NON-IDEAL REACTORS

* Temperature changes are a part & parcel of life in the chemical industry.

- isothermal
  - ① steady state eg. CSTR
  - ② unsteady-state eg. batch reactors
- non-isothermal
  - ③ steady-state
  - ④ unsteady-state

We have covered ① & ② thus far.

Let's first look @ ③.

Temperatures ≡ energy flows

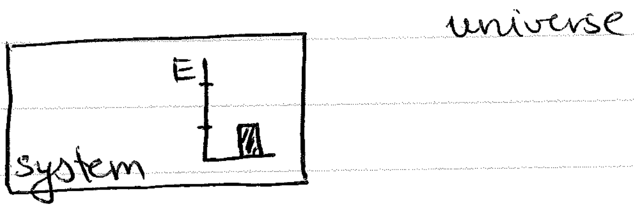
We need an <u>energy balance</u>!

1st law of thermodynamics:

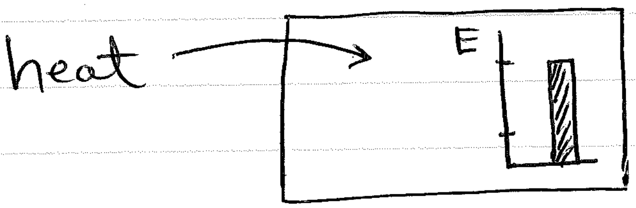
Consider a system that is closed.

- system → open (material + energy flows)
- system → closed (energy flows)
- system → insulated (completely isolated)

The system has an energy content.

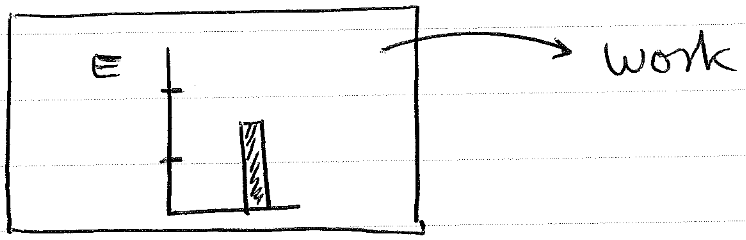


Now, we add heat to this system.



The addition of heat energises the system.

The energised system does <u>work</u>.



By doing work, the energy content of the system drops.

$$\therefore \left\{\begin{matrix}\text{heat added}\\ \text{to system}\end{matrix}\right\} = \left\{\begin{matrix}\text{change in}\\ \text{energy}\\ \text{content}\end{matrix}\right\} + \left\{\begin{matrix}\text{work}\\ \text{done}\\ \text{by}\\ \text{system}\end{matrix}\right\}$$

In this eqn., each quantity is positive.

If you remember, there are 2 types of thermodynamic quantities

→ path functions ≡ work, heat

→ state function ≡ energy, entropy

* path functions are the reason for phenomena like reversible / irreversible work, hysteresis etc.

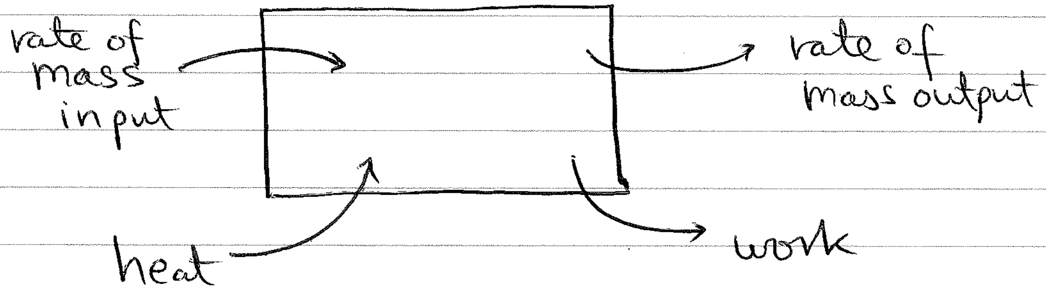
$$\therefore \underbrace{\left\{\begin{matrix}\text{heat added}\\ \text{to system}\end{matrix}\right\}}_{\text{path function}} = \underbrace{\left\{\begin{matrix}\text{change in its}\\ \text{energy content}\end{matrix}\right\}}_{\text{state function}} + \underbrace{\left\{\begin{matrix}\text{work done}\\ \text{by system}\end{matrix}\right\}}_{\text{path function}}$$

