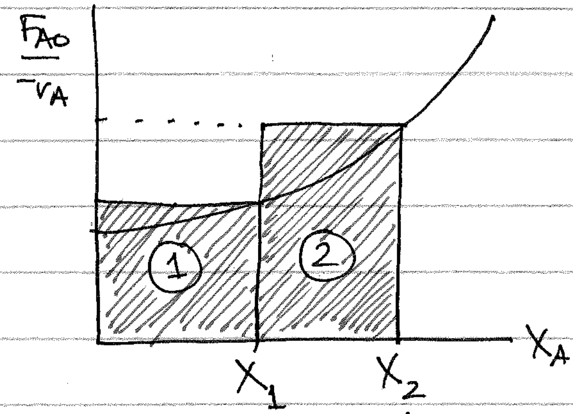
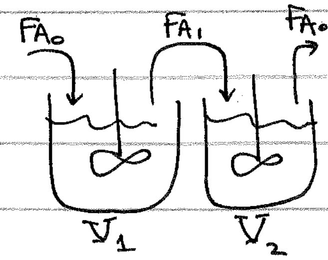
Let's look @ a general CSTR-in-series case:

Recollect:





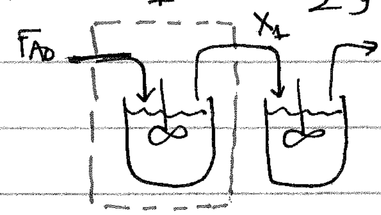
what does this mean?

and this?

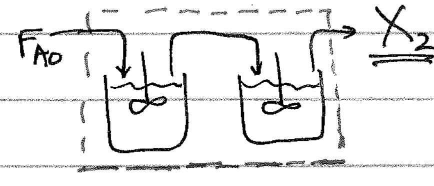
The X refers to conversion <u>until</u> that point.

∴ If reactor volumes are V_1 & V_2,

$$X_{A_1} = \frac{F_{A0} - F_{A1}}{F_{A0}}$$



$$X_{A_2} = \frac{F_{A0} - F_{A2}}{F_{A0}}$$



$$\therefore \quad F_{A1} = F_{A0}\left(1 - X_{A1}\right)$$

$$F_{A_2} = F_{A0}\left(1 - X_{A_2}\right)$$

Design eqns:

* Don't panic, just write them out systematically

$$F_{A0} - F_{A1} = (-r_A)_1 \, V_1$$

$$F_{A1} - F_{A2} = (-r_A)_2 \, V_2$$

why did we subscript $-r_A$?
The conditions could be different (eg. T)

If isothermal (which is what we have been assuming):

$$F_{A0} - F_{A1} = (-r_A) V_1$$

$$F_{A1} - F_{A2} = (-r_A) V_2$$

$$\therefore \quad F_{A0} X_{A1} = (-r_A) V_1$$

$$\& \quad F_{A0}(1 - X_{A1}) - F_{A0}(1 - X_{A2}) = (-r_A) V_2$$

$$\therefore \quad F_{A0} X_{A1} = (-r_A) V_1$$

$$F_{A0}(X_{A2} - X_{A1}) = (-r_A) V_2$$

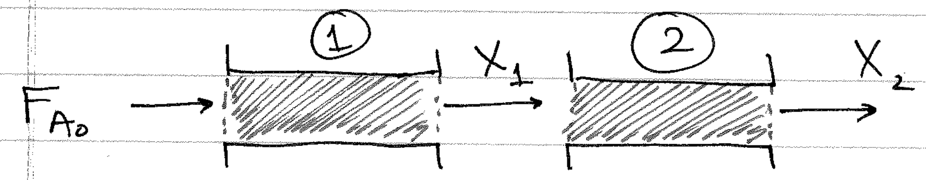
$$\therefore \quad \boxed{\frac{C_{A_1} V_1}{C_{A_2} V_2} = \frac{X_{A1}}{X_{A_2} - X_{A1}}}$$ ... isothermal case

