Université d'Ottawa | University of Ottawa

MAT 1330C : Instructor: Dr. Xinhou Hua
Thursday, October 5, 2017 : Test #1

Duration: 75 minutes

Family name: ______________________

First name: __________

Student number : ______________________

Faculté des sciences
Mathématiques et statistique

Faculty of Science
Mathematics and Statistics

613–562–5864
613–562–5776
www.uOttawa.ca
585 King Edward
Ottawa
ON K1N 6N5

For picking up your graded test:
Circle the DGD you will attend to pick up your test (whether you are registered or not).

| DGD on Monday | DGD on Thursday |

Please read the following instructions carefully.

- You have 75 minutes to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*), no notes, cell phones, smartwatches or related devices of any kind are permitted. All such devices, including cell phones, **must be stored in your bag under your desk for the duration of the exam.**
- Read each question carefully — you will save yourself time and grief later on.
- Questions 1 through 4 are multiple choice, worth 1 point each. **Record your answers to the multiple choice questions in the boxes provided.**
- Questions 5 through 9 are short answer, with number of points as indicated. **You must show your work, your work must be legible, and you must record your answers in the boxes provided.**
- Where it is possible to check your work, do so.
- Good luck!

Marker's use only:

Question	Marks
1–4(/4)	
5, 6 (/4)	
7 (/7)	
8 (/5)	
9 (/4)	
Total (/24)	

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1. (1 point) Suppose that a pond is restocked with 50 fish per m^2 of surface area each spring and that fishing and natural causes kill 55% of the fish during each year. If one year, the number of fish per m^2 (counted shortly after the restocking) is 42 fish per m^2, which of the following Discrete-Time Dynamical Systems describe the dynamics of the population x_t of fish (per m^2, t years later)?

A. $x_{t+1} = .45x_t + 42$, with $x_0 = 50$

B. $x_{t+1} = .55x_t + 42$, with $x_0 = 50$

C. $x_{t+1} = .50x_t + 45$, with $x_0 = 42$

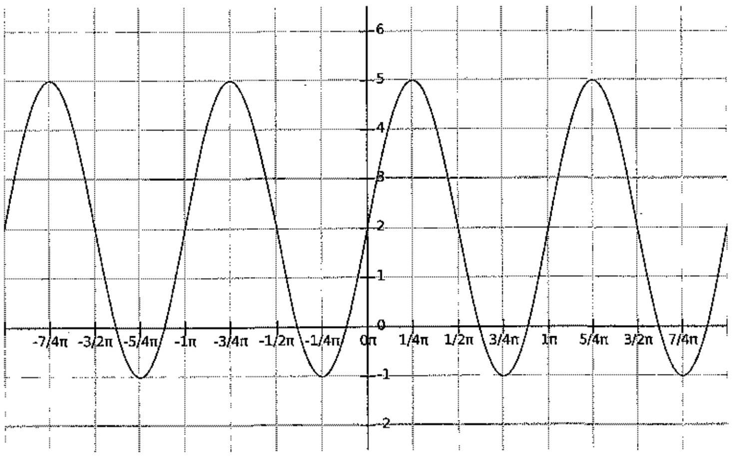
D. $x_{t+1} = .45x_t + 50$, with $x_0 = 42$

E. $x_{t+1} = .55x_t + 42$, with $x_0 = 45$

F. $x_{t+1} = 42x_t + 50$, with $x_0 = 45$

Your answer: D

2. (1 point) The following is the graph of a function $y = f(x)$.



Which of the following is a formula for $f(x)$?

A. Amplitude $A = 2$, Period $P = 2\pi$

B. Amplitude $A = 3$, Period $P = 2$

C. Amplitude $A = 2$, Period $P = 1$

D. Amplitude $A = 2$, Period $P = \pi$

E. Amplitude $A = 3$, Period $P = \pi$

F. Amplitude $A = 3$, Period $P = \pi/2$

Your answer: E

3. (1 point) Find the set of all solutions to the inequality

$$\frac{2}{x-3} > 4.$$

A. $(-\infty, \frac{7}{2})$
B. $(3, \frac{7}{2})$
C. $(\frac{7}{2}, \infty)$
D. $(3, \infty)$
E. $(-\infty, 3) \cup (\frac{7}{2}, \infty)$
F. $(-\infty, 3)$

Your answer: B

Note that $x-3>0$. $\therefore$ $x-3<\frac{2}{4}$ & $x>3$

$\Rightarrow 3<x<3+\frac{1}{2}$

4. (1 point) Find all x for which the following equality holds:

$$|5-2x^2| = 9.$$

A. $x = 0$
B. $x = \sqrt{7}$
C. $x = \pm\sqrt{7}$
D. $x = \pm\sqrt{7}$ or $x = \pm\sqrt{2}$
E. $x = 3 \pm \sqrt{5/2}$
F. there are no solutions

Your answer: C

$5-2x^2=9$ no sol.