$$\therefore \quad \delta Q = dE + \delta W$$

$$\boxed{\begin{array}{l}\text{path fn.} \equiv \delta\,,\ \oint\\ \text{state fn.} \equiv d\,,\ \int\end{array}} \Longrightarrow \text{math nomenclature}$$

$$\boxed{\therefore \quad dE = \delta Q - \delta W} \longrightarrow ①$$

For an <u>open</u> system involving material flows,



* mass flows / mole flows can be associated with energy

This time, we incorporate <u>rates</u> in our equation.

$$\begin{Bmatrix}\text{rate of}\\ \text{change of}\\ \text{energy}\end{Bmatrix} = \begin{Bmatrix}\text{rate of}\\ \text{heat}\\ \text{flow in}\end{Bmatrix} - \begin{Bmatrix}\text{rate of}\\ \text{work}\\ \text{done}\end{Bmatrix} + \begin{Bmatrix}\text{energy}\\ \text{added}\\ \text{due to}\\ \text{mass flow}\\ \text{in}\end{Bmatrix}$$

$$- \begin{Bmatrix}\text{energy}\\ \text{removed}\\ \text{by mass flow}\\ \text{out}\end{Bmatrix}$$

$$\therefore \frac{dE}{dt} = \delta\dot{Q} - \delta\dot{W} + F_{in}E_{in} - F_{out}E_{out}$$

More generally, $\delta\dot{Q}$ is written as $\dot{Q}$

$$\therefore \underbrace{\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}}_{\left(\frac{\text{Joules}}{\text{time}}\right)} + \underbrace{F_{in}E_{in}}_{\left(\frac{\text{moles}}{s}\right)\left(\frac{J}{\text{mole}}\right)} - F_{out}E_{out}$$

The F_{in} & F_{out} terms can be separated out into all the species.

i.e. $$F_{in}E_{in} = \sum_{i=1}^{n} E_i F_i \Big|_{in}$$

$$F_{out}E_{out} = \sum_{i=1}^{n} E_i F_i \Big|_{out}$$

$$\therefore \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{n} E_i F_i \Big|_{in} - \sum_{i=1}^{n} E_i F_i \Big|_{out}$$

Remember: mole balance = moles/s

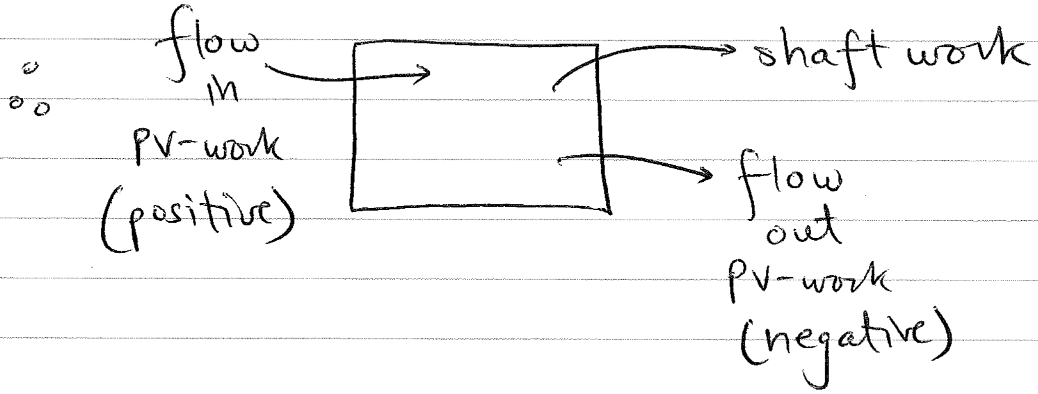
energy balance = Joules/s (or Watts)

Let's take a closer look at each of the terms.

work → PV-work (due to flow)

→ shaft work (impeller etc.)

