MAT 2379, Introduction to biostatistics

Solution to Assignment 4
(Total = 20 points)

[2] **Problem 8.4** A 95% confidence interval for the mean PVR for patients with heart failure associated to left ventricular dysfunction is

$$\overline{x} \pm t\,\frac{s}{\sqrt{n}} = [183.26; 285.94],$$

where $t = t_{0,025;29} = 2.045$, $\overline{x} = 234.6$, $s = 137.5$ and $n = 30$.

Problem 8.6
(a) A 95% confidence interval for the mean μ is of the form $\overline{x} \pm 1.96\, s/\sqrt{n}$. Subtracting the upper limit from the lower limit, we get:

$$6.5921 - 5.3279 = 2\,(1.96\, s/\sqrt{n}) \quad \Rightarrow \quad s/\sqrt{n} = 0.3225.$$

Therefore, the estimated standard error for the estimate of the mean is 0.3225 mg/100 cc.
(b) The sample standard deviation is $s = \sqrt{n}(s/\sqrt{n}) = \sqrt{150}(0.3225) = 3.9498$ mg/100 cc.
(c) Since we want a larger percentage of the confidence intervals to contain the value of the population mean, then it should be natural to expect that a 97% confidence interval will be larger than a 95% confidence interval.
(d) A 97% confidence interval for the mean μ is of the form $\overline{x} \pm z\ s/\sqrt{n}$, where z satisfies $0.97 = P(-z < Z < z)$ or equivalently $P(Z < z) = 0.985$. From Table 18.3, we get $z = 2.17$. So the 97% confidence interval for μ is

$$5.96 \pm 2.17\,(0.3225) = [5.26; 6.66].$$

Problem 9.2 We denote by p the proportion of patients treated with the new drug who will have a recurrent UTI. We would like to test $p = 0.1$ against $p < 0.1$. A point estimate for p is $\hat{p} = 29/347 = 0.0836$. The observed value of the test statistic is:

$$z_0 = \frac{\hat{p} - 0.10}{\sqrt{(0.1)(0.9)/347}} = -1.02$$

From Table 18.2, we see that p-value= $P(Z < -1.02) = 0.1539$. Since the p-value is larger than $\alpha = 0.05$, we do not reject H_0. There is not enough evidence that p is smaller than 0.10.

Problem 9.6 Let p be the true success rate of PN in treating kidney stones.
[2] (a) A point estimate for p is $\hat{p} = 289/350 = 0.8257$ and its estimated standard error is $s\{\hat{p}\} = \sqrt{\hat{p}(1-\hat{p})/n} = 0.02028$.
[4] (b) We want to test $H_0 : p = 0.78$ against $H_1 : p \neq 0.78$. The observed value of the test statistic is

$$z_0 = \frac{\hat{p} - 0.78}{\sqrt{.78(1 - .78)/350}} = 2.06.$$

The p-value is $2\,P(Z > 2.06) = 2\,(1 - 0.9803) = 0.0394$. At a level of significance of 1%, the evidence that the success rate of PN in treating kidney stones is different than the success rate of open surgery is not significant.
[2] (c) A 95% confidence interval for p is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = [0.786; 0.865].$$

We are 95% confident that the success rate of PN in treating kidney stones is between 78.6% and 86.5%. We are 95% confident that PN is more successful for treating kidney stones than open surgery.

[4] **Problem 9.10** We want to test $H_0 : \mu = 14$ versus $H_1 : \mu > 14$. The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{14.4 - 14}{\sqrt{0.025/5}} = 5.656$$

From Table 18.4, we see that the p-value= $P(T_4 > 5.565)$ is smaller than 0.005. This means that the p-value is also smaller than 0.05. Hence, we reject H_0, and conclude that the concern is justified.

Additional question:

$$n \geq \left(\frac{z_{\alpha/2}\sigma}{0.01}\right)^2 = (\frac{1.645 \times 0.5}{0.01})^2 = 6765.062.$$

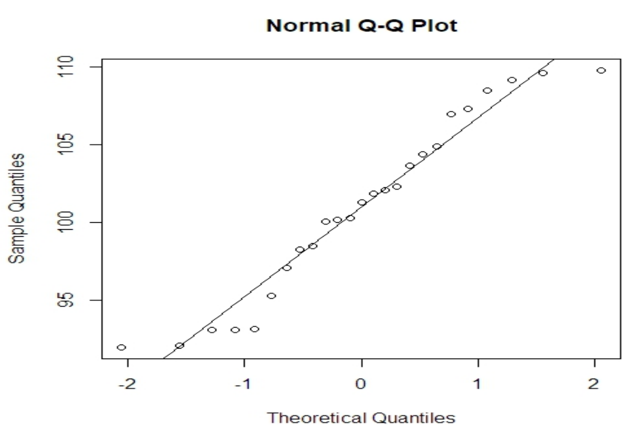
We round it up to $n = 6766$.

Part (II)

[2] 1. **a)**

```
> carbon= read.table(file.choose(),header=TRUE,sep="\t")
> names(carbon)
[1] "Carbon.ppm."
> x=carbon$Carbon.ppm.
> qqnorm(x)
> abline(mean(x),sd(x))
```

Here is the quantile-quantile plot. There is a linear tendency in the QQ-plot with slight deviances in the tails. It is reasonable to assume that the carbon monoxide concentration is normally distributed.



[1] **b)** Let μ be the mean carbon monoxide concentration. We want to test $H_0 : \mu = 100$ against $H_1 : \mu > 100$.

[**2**] **c)**

```
> t.test(x,mu=100,alternative="greater",conf.level=0.9)

One Sample t-test

data:  x
t = 0.8778, df = 24, p-value = 0.1944
alternative hypothesis: true mean is greater than 100
90 percent confidence interval:
99.49269      Inf
sample estimates:
mean of x
101.012
```

The p-value is 19.44% and at a level of significance of 10%, the evidence against the null hypothesis is not significant. We cannot conclude that the mean monoxide concentration is larger than 100 ppm.

[**1**] **d)** We failed to reject the null hypothesis, this means that we committed an error of type II, if we did indeed commit an error.