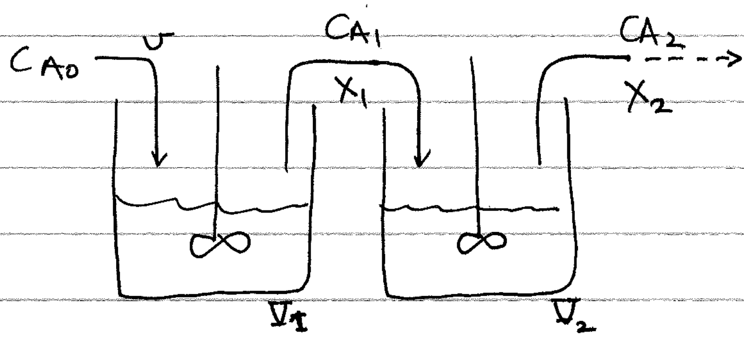
CSTRs in series



system is @ steady-state & $-r_A = kC_A^n$

$$X_{A_1} = \frac{C_{A0} - C_{A1}}{C_{A0}}$$

$$X_{A_2} = \frac{C_{A0} - C_{A2}}{C_{A0}}$$

$$\tau_1 = \frac{V_1}{v} \quad , \quad \tau_2 = \frac{V_2}{v} \quad \text{\& so on...}$$

Let us write the mole balance for reactor 1:

$$F_{A0} - F_{A_1} = (-r_A)\,V$$

$$C_{A0} - C_{A_1} = (-r_A)\,\tau$$

$$\therefore \; C_{A0} - C_{A_1} = k\tau_1 C_{A_1}^n$$

For reactor 2,

$$C_{A_1} - C_{A_2} = k\tau_2 C_{A_2}^{\,n}$$

Rewriting the system in terms of conversion:

$$\boxed{C_{A_0} X_{A_1} = k\tau_1 C_{A_1}^{\,n}}$$

$$\& \quad C_{A_0}(1 - X_{A_1}) - C_{A_0}(1 - X_{A_2}) = k\tau\, C_{A_2}^{\,n}$$

$$\therefore \quad \boxed{C_{A_0}(X_{A_2} - X_{A_1}) = k\tau_2 C_{A_2}^{\,n}}$$

$$\therefore \quad X_{A_1} = \frac{k\tau_1}{C_{A_0}} C_{A_0}^{\,n} (1 - X_{A_1})^n$$

$$\& \quad X_{A_2} - X_{A_1} = \frac{k\tau_2}{C_{A_0}} C_{A_0}^{\,n} (1 - X_{A_2})^n$$

$$\therefore \quad X_{A_1} = k\tau_1 C_{A_0}^{\,n-1} (1 - X_{A_1})^n$$

$$\& \quad X_{A_2} - X_{A_1} = k\tau_2 C_{A_0}^{\,n-1} (1 - X_{A_2})^n$$

Remember: $\boxed{k\tau\, C_{A_0}^{\,n-1} = Da}$

$$\therefore \quad X_{A_1} = Da_1(1-X_{A_1})^n$$ ... Da will be different

$$X_{A_2} - X_{A_1} = Da_2(1-X_{A_2})^n$$

Similarly,

$$\boxed{X_{A_i} - X_{A_{i-1}} = Da_i(1-X_{A_i})^n}$$ i = reactor #

Depending on the flow patterns, you can find the intermediate conversions for a given isothermal, SS CSTR.

Imagine if you have to do this for a series of reactors. It is a lot of work, isn't it?

Let's rewrite the equations:

$$F_{A0} - F_A = (-r_A)V$$

$$\therefore \quad C_{A0}X_{A_1} = k\tau_1 C_{A_1}^n$$

$$\therefore \quad C_{A0} - C_{A_1} = k\tau_1 C_{A_1}^n$$

$$\therefore \quad C_{A0} = C_{A_1} + k\tau_1 C_{A_1}^n$$

$$\therefore \quad C_{A_1} = \frac{C_{A0}}{1 + k\tau_1 C_{A_1}^{n-1}}$$

$$\text{OR} \quad \left(\frac{C_{A0}}{C_{A1}}\right) = 1 + k\tau_1 C_{A_1}^{n-1}$$

BTW, What happens if volumes change?

Recollect: $\upsilon = \upsilon_0 \left(\frac{P_0 T}{P T_0}\right)(1 + \varepsilon X_A)$ .... for a PFR

In a CSTR, $V = V_0 \left(\frac{P_0 T}{P T_0}\right)(1 + \varepsilon X_A)$

For const. P & T, $V = V_0 (1 + \varepsilon X_A)$

$\therefore \quad N_A = N_{A0}(1 - X_A)$ & $C_A = \frac{N_A}{V}$

$$\therefore \quad C_A = \frac{N_{A0}(1 - X_A)}{V_0 (1 + \varepsilon X_A)}$$

$$\boxed{\therefore \quad C_A = C_{A0}\left(\frac{1 - X_A}{1 + \varepsilon X_A}\right)}$$

$\Rightarrow$ subst. this in design equation

$\Downarrow$

analyse!