Remember the convention ⇒ work done by system <u>on</u> surroundings was <u>negative</u> in the energy balance.



∴
$$\underbrace{-\dot{W}}_{\text{term as it appears in energy balance}} = \sum_{i=1}^{n} \underset{\text{mol/s}}{F_i}\,\underset{\text{Pascal}}{P}\,\underset{m^3/\text{mol}}{V_i}\Big|_{in} - \sum_{i=1}^{n} F_i P V_i\Big|_{out} - \dot{W}_S$$

∴
$$\frac{dE_{sys}}{dt} = \dot{Q} + \sum_{i=1}^{n} F_i P V_i\Big|_{in} - \sum_{i=1}^{n} F_i P V_i\Big|_{out} - \dot{W}_S + \sum_{i=1}^{n} F_i E_i\Big|_{in} - \sum_{i=1}^{n} F_i E_i\Big|_{out}$$

$$\therefore \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s + \sum_{i=1}^{n} F_i(E_i + PV_i)\Big|_{in} - \sum_{i=1}^{n} F_i(E_i + PV_i)\Big|_{out}$$

Let's look at E_i

energy associated with an object or species → internal energy
→ kinetic energy
→ potential energy
→ other

* Generally, kinetic & potential energy terms are negligible (unless you are pumping species to great heights!)

$$\therefore \underset{\left(\frac{\text{Joules}}{\text{mole}}\right)}{E_i} = U_i + \underbrace{\frac{u_i^2}{2} + gz_i + \text{other}}_{\text{negligible}}$$

$\therefore\ E_i = U_i$ ----- for most cases

$$\therefore\ E_i + PV_i = \underbrace{U_i + PV_i}_{\text{definition of enthalpy}}$$

$$\therefore\ \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s + \sum_{i=1}^{n} F_i H_i \Big|_{in} - \sum_{i=1}^{n} F_i H_i \Big|_{out}$$

Using the convention we have used thus far,

$$\boxed{\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s + \sum_{i=1}^{n} F_{io} H_{io} - \sum_{i=1}^{n} F_i H_i}$$

For steady-state, $\frac{dE_{sys}}{dt} = 0$
(energetically)

For unsteady-state, $\frac{dE_{sys}}{dt} \neq 0$

Consider $aA + bB \longrightarrow cC + dD$

$$\therefore \quad A + \frac{b}{a}B \longrightarrow \frac{c}{a}C + \frac{d}{a}D$$

General energy balance equation:

$$\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_S + \sum_{i=1}^{n} F_{i0}H_{i0} - \sum_{i=1}^{n} F_i H_i$$

Let's look @ the enthalpy terms:

$$\sum_{i=1}^{n} F_{i0}H_{i0} = F_{A0}H_{A0} + F_{B0}H_{B0} + F_{C0}H_{C0} + F_{D0}H_{D0}$$

$$\sum_{i=1}^{n} F_i H_i = F_A H_A + F_B H_B + F_C H_C + F_D H_D$$

Stoichiometry table

Species	Initial	Change	Final
A	F_{A0}	$-F_{A0}X_A$	$F_{A0}(1-X_A)$
B	$F_{B0} = \Theta_B F_{A0}$	$-\frac{b}{a}F_{A0}X_A$	$F_{A0}(\Theta_B - \frac{b}{a}X_A)$
C	$F_{C0} = \Theta_C F_{A0}$	$c/a\, F_{A0}X_A$	$F_{A0}(\Theta_C - c/a\, X_A)$
D	$F_{D0} = \Theta_D F_{A0}$	$d/a\, F_{A0}X_A$	$F_{A0}(\Theta_D - d/a\, X_A)$
I	$F_{I0} = \Theta_I F_{A0}$	0	$\Theta_I F_{A0}$