$5-2x^2=-9 \Rightarrow x^2=7$, $x=\pm\sqrt{7}$

5. (2 points) Find all solutions x of the following equation. Show your work.

$$\ln(x+1) + \ln(x-3) = \ln(x+15)$$

Your work:

$$\ln[(x+1)(x-3)] = \ln(x+15)$$

$$(x+1)(x-3) = x+15$$

$$x^2 - 3x - 18 = 0 \Rightarrow x = 6, -3$$

Note that $-3 \notin D$

$$\therefore x = 6$$

Your answer: $\boxed{6}$

6. (2 points) For which value of the parameter a is the following function continuous at $x = 5$? Justify your answer by explaining what is required for continuity and solving for it.

$$f(x) = \begin{cases} 3\cos(\pi x) & \text{if } x < 5 \\ \frac{x}{2} - 3a & \text{if } x \geq 5. \end{cases}$$

Your work:

$$\lim_{x \to 5} f(x) = f(5) = \frac{5}{2} - 3a$$

$$\lim_{x \to 5^-} f(x) = 3\cos(5\pi) = -3$$

$$\lim_{x \to 5^+} f(x) = \frac{5}{2} - 3a$$

$$-3 = \frac{5}{2} - 3a \Rightarrow a = \frac{11}{6}$$

$a = \boxed{\frac{11}{6}}$

7. (7 points) A drug is administered to a patient on a daily basis. We have determined that the DTDS governing the concentration x_t of drug (in mg/L) in the patient's body on day t is given by

$$x_{t+1} = 0.8x_t + 4.8.$$

(a) (1 point) Give the updating function f for this DTDS. $f(x) = \boxed{0.8x + 4.8}$

(b) (1 point) Find the fixed point x^* of this DTDS.

$$0.8x^* + 4.8 = x^*$$

$$x^* = 24$$

$x^* = \boxed{24}$

(c) (1 point) Suppose that on day zero the concentration is measured as 15 mg/L. Give the general solution formula to this DTDS.

$$x_t = \left(x_0 + \frac{4.8}{0.8-1}\right)0.8^t + \frac{4.8}{1-0.8} = (15-24)0.8^t + 24$$

General solution formula: $\boxed{-9(0.8)^t + 24}$

(d) (1 point) Find the concentration in two days.

Your answer: $\boxed{16.71}$

(e) (3 points) Determine the (whole) number of days necessary until the concentration of drug in the body is within 0.9 of the fixed point, that is, until $|x_t - x^*| < 0.9$. **Show your work. Your answer must be clear and well-justified to earn full marks.**

$$|-9(0.8)^t + 24 - 24| < 0.9$$

$$0.8^t < 0.1$$

$$t > \frac{\ln 0.1}{\ln 0.8} \approx 10.3$$

$$\therefore t_{min} = 11$$

8. **(5 points)** The DTDS $x_{t+1} = 0.3x_t(9.1 - x_t)$ models a certain population.

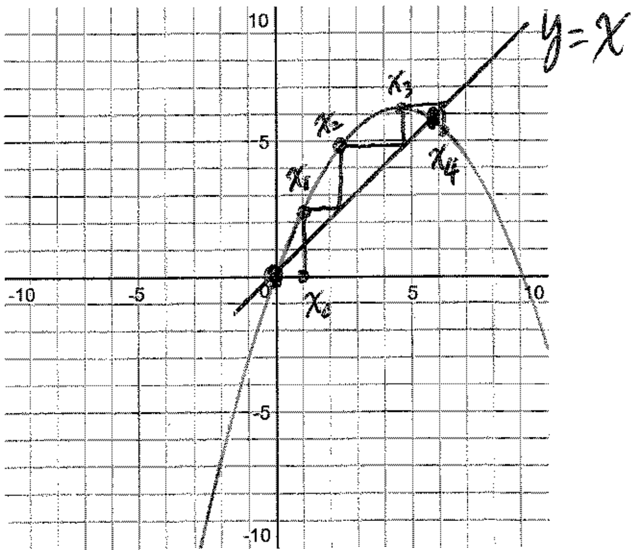
(a) **(2 points)** Solve for all fixed points of the DTDS exactly.

Your work:

$$x = 0.3x(9.1 - x)$$
$$x(1.73 - 0.3x) = 0$$
$$x = 0, \frac{17.3}{3}.$$

Your answer: $0, \frac{17.3}{3}$

(b) **(2 points)** The graph of the updating function of this DTDS is given below. Suppose the initial value is $x_0 = 1$. Draw a cobweb diagram on the graph below for this DTDS with at least 4 steps. Label the axes, the functions, the fixed points and the points $x_0 \cdots x_4$.



(c) **(1 point)** Write a sentence to explain what happens in the long term if $x_0 = 1$. Your sentence should include the word "stable" or "unstable", as well as the exact fixed point from (a) which is relevant.

Long term will approach $\frac{17.3}{3} \approx 5.7667$, which is stable.

9. (4 points) Decide if the following limits exists. For each one, if it exists, evaluate the limit exactly using algebraic methods, showing all your steps. If it does not exist, justify your answer clearly using mathematical reasoning.

(a) **(2 points)** $\lim_{x \to 3} \frac{x^2 + 2x - 15}{|x - 3|}$

$$\lim_{x \to 3^+} \frac{x^2+2x-15}{|x-3|} = \lim_{x \to 3^+} \frac{(x-3)(x+5)}{x-3} = \lim_{x \to 3^+} (x+5) = 8$$

$$\lim_{x \to 3^-} \frac{x^2+2x-15}{|x-3|} = \lim_{x \to 3^-} \frac{(x-3)(x+5)}{-(x-3)} = \lim_{x \to 3^-} -(x+5) = -8$$

$$\because \lim_{x \to 3^-} \frac{x^2+2x-15}{|x-3|} \neq \lim_{x \to 3^+} \frac{x^2+2x-15}{|x-3|}, \text{ The limit } \nexists$$

Your answer: $\nexists$

(b) **(2 points)** $\lim_{x \to \infty} \frac{5x^2}{\sqrt{16x^4 + 7}}$

$$= \lim_{x \to \infty} \frac{5x^2}{\sqrt{16x^4 (1 + \frac{7}{16x^4})}} = \lim_{x \to \infty} \frac{5x^2}{4x^2 \sqrt{1 + \frac{7}{16x^4}}}$$

$$= \lim_{x \to \infty} \frac{5}{4\sqrt{1 + \frac{7}{16x^4}}} = \frac{5}{4}$$

Your answer: $\frac{5}{4}$