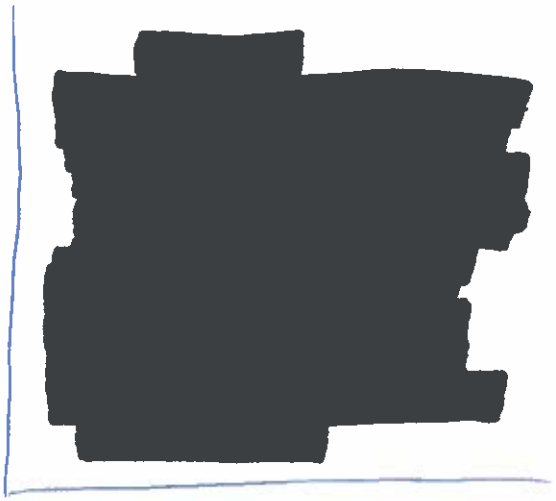
Assignment 1

In exercises 1, 2 and 3, p, q and r denote propositions.

1. Construct a truth table for the following compound propositions. (Make sure that the rows follow the reverse-alphabetical order as we agreed upon in class: The first row starts with "TTT", the second row starts with "TTF", etc.)

a) (4 points) $(p \vee q) \rightarrow (\neg q \wedge r)$

p	q	r	$p \vee q$	$\neg q \wedge r$	$(p \vee q) \rightarrow (\neg q \wedge r)$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T
F	F	F	F	F	T



b) (4 points) $(q \leftrightarrow p) \oplus (r \rightarrow p)$

p	q	r	$q \leftrightarrow p$	$r \rightarrow p$	$(q \leftrightarrow p) \oplus (r \rightarrow p)$
T	T	T	T	T	F
T	T	F	T	T	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	T	F	T
F	F	F	T	T	F

2. (3 points) Use a truth table to determine if the given compound proposition is a tautology, a contradiction or a contingency. Give a very short explanation for your conclusion by referring to the relevant feature of the corresponding truth table.

$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \wedge (p \rightarrow q)$	$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	T	T	F
F	F	T	T	T	T

This is a contingency because this column has instances for both T & F.

Notes ① Ordering of the truth table is important

[redacted]

3. (4 points) Use truth tables to show that $p \leftrightarrow q$ is logically equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$.

p	q	$p \leftrightarrow q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	T	T	F	T
T	F	F	F	F	F
F	T	F	F	F	F
F	F	T	F	T	T

These two columns are identical, hence $p \leftrightarrow q$ is equivalent to $(p \wedge q) \vee (\neg p \wedge \neg q)$.

[redacted]

4. (1 point each) Let W, O, G, Q, J, I denote the following propositional variables:

W : "The mouse survives the winter."
O : "The owl hunts over the field."
G : "The mouse gathers food in the field."
Q : "Pat is qualified for the job."
J : "Pat got the job."
I : "The interviewer asked tough questions."

Translate each of the following sentences into compound propositions using the above propositional variables, logical connectives, and parentheses when appropriate. Make sure to take into account the conventions of precedence of logical connectives when writing your solutions.

a) Either the mouse gathers food in the field or the mouse does not survive the winter.

$G \vee \neg W$

b) A sufficient condition for the mouse to survive the winter is that the owl does not hunt over the field.

$\neg O \rightarrow W$

c) Pat is not qualified for the job but the interviewer did not ask tough questions.

$\neg Q \wedge \neg I$

d) Pat got the job if the owl hunts over the field or the mouse does not gather food in the field.

OR before IF

$(O \vee \neg G) \rightarrow J$

5. There are two types of people on the island of Knights and Knaves. Knights always speak the truth, knaves always lie. There you meet two people: A and B. (People on the island of Knights and Knaves are known for their conveniently short names!)
A says: If B is a knight, then I'm a knight.
B says: I'm a knight and A is a knave.

Follow the steps below to find out the possible scenarios for the identities of A and B.

Let p be the proposition "A is a knight", and q be the proposition "B is a knight".

Observation: (Convince yourself!) The truth value of any statement made by A must coincide with the truth value of p. Similarly the truth value of any statement made by B must coincide with the truth value of q.

a) (2 points) Express the statements made by A and B as compound propositions using p and q together with logical connectives. Denote the first of these compound proposition as r, the second as s.

r: $q \to p$

s: $q \wedge \neg p$

b) (2 points) Assign truth values T and F to p and q to make a truth table that has four rows, and a column for each of p, q, r, s. $(q \to p)$ $(q \wedge \neg p)$

p	q	r	s
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	F

c) (1 point) (Now we make use of the observation.) For which row(s) in the truth table you found in part b) is it true that the truth value of p coincides with r, and at the same time the truth value of q coincides with s?

p	q	r	s
(T)	T	(T)	F
(T)	[F]	(T)	[F]
(F)	[T]	(F)	[T]
F	[F]	T	[F]

= Rows 2 & 3

[redacted]

d) (1 point) Use the rows you found in part c) to determine the possible cases for the identities of A and B. (Hint: There are two possible scenarios.)

Case 1: (Row 2) A is knight and B is a knave

Case 2: (Row 3) A is a knave and B is a knight.

[redacted]