there could be inerts present

make these substs. in the energy balance

$$\therefore \sum_{i=1}^{n} F_{io} H_{io} = F_{A0} H_{A0} + \Theta_B F_{A0} H_{B0} + \Theta_C F_{A0} H_{C0} + \Theta_D F_{A0} H_{D0} + \Theta_I F_{A0} H_{I0}$$

$$\& \sum_{i=1}^{n} F_i H_i = F_{A0}(1-X_A) H_A + F_{A0}\left(\Theta_B - \frac{b}{a} X_A\right) H_B + F_{A0}\left(\Theta_C - \frac{c}{a} X_A\right) H_C + F_{A0}\Theta_I H_I + F_{A0}\left(\Theta_D - \frac{d}{a} X_A\right) H_D$$

$\therefore \sum_{i=1}^{n} F_{io} H_{io} - \sum_{i=1}^{n} F_i H_i \Rightarrow$ let's look @ individual terms for simplicity

species A : $F_{A0} H_{A0}$ = in

$F_{A0}(1-X_A) H_A$ = out

$\}$ subtraction gives:

$$= F_{A0} H_{A0} - \left[F_{A0} H_A - F_{A0} X_A H_A\right]$$

$$= F_{A0} H_{A0} - F_{A0} H_A + F_{A0} X_A H_A$$

$$= F_{A0}\left[H_{A0} - H_A\right] + F_{A0} X_A H_A \longrightarrow$$ ①

species B: difference $= \Theta_B F_{A0} H_{B0} - \left[F_{A0}\left(\Theta_B - \frac{b}{a} X_A\right) H_B\right]$

$$= \Theta_B F_{A0} H_{B0} - \left[\Theta_B F_{A0} H_B - \frac{b}{a} F_{A0} X_A H_B\right]$$

$$= \Theta_B F_{A0}\left(H_{B0} - H_B\right) + \frac{b}{a} F_{A0} X_A H_B \longrightarrow$$ ②

Similarly: species C $= \Theta_C F_{A0}\left(H_{C0} - H_C\right) - \frac{c}{a} F_{A0} X_A H_C \longrightarrow$ ③

species D $= \Theta_D F_{A0}\left(H_{D0} - H_D\right) - \frac{d}{a} F_{A0} X_A H_D \longrightarrow$ ④

$$\therefore \sum_{i=1}^{n} F_{io}H_{io} - \sum_{i=1}^{n} F_i H_i = F_{Ao}\left[H_{Ao} - H_A\right] + F_{Ao}X_A H_A$$

$$+ F_{Ao}\Theta_B\left[H_{Bo} - H_B\right] + \frac{b}{a}F_{Ao}X_A H_B$$

$$+ F_{Ao}\Theta_C\left(H_{Co} - H_C\right) + \left\{-\frac{c}{a}F_{Ao}X_A H_C\right\}$$

$$+ F_{Ao}\Theta_D\left(H_{Do} - H_D\right) + \left\{-\frac{d}{a}F_{Ao}X_A H_D\right\}$$

$$+ F_{Ao}\Theta_I\left(H_{Io} - H_I\right)$$

$$\therefore \sum_{i=1}^{n} F_{io}H_{io} - \sum_{i=1}^{n} F_i H_i = F_{Ao}\left[\left(H_{Ao} - H_A\right) + \Theta_B\left(H_{Bo} - H_B\right) + \Theta_C\left(H_{Co} - H_C\right) + \Theta_D\left(H_{Do} - H_D\right) + \Theta_I\left(H_{Io} - H_I\right)\right]$$

$$+ F_{Ao}X_A\left[H_A + \frac{b}{a}H_B - \frac{c}{a}H_C - \frac{d}{a}H_D\right]$$

Re-consider $A + \frac{b}{a}B \longrightarrow \frac{c}{a}C + \frac{d}{a}D$

What is ΔH_{rxn}?