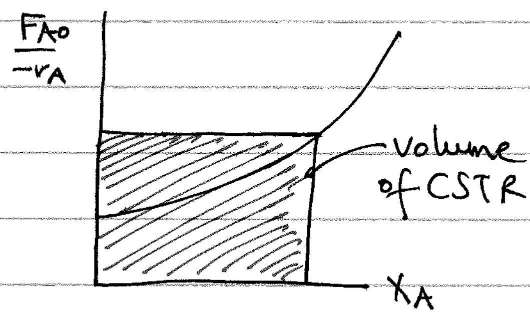
For simplicity, let's assume volumes are the same.

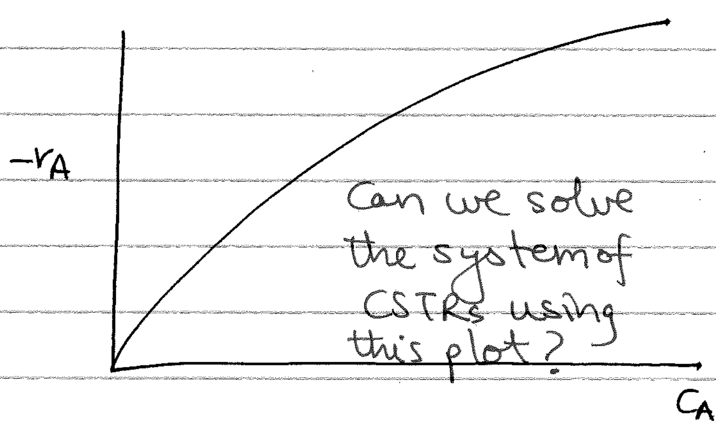
$$C_{A_1} = \frac{C_{A0}}{1 + k\tau_1 C_A^{n-1}}$$

For $n = 1$ (1st order),

$$C_{A1} = \frac{C_{A0}}{1 + k\tau_1}$$

Recollect:





This is how the plot of $-r_A$ vs C_A looks like

$$C_{A1} = \frac{C_{A0}}{1 + k\tau_1 C_A^{n-1}} \rightarrow \text{looks big \& complex}$$

$\Downarrow$

let's simplify this

$$C_{A0} - C_{A1} = k C_A^n \frac{V}{v}$$

$$\therefore \; C_{A0} - C_{A1} = k\tau C_A^n$$

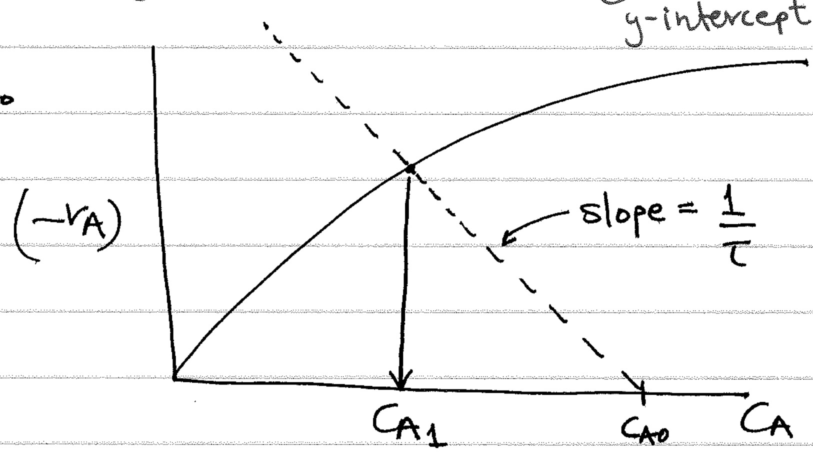
$$\therefore \; C_{A0} - C_{A1} = \tau(-r_A)$$

$$\therefore \; (-r_A) = \frac{C_{A0}}{\tau} - \frac{C_{A1}}{\tau}$$

$$\therefore \; \underbrace{(-r_A)}_{\text{y-axis}} = \overbrace{\frac{1}{\tau}}^{\text{slope}} \underbrace{C_{A0}}_{\text{x-axis}} - \underbrace{\left(\frac{1}{\tau} C_{A1}\right)}_{\text{y-intercept}}$$

