

**MAT 2379, Introduction to Biostatistics**

**Assignment 1 solutions (Total = 35 )**  
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**2.2** Let  $A$  be the event that the day is frigid and  $B$  the event that there is light snow that day. We know that  $P(A) = 0.51$ ,  $P(B) = 0.68$  and  $P(A \cap B) = 0.35$ . By the addition rule,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.51 + 0.68 - 0.35 = 0.84.$$

The desired probability is

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.84 = 0.16.$$

[10] **2.4** Let  $A$  be the event that a randomly chosen woman in this study suffers from inoperable locoregional recurrent disease, and  $B$  the event that she has distant metastases. We know that

$$P(A) = \frac{87}{268} = 0.325, \quad P(B) = \frac{140}{268} = 0.522 \quad \text{and} \quad P(A \cap B) = \frac{41}{268} = 0.153.$$

(a) The probability that a randomly chosen woman in this study suffers from inoperable locoregional recurrent disease, but does not have distant metastases is:

$$P(A \cap B') = P(A) - P(A \cap B) = \frac{87 - 41}{268} = \frac{46}{268} = 0.172.$$

b) The probability that a randomly chosen woman in this study suffers from distant metastases but does not have inoperable locoregional recurrent disease is:

$$P(A' \cap B) = P(B) - P(A \cap B) = \frac{140 - 41}{268} = \frac{99}{268} = 0.369.$$

[5] **2.6** Let  $A$  be the event that the tree has been affected by a fire and  $B$  be the event that the tree has been affected by budworms. We know that  $P(A' \cap B') = 0.75$ ,  $P(A \cap B') = 0.12$ , and  $P(B \cap A') = 0.05$ .

Since,  $P(A' \cap B') = 0.75$ , then  $P(A \cup B) = 1 - 0.75 = 0.25$ . From a Venn diagram, we observe that

$$\begin{aligned} 0.25 &= P(A \cup B) = P(A \cap B') + P(B \cap A') + P(A \cap B) \\ &= 0.12 + 0.05 + P(A \cap B) \end{aligned}$$

Hence,

$$P(A \cap B) = 0.25 - 0.12 - 0.05 = 0.08$$

**3.2** We denote by  $D$  the event that a patient has sleep apnea, by  $A$  the event that the patient has symptom  $A$  and by  $B$  the event that the patient has symptom  $B$ . The four events  $G_1 = A \cap B$ ,  $G_2 = A \cap B'$ ,  $G_3 = A' \cap B$  and  $G_4 = A' \cap B'$  form a partition of the sample space  $S$ . We know that

$$P(G_1) = 0.56, \quad P(G_2) = 0.21, \quad P(G_3) = 0.19, \quad P(G_4) = 0.04$$

and

$$P(D|G_1) = 0.60, \quad P(D|G_2) = 0.45, \quad P(D|G_3) = 0.35, \quad P(D|G_4) = 0.03.$$

[5] (a) By the total probability rule,

$$\begin{aligned} P(D) &= P(D|G_1)P(G_1) + P(D|G_2)P(G_2) + P(D|G_3)P(G_3) + P(D|G_4)P(G_4) \\ &= (0.60)(0.56) + (0.45)(0.21) + (0.35)(0.19) + (0.03)(0.04) = 0.4982 \end{aligned}$$

[10] (b) We have to calculate  $P(A|D)$ . Note that

$$P(A|D) = P(G_1|D) + P(G_2|D).$$

By Bayes' rule,

$$\begin{aligned} P(G_1|D) &= \frac{P(D|G_1)P(G_1)}{P(D)} \\ &= \frac{(0.60)(0.56)}{0.4982} = 0.6744 \end{aligned}$$

and

$$\begin{aligned} P(G_2|D) &= \frac{P(D|G_2)P(G_2)}{P(D)} \\ &= \frac{(0.45)(0.21)}{0.4982} = 0.1896. \end{aligned}$$

Hence,  $P(A|D) = 0.6744 + 0.1896 = 0.864$ .

**3.8** Let  $G$  and  $C$  be the respective events of guessing the answer and correctly choosing the right answer. We know that  $P(C|G) = 1/5 = 0.2$ ,  $P(C|G') = 0.8$  and  $P(G) = 0.15$ .

(a) We want

$$P(C) = P(C|G')P(G') + P(C|G)P(G) = 0.8(0.85) + 0.2(0.15) = 0.71.$$

(b) We want

$$P(G'|C) = \frac{P(G' \cap C)}{P(C)} = \frac{P(C|G')P(G')}{P(C)} = \frac{0.8(0.85)}{0.71} = 0.9577.$$

[5] **3.10** Let  $A$  be the event that the person is a non-smoker and  $B$  be the event that the person has emphysema. We know that  $P(A) = 0.85$  and  $P(A \cap B) = 0.24$ . The desired probability is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.24}{0.85} = 0.282.$$

**3.3** (Adjusted with the given table)

(a) The false positive rate is:  $P(\text{test} + | \text{true} -) = 56/993 = 0.056$ . The false negative rate is:  $P(\text{test} - | \text{true} +) = 3/7 = 0.43$ .

(b) The sensitivity is  $P(\text{test} + | \text{true} +) = 4/7 = 0.571$ . The specificity is  $P(\text{test} - | \text{true} -) = 937/993 = 0.943$

(c) The positive predictive value is  $P(\text{true} + | \text{test} +) = 4/60 = 0.067$ . The positive predictive value is  $P(\text{true} - | \text{test} -) = 937/940 = 0.996$ .

**AQ** (Additional question)

Let B be the even that the randomly chosen student is a boy, therefore  $P(B) = 40/100 = 0.4$ .

Let E be the even that the randomly chosen student wears eyes glasses, therefore  $P(E) = 0.4$ .

$$P(E|B) = \frac{P(B \cap E)}{P(B)} = \frac{\frac{16}{100}}{0.4} = 0.4$$