ΔH_{rxn} = enthalpies of products − enthalpies of reactants

include stoichiometry!

$$\therefore \Delta H_{rxn} = \frac{c}{a}\Delta H_f^{C} + \frac{d}{a}\Delta H_f^{D} - \frac{b}{a}\Delta H_f^{B} - \Delta H_f^{A}$$

One can approximate this as:

$$\Delta H_{rxn} = \frac{c}{a} H_C + \frac{d}{a} H_D - \frac{b}{a} H_B - H_A$$

or, more specifically:

we'll look @ this in more detail later

$$\boxed{\Delta H_{rxn}(T) = \frac{c}{a} H_c(T) + \frac{d}{a} H_D(T) - \frac{b}{a} H_B(T) - H_A(T)}$$

$$\therefore \sum_{i=1}^{n} F_{io} H_{io} - \sum_{i=1}^{n} F_i H_i = F_{A0} \sum_{i=1}^{n} \Theta_i (H_{io} - H_i) - F_{A0} X_A \Delta H_{rxn}(T)$$

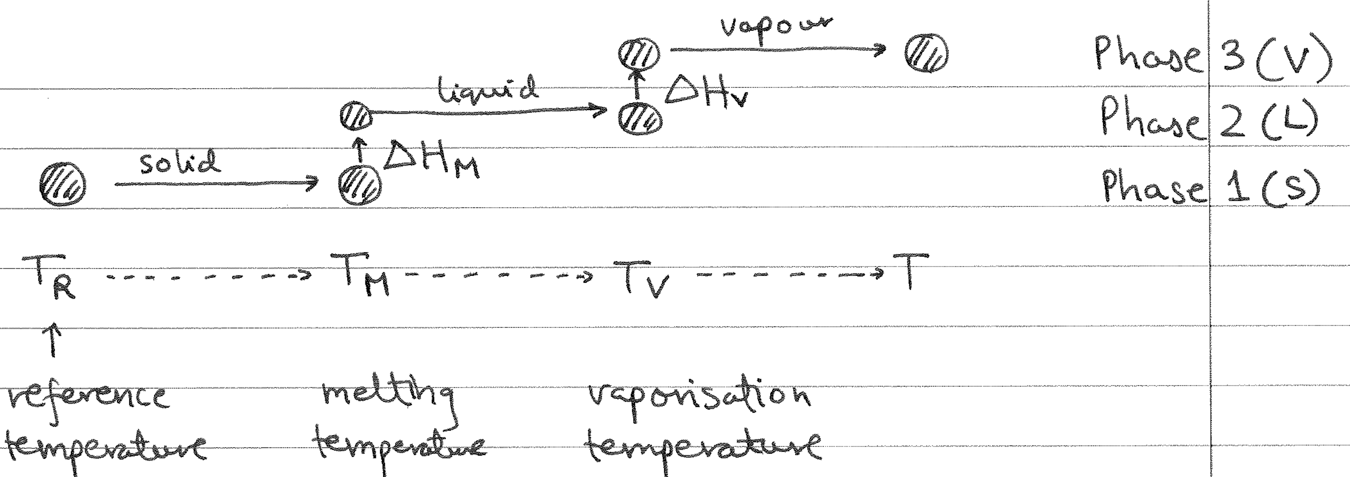
$$\boxed{\therefore \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s + F_{A0} \sum_{i=1}^{n} \Theta_i (H_{io} - H_i) - F_{A0} X_A \Delta H_{rxn}(T)}$$

↳ $\underline{H_{io} @ T_{io}}$, $\underline{H_i @ T}$

So far we have looked @ work (PV & shaft work) and the enthalpy of reaction.

However, enthalpies are <u>temperature-dependent</u>.