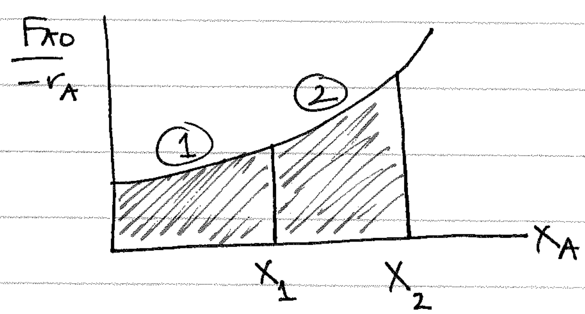
For non-isothermal case (i.e. $T_1 \neq T_2$):

$$\boxed{\frac{k_1 V_1 C_{A_1}}{k_2 V_2 C_{A2}} = \frac{X_{A1}}{X_{A_2} - X_{A1}}}$$

other cases such as $V_1 = V_2$ etc. can be derived appropriately

What about PFRs in series?





$$\frac{dF_A}{dV} = r_A$$ .... start with the design equation

$$F_{A0}\frac{dX_A}{dV} = -r_A$$

$$dV = \left(\frac{F_{A0}}{-r_A}\right) dX_A$$

For reactor 1 $\quad dV\Big|_1 = \left(\frac{F_{A0}}{-r_{A_1}}\right) dX_A$

$$\therefore \int dV_1 = \int_0^{X_{A_1}} \left(-\frac{F_{A0}}{r_A}\right) dX_A$$

$$\therefore V_1 = \int_0^{X_{A_1}} \left(\frac{F_{A0}}{-r_A}\right) dX_A$$

$$V_2 = \int_{X_{A_1}}^{X_{A_2}} \left(\frac{F_{A0}}{-r_A}\right) dX_A \quad \text{.... } F_{A0} \text{ const. throughout}$$

$$\therefore \underbrace{V_1 + V_2}_{\text{just `doubling' the pipe (or elongating it)}} = \int_0^{X_{A_2}} \left(\frac{F_{A0}}{-r_A}\right) dX_A$$

if $V_1 = V_2$

$$\int_0^{X_{A_1}} \left(\frac{F_{A0}}{-r_A}\right) dX_A = \int_{X_{A_1}}^{X_{A_2}} \left(\frac{F_{A0}}{-r_A}\right) dX_A$$

$\rightarrow$ give X_{A_2}, find X_{A_1}.

Residence time & space velocity

$v \equiv \frac{\text{volume}}{s}$ $\qquad V \equiv$ volume

$\frac{V}{v} \equiv$ seconds → time spent by a molecule or unit reacting element in the reactor

$$\boxed{\begin{Bmatrix}\text{residence}\\ \text{time}\end{Bmatrix} = \frac{V}{v} = \tau}$$

$$\frac{F_{A0}}{-r_A} = \frac{C_{A0}\, v}{k f(C_A)} \implies F_{A0} X_A = (-r_A)\, V$$

$$\therefore \quad X_A = \frac{(-r_A)}{F_{A0}} \cdot V$$

$$\therefore \quad X_A = \frac{k f(C_A)\, V}{C_{A0}\, v}$$

$$\boxed{\therefore \quad X_A = \frac{k \tau f(C_A)}{C_{A0}}}$$

$$\underbrace{X_A}_{\text{dimensionless}} = \underbrace{\frac{(k\tau)\, f(C_A)}{C_{A0}}}_{\text{dimensionless}}$$

k = units depend on order

$$k = \frac{1}{(\text{conc.})^{n-1} \times \text{time}}$$

order of the rxn.

$\therefore$ For a first order rxn., $k = \dfrac{1}{\text{time}}$ & so on.

In reactor design, we introduce another dimensionless quantity

→ Dämköhler Number = Da

1[st] order $\quad \underline{\underline{Da = k\tau}}$

2[nd] order $\quad \underline{\underline{Da = k\tau C_{A0}}}$ & so on....

$$\boxed{\text{Space velocity} \equiv \frac{1}{\tau} \quad \text{.... rarely used}}$$