of wave length 650 nm appear green to him, with a wavelength of 520nm. The police officer writes out a trffic citation for speeding. How fast was the physicist traveling, according to his own testimony?

Ans: (6.59×10^7 m/s)

The Relativistic Doppler Effect

- An important Consequence of time dilation is the shift in frequency found for light emitted by atoms in motion as opposed to light emitted by atoms at rest. In Case of sound, the motion of the Source with respect to the medium of propagation Can be distinguished from the motion of the observer with respect to the medium.

- Light waves must be analyzed differently, because they require no medium of propagation, and no method exists for distinguishing the motion of a light source from the motion of observer.

If light source and an observer approach each other with relative speed v, the frequency f_{obs} measured by the observer is given by:

$$\boxed{f_{obs} = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}\, f_{Source}} \qquad (1)$$

where, f_{source} is the frequency of the source measured in the rest frame. Note that Eq.(1) depends only on the relative speed v of the source and observer, and holds for relative speeds as great as c. The equation predicts that $f_{obs} > f_{source}$ when the source and observer approach each other.

Solution: f_{obs} by the physicist is given by from (1).

(approaching) $$f_{obs} = \frac{c}{\lambda_{obs}} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\, f_{Source} = \sqrt{\frac{1+v/c}{1-v/c}} \cdot \frac{c}{\lambda_{Source}}$$

$$\therefore \quad \frac{\lambda_{Source}}{\lambda_{obs}} = \sqrt{\frac{1+v/c}{1-v/c}} = \frac{650}{520} = 1.25$$

$$\therefore \quad 1+\frac{v}{c} = (1.25)^2\left(1-\frac{v}{c}\right)$$

$$\therefore \quad \frac{v}{c}\left\{1+(1.25)^2\right\} = (1.25)^2 - 1$$

$$\therefore \quad \frac{v}{c} = \frac{0.562}{2.562} = 0.220$$

$$\therefore \quad v = 0.22c = 6.59 \times 10^7 \text{ m/s}$$

Q18 In a typical color television picture tube, the electrons are accelerated through a potential difference of 25,000 V, (a) What speed do the electrons have when they strike the screen?, (b) Wat is their kinetic energy in joules? Ans: (0.302c, 4.00×10^{-15} J)

Solution: (b) Accelerating potential difference $\rightarrow \Delta V = 25\times10^{3}$ V.

K.E of electrons $\rightarrow$ $T = e\Delta V = (1.6\times10^{-19})(25\times10^{3}) = 4.0\times10^{-15}$ J

(a) Also, $T = (mc^2 - m_0c^2) = m_0c^2\left\{\frac{1}{\sqrt{1-v^2/c^2}} - 1\right\} = 4.0\times10^{-15}$ J

$$= 9.11\times(3.0\times10^{8})^2\left\{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right\} = 4.0\times10^{-15} \text{ J}$$

or $$(81.99\times10^{-15}\text{ J})\left\{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right\} = 4.0\times10^{-15} \text{ J}$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1 + \frac{4.0\times10^{-15}}{81.99\times10^{-15}}$$

or $\frac{v^2}{c^2} = 1 - \left(\frac{81.99}{85.99}\right)^2$ or $v = c\sqrt{1-\left(\frac{81.99}{85.99}\right)^2} = 0.301c$

Q19 When 1.00 g of hydrogen combines with 8.00 g of oxygen, 9.00 g of water is formed. During this chemical reaction, 2.86×10^5 J of energy is released. How much mass do the constituents of this reaction lose? Is the loss of mass likely to be detectable?

Ans: (3.18×10^{-12} kg)

Solution:

Energy released $\rightarrow$ $E = 2.86\times10^{5}$ J

From $E = mc^2$ relation, the loss of mass is given by, $m = \frac{E}{c^2} = \frac{2.86\times10^{5}}{(3\times10^{8})^2} = 3.18\times10^{-12}$ kg

The loss of mass is so small that it will not be detectable

Q20 Imagine that you are at rest in a frame [O'X'Y'Z'] which moves horizontally past an inertial frame [O X Y Z] at a speed of 0.6c. A boy in the latter frame drops a ball which, according to your clock, falls for 0.5 sec. How long would the ball fall as timed by an observer at rest in the [O X Y Z] frame? Ans: (0.4 sec.)