★ Let us dissect this dependence a little further.



$$\therefore \quad H_i(T) = H_i^\circ(T_R) + C_{p_i}^{s}(T_M - T_R) + \Delta H_{M_i} + C_{p_i}^{L}(T_V - T_M) + \Delta H_{V_i} + C_{p_i}^{V}(T - T_V)$$

for any species

reference enthalpy (standard enthalpy)

$C_{p_i}^{s}$ ≡ heat capacity of solid i

$C_{p_i}^{L}$ ≡ heat capacity of liquid i

$C_{p_i}^{V}$ ≡ heat capacity of vapour i

C_p can be a function of temperature too!

$\therefore$, a more general form of enthalpy is:

$$H_i(T) = H_i^\circ(T_R) + \int_{T_R}^{T_M} C_{p_i}^{s}\, dT + \Delta H_{M_i} + \int_{T_M}^{T_V} C_{p_i}^{L}\, dT + \Delta H_{V_i} + \int_{T_V}^{T} C_{p_i}^{V}\, dT$$

Phase changes are rare in the industry.

In this course, as well as an overwhelming number of real-life case, we do not consider phase changes.

$$\boxed{\therefore\ H_i(T) = H_i^\circ(T_R) + \int_{T_R}^{T} C_p \, dT}$$

the phase of the reaction

Generally, $C_p = \alpha + \beta T + \gamma T^2 + \cdots\cdots$

you can derive these expressions from a source like Perry's Handbook

Therefore,

$$H_{io} - H_i = \left\{ H_i^\circ(T_R) + \int_{T_R}^{T_{io}} C_{pi}\, dT \right\} - \left\{ H_i^\circ(T_R) + \int_{T_R}^{T} C_{pi}\, dT \right\}$$

@ T

Why Tio? Because different species can enter the system @ different temperatures.

$$\therefore\ H_{io} - H_i = H_i^\circ(T_R) + \int_{T_R}^{T_{io}} C_{p_i}\,dT - H_i^\circ(T_R) - \int_{T_R}^{T} C_{p_i}\,dT$$

$$= \int_{T_R}^{T_{io}} C_{p_i}\,dT - \int_{T_R}^{T} C_{p_i}\,dT$$

$$= -\int_{T_{io}}^{T} C_{p_i}\,dT$$

$$\therefore\ \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s + F_{A0}\sum_{i=1}^{n}\Theta_i\left\{-\int_{T_{io}}^{T} C_{p_i}\,dT\right\} - F_{A0}X_A\,\Delta H_{rxn}(T)$$

$$\therefore\ \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s - F_{A0}\sum_{i=1}^{n}\Theta_i\int_{T_{io}}^{T} C_{p_i}\,dT - F_{A0}X_A\,\Delta H_{rxn}(T)$$

Let us treat ΔH_{rxn} similarly.

$$\Delta H_{rxn} = \frac{c}{a}H_C(T) + \frac{d}{a}H_D(T) - \frac{b}{a}H_B(T) - H_A(T)$$

no inerts here!

$$\text{But}\quad H_C(T) = H_C^\circ(T_R) + \int_{T_R}^{T} C_{p_C}\,dT$$

$$\therefore \Delta H_{rxn} = \left[\frac{c}{a} H_C^\circ(T_R) + \frac{d}{a} H_D^\circ(T_R) - \frac{b}{a} H_B^\circ(T_R) - H_A^\circ(T_R)\right] + \left[\frac{c}{a}\int_{T_R}^{T} C_{p_C}\,dT + \frac{d}{a}\int_{T_R}^{T} C_{p_D}\,dT - \frac{b}{a}\int_{T_R}^{T} C_{p_B}\,dT - \int_{T_R}^{T} C_{p_A}\,dT\right]$$

$$\therefore \Delta H_{rxn} = \Delta H_{rxn}^\circ(T_R) + \int_{T_R}^{T} \underbrace{\left(\frac{c}{a} C_{p_C} + \frac{d}{a} C_{p_D} - \frac{b}{a} C_{p_B} - C_{p_A}\right)}_{\text{let's define this as } \Delta C_{p_{rxn}}} dT$$

$$\therefore \Delta H_{rxn} = \Delta H_{rxn}^\circ(T_R) + \int_{T_R}^{T} \Delta C_{p_{rxn}}\,dT$$

$$\therefore \frac{dE_{sys}}{dt} = \dot{Q} - \dot{W}_s - F_{A0}\sum_{i=1}^{n} \Theta_i \int_{T_{i0}}^{T} C_{p_i}\,dT - F_{A0} X_A \left\{\Delta H_{rxn}^\circ(T_R) + \int_{T_R}^{T} \Delta C_{p_{rxn}}\,dT\right\}$$

applies to all systems

Remember, since entholpy is a state function, we can solve the energy balance by merely looking at the entry & exit streams.

