University of Ottawa **PHY1322**
Principles of Physics
Professor: A.Czajkowski

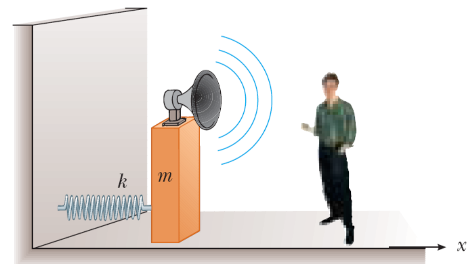
ID:______________________________

NAME: ___________________________

ASSIGNMENT 5: Doppler Effect. Optics I

Assigned Fri Feb 10 Due: Thu Feb 16 6PM SHARP!

1 A block with a speaker bolted to it is connected to a spring having spring constant k = 50.0 N/m as shown. The total mass of the block and speaker is 15.00 kg, and the amplitude of this unit's motion is 1 m. The speaker emits sound waves of frequency 400 Hz. Determine the highest and lowest frequencies heard by the person to the right of the speaker. Take the speed of sound to be 343 m/s.



(a) The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0\ \text{N/m}}{5.00\ \text{kg}}}(0.500\ \text{m}) = 1.00\ \text{m/s}$$

The frequencies heard by the stationary observer range from

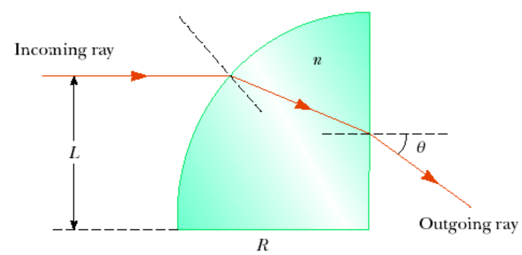
$$f'_{\min} = f\left(\frac{v}{v+v_{\max}}\right) \text{ to } f'_{\max} = f\left(\frac{v}{v-v_{\max}}\right)$$

where v is the speed of sound.

$$f'_{\min} = 440\ \text{Hz}\left(\frac{343\ \text{m/s}}{343\ \text{m/s}+1.00\ \text{m/s}}\right) = \boxed{439\ \text{Hz}}$$

$$f'_{\max} = 440\ \text{Hz}\left(\frac{343\ \text{m/s}}{343\ \text{m/s}-1.00\ \text{m/s}}\right) = \boxed{441\ \text{Hz}}$$

2 A material having an index of refraction n is surrounded by a vacuum and is in the shape of a quarter circle of radius R. A light ray parallel to the base of the material is incident from the left at a distance L above the base and emerges from the material at the angle θ. Determine an expression for θ using n and R



SOLVED DURING LECTURE

3 Follow the Procedure outlined in class and show that Law of Reflection may be obtained from Fermat Principle /Use the opposite page for full solution with large diagram)

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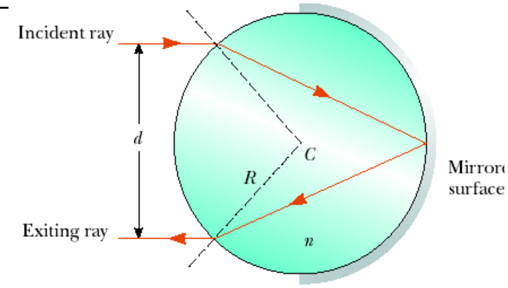
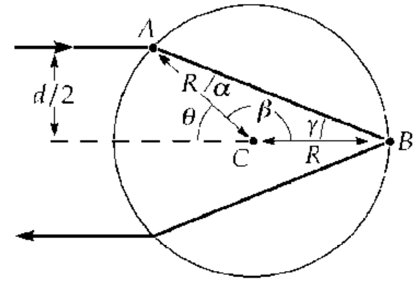
4 A transparent cylinder of radius R = 2.00 m has a mirrored surface on its right half. A light ray traveling in air is incident on the left side of the cylinder. The incident light ray and exiting light ray are parallel and d = 2.00 m. Determine the index of refraction of the material.





As shown in the sketch, the angle of incidence at point A is:

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°.$$

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline CB of the cylinder. In the isosceles triangle ABC,

$$\gamma = \alpha \quad \text{and} \quad \beta = 180° - \theta.$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

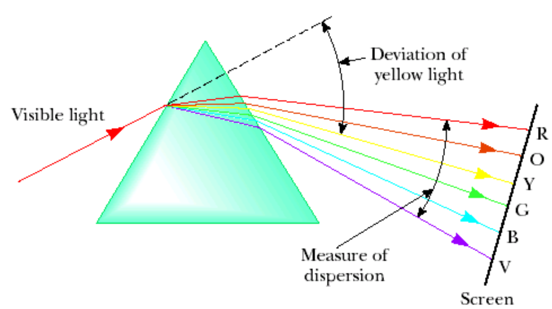
or $\alpha = \dfrac{\theta}{2} = 15.0°$.

Then, applying Snell's law at point A,

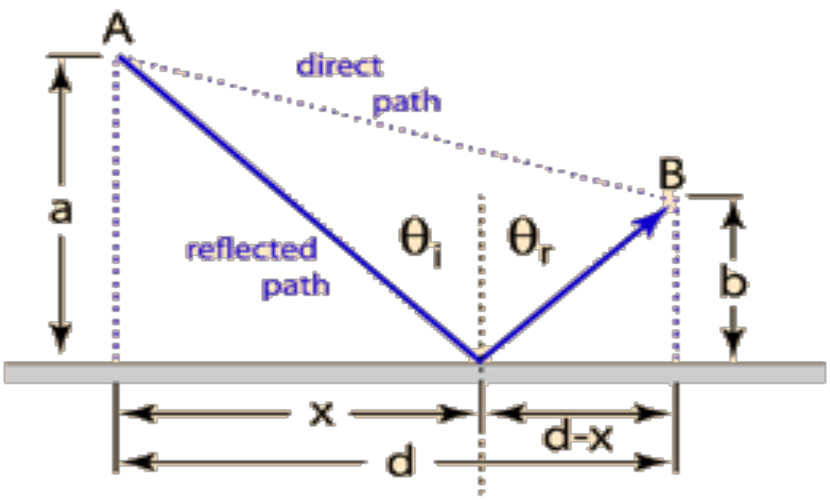
$$n \sin\alpha = 1.00 \sin\theta$$

or $n = \dfrac{\sin\theta}{\sin\alpha} = \dfrac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$.

6 The index of refraction for violet light in silica flint glass is 1.66, and that for red light is 1.62. What is the angular dispersion of visible light passing through a prism of apex angle 60.0° if the angle of incidence is 50.0°?



PROBLEM 3



$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{\sqrt{a^2 + x^2}}{v} + \frac{\sqrt{b^2 + (d - x)^2}}{v}$$

Fermat Principle tells us that the light will take the path that will make this time to be shortest!