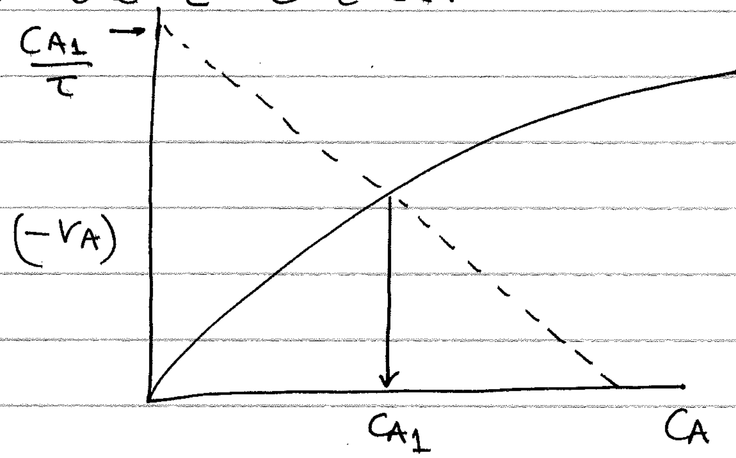
shows decrease

$\therefore$

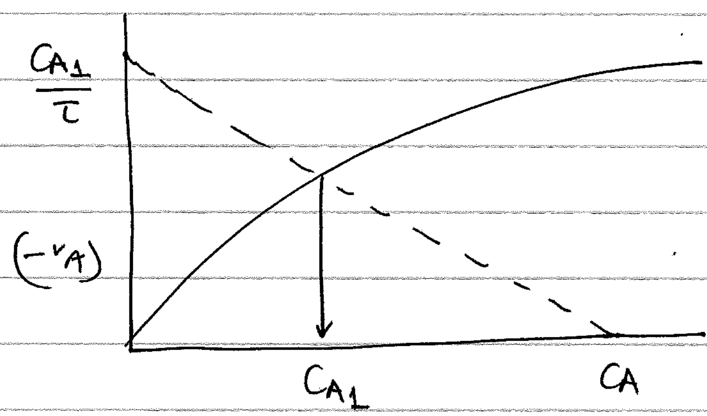


Now, let's add a series of reactors

For the $\varepsilon = 0$ case:



For the $\varepsilon \neq 0$ case:



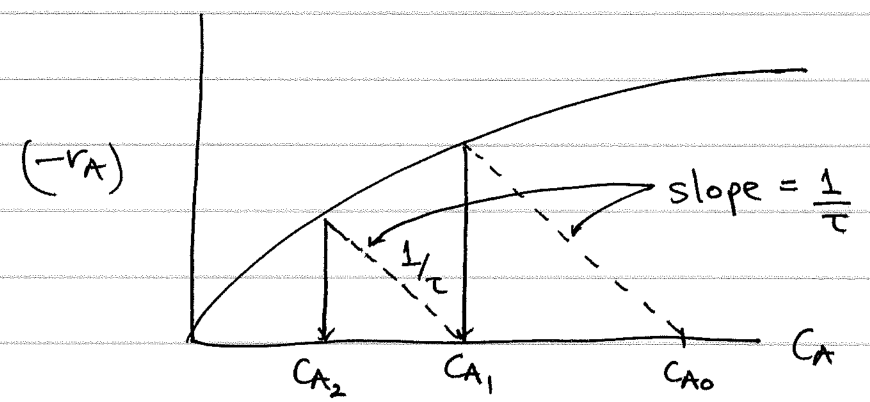
remains the same!

Reactors in series:

$$C_{A0} - C_{A1} = \tau_1 (-r_A)_1$$

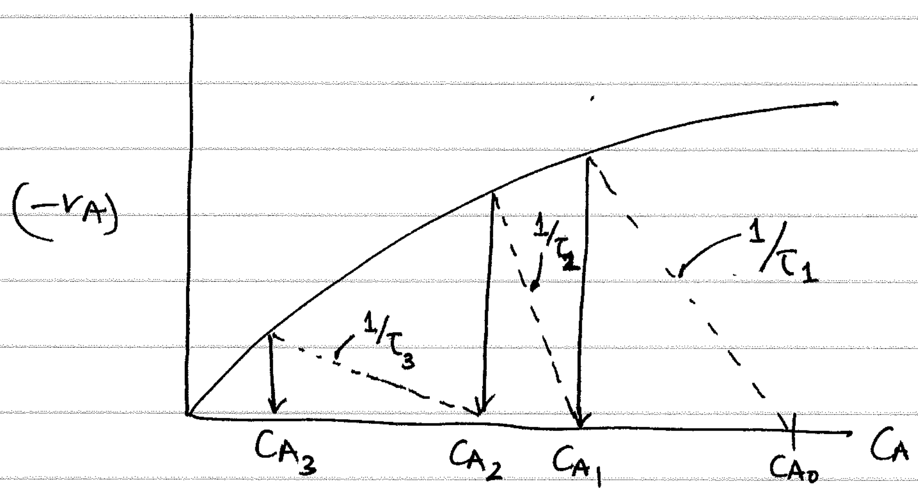
$$C_{A1} - C_{A2} = \tau_2 (-r_A)_2 \quad \text{\& so on.}$$

you need not have the same τ



τ_s are same

<u>OR</u> if τ_s are different:



This method is incredibly useful to solve CSTR-in-series problems with variable volumes.

Process: Plot C_A vs. kC_A^n → 1

Use operating lines → 2

Deduce sizes → 3