Let's consider some simplifications:

① → If steady-state, $\frac{dE_{sys}}{dt} = 0$

→ If adiabatic, $\dot{Q} = 0$

→ Assume $\dot{W}_s$ is negligible (good assumption for PFRs)

→ Also assume C_p is constant & independent of temperature for all species.

For such a system,

$$0 = -F_{A0}\sum_{i=1}^{n}\Theta_i C_{pi}(T - T_{i0}) - F_{A0}X_A\left[\Delta H^\circ_{rxn}(T_R) + \Delta C_{p\,rxn}(T - T_R)\right]$$

Remember, this includes inerts

no inerts involved

$$\therefore\ F_{A0}\sum_{i=1}^{n}\Theta_i C_{pi}(T - T_{i0}) = -F_{A0}X_A\left[\Delta H^\circ_{rxn}(T_R) + \Delta C_{p\,rxn}(T - T_R)\right]$$

$$\boxed{\therefore\ X_A = \frac{\sum_{i=1}^{n}\Theta_i C_{pi}(T - T_{i0})}{-\left[\Delta H^\circ_{rxn}(T_R) + \Delta C_{p\,rxn}(T - T_R)\right]}}$$

Since this conversion is derived from the energy balance,

$$X_A^{EB} = \frac{\sum_{i=1}^{n} \Theta_i C_{pi} (T - T_{i0})}{-\left[\Delta H^\circ_{rxn}(T_R) + \Delta C_{prxn}(T - T_R)\right]}$$

For a PFR that is operated adiabatically & with similar assumptions in case ① discussed previously:

$$F_{A0} \frac{dX_A}{dV} = -r_A (X_A, T) \quad \text{.... mass balance}$$

We need to <u>solve these equations simultaneously</u> to design our reactor.

Let us look @ an actual example.

Consider an elementary, gas-phase rxn.

$$A \xrightarrow{k} B$$

Assumptions = adiabatic, no shaft work, all species enter at same temperature, temperature independence of C_p's, liquid-phase

* Also assume negligible thermal expansivity. $\hookrightarrow$ i.e. $\underline{\underline{v \approx v_0}}$.

2 design equations:

$$\text{EB} \Rightarrow \quad X_A^{EB} = \frac{\sum_{i=1}^{n} \Theta_i C_{p_i} (T - T_{i0})}{-\left[\Delta H^\circ_{rxn}(T_R) + \Delta C_{p\,rxn}(T - T_R)\right]} \longrightarrow \text{Ⓐ}$$

$$\text{MB} \Rightarrow \quad F_{A0} \frac{dX_A^{MB}}{dV} = -r_A \longrightarrow \text{Ⓑ}$$

Also, $-r_A = kC_A$

$$k = A \exp\left(\frac{-E_a}{RT}\right)$$

Let's simplify the two equations

$$\therefore X_A^{EB} = \frac{\sum_{i=1}^{n} \Theta_i C_{p_i} (T - T_{i0})}{-\left[\Delta H^\circ_{rxn}(T_R) + \Delta C_{p\,rxn}(T - T_R)\right]}$$

$$\therefore X_A^{EB} = \frac{(C_{pA} + C_{pB}\Theta_B)(T - T_0)}{-\left[\Delta H^\circ_{rxn}(T_R) + (C_{pB} - C_{pA})(T - T_R)\right]} \longrightarrow ①$$

$\uparrow$ Algebraic eqn.