Solution:

Since the clock is in the moving frame, the time of an event in the inertial frame will be dilated in the moving frame-clock. Therefore, if t_p is the proper time in the rest frame, time t' in the moving frame will be

$$t' = \frac{t_p}{\sqrt{1-\frac{v^2}{c^2}}}$$

For $v = 0.6c$, $\quad t_p = t'\sqrt{1-\frac{v^2}{c^2}} = 0.5\sqrt{1-(0.6)^2}$

$$= 0.4 \text{ Sec.}$$

Q 21 An unstable particle at rest breaks into two fragments of unequal mass. The mass of the first fragment is 2.50×10^{-28} kg and that of the other is 1.67×10^{-27} kg. If the lighter fragment has a speed of 0.893c after the breakup, what is the speed of the heavier fragment? Ans:(0.285c)

Solution: Relativistic momentum of the system of fragments must be conserved. If m_1, v_1 is the mass and velocity of lighter fragment and m_2, v_2 the mass and velocity of heavier fragment,

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad \frac{m_{10} v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{m_{20} v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} \quad (1)$$

Rest mass

↓

$m_{10} = 2.5 \times 10^{-28}$, $v_1 = 0.893c$, $m_{20} = 1.67 \times 10^{-27}$, $v_2 = ?$

From(1) $$\frac{v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{m_{10}}{m_{20}} \cdot \frac{v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{(2.5 \times 10^{-28})}{(1.67 \times 10^{-27})} \cdot \frac{(0.893c)}{\sqrt{1-(0.893)^2}} = 0.297c$$

$$\therefore v_2^2 = (0.297c)^2 (1 - v_2^2/c^2) \quad \text{or} \quad v_2 = \frac{0.297c}{\sqrt{1+(0.297)^2}} = 0.285c$$

Assignment 6
Physics 206
(Do any 12 problems)

1. Find the angle of incidence for which the light reflected from water of refractive index 1.333 is plane polarized.
   Ans: 53°

2. Light reflected from a flint-glass surface is plane-polarized when the angle of incidence is 59°. What is the index of refraction of the glass?
   Ans: (1.66)

3. Two Nocol prisms have their planes of vibration parallel. One of the prisms is then turned so that its plane of vibration makes an angle of 35° with that of the other. What fraction of the amplitude incident on the second Nicol prism is transmitted? What percentage of light incident on the second Nicol prism is transmitted?
   Ans: (0.819, 67 %)

4 Two polaroids are crossed. If the analyzer is now rotated through an angle of 50°, what percentage of the plane polarized light from the polarizer is transmitted, assuming the Polaroid is perfect transmitter of one polarization and a perfect absorber of the other? What percentage of the ordinary light incident on the polarizer is transmitted through the analyzer?
   Ans: (58.7%, 29.3 %)

5 A river 8 km wide has a current with a speed of 3 km/hr. How much longer will it take a boat with a speed of 5 km/hr to go upstream 8 km and return than go directly across the river and return to the same point?
   Ans: (1.0 hr)

6 Find the mass of an electron traveling at 0.6 times the speed of light. How many times as great as the rest mass is this value?
   Ans: (1.14×10^{-30} kg, 1.25)

7 Find the work, which must be done on an electron to increase its speed from 0.5 c to 0.9 c, where c is the speed of light.
   Ans: (9.33×10^{-14} J)

8 How many electron volts of energy must an electron gain to bring its mass to (a) 1.05 m_0, (b) 2 m_0? In each case what is the speed of electron? Ans: {(a) 25500, 9.2×10^7 m/s, (b) 511000, 2.6×10^8}

9 A proton of rest mass 938 MeV (1.67×10^{-27} kg) is given a kinetic energy of 9.38×10^9eV in a proton synchrotron. Find the mass of proton at this speed. What must be the radius of the orbit if the vertical magnetic field is 1.5 webers/m^2?
   Ans: (1.84×10^{-26} kg, 23m)

10. An arrow passes an observer with a speed 0.6 times that of light If the rest length of the arrow is 0.75 m, find its apparent length as it passes the observer.
    Ans: (0.6 m)

11. An electron moving to the right with a speed of 2.5×10^8 m/s passes an electron moving to the left with a speed of 2.8×10^8 m/s. Find the speed of one electron relative to the other as predicted by (a) the Newtonian-Galilean transformation and (b) the relativistic transformation.
    {5.3×10^8 m/s, 2.98×10^8 m/s}