The material balance becomes:

$$F_{A0}\frac{dX_A^{MB}}{dV} = -r_A$$

$$\therefore F_{A0}\frac{dX_A^{MB}}{dV} = kC_A$$

due to negligible expansivity

$$\therefore v_0 C_{A0}\frac{dX_A^{MB}}{dV} = \left\{A\exp\left(-\frac{E_a}{RT}\right)\right\}C_{A0}\left(1-X_A^{MB}\right)$$

$$\therefore v_0\frac{dX_A^{MB}}{dV} = \underbrace{\left\{A\exp\left(\frac{-E_a}{RT}\right)\right\}}\left(1-X_A^{MB}\right)$$

you can either be given this expression

or $k = k_R \exp\left(\frac{E_a}{RT_R}-\frac{E_a}{RT}\right)$

reference point

$$\therefore \frac{dX_A^{MB}}{d\tau} = \left\{A\exp\left(\frac{-E_a}{RT}\right)\right\}\left(1-X_A^{MB}\right) \longrightarrow \text{②}$$

There are 2 types of problems

Type 1 : * You are given v_0 & V i.e. τ & inlet flow conditions.

* You are also given all the other parameters
* You don't know the exit temperature

Goal : Solve for X_A

The solutions to X_A^{MB} & X_A^{EB} must be equivalent

$$\therefore \quad X_A^{EB} = \frac{\left(C_{pA} + C_{pB}\Theta_B\right)\left(T - T_0\right)}{-\Delta H^\circ_{rxn}(T_R) - \left(C_{pB} - C_{pA}\right)\left(T - T_R\right)}$$

$$\therefore \quad X_A = \frac{\left(C_{pA} + C_{pB}\Theta_B\right)T - \left(C_{pA} + C_{pB}\Theta_B\right)T_0}{-\Delta H^\circ_{rxn}(T_R) - \left(C_{pB} - C_{pA}\right)T + \left(C_{pB} - C_{pA}\right)T_R}$$

$$\therefore \left(C_{pA} + C_{pB}\Theta_B\right)T - \left(C_{pA} + C_{pB}\Theta_B\right)T_0 = -\Delta H^\circ_{rxn}(T_R)X_A - \left(C_{pB} - C_{pA}\right)T X_A + \left(C_{pB} - C_{pA}\right)T_R X_A$$

$$\therefore T\left\{\left(C_{pA} + C_{pB}\Theta_B\right) + \left(C_{pB} - C_{pA}\right)X_A\right\} = -\Delta H^\circ_{rxn}(T_R)X_A + \left(C_{pA} + C_{pB}\Theta_B\right)T_0 + \left(C_{pB} - C_{pA}\right)T_R X_A$$

$$\therefore\; T = \frac{-\Delta H^{\circ}_{rxn}(T_R)X_A + (C_{pA} + \Theta_B C_{pB})T_0 + X_A((C_{pB} - C_{pA})T_R}{(C_{pA} + C_{pB}\Theta_B) + (C_{pB} - C_{pA})X_A} \longrightarrow ③$$

The generic form is:

$$T = \frac{-\Delta H^{\circ}_{rxn}(T_R)X_A + \sum_{i=1}^{n} \Theta_i C_{pi} T_0 + X_A \Delta C_{p\,rxn} T_R}{\sum_{i=1}^{n} \Theta_i C_{pi} + \Delta C_{p\,rxn} X_A}$$

Subst. this value of T into the MB.

$$\text{i.e.}\quad \frac{dX_A}{d\tau} = \left[A\exp\left(\frac{-E_a}{RT}\right)\right](1 - X_A)$$

$$\therefore\quad d\tau = \frac{dX_A}{\left\{A\exp\left(\frac{-E_a}{RT}\right)\right\}(1 - X_A)}$$

$$\therefore\quad \tau = \int_0^{X_{Af}} \frac{dX_A}{\left\{A\exp\left(\frac{-E_a}{RT}\right)\right\}(1 - X_A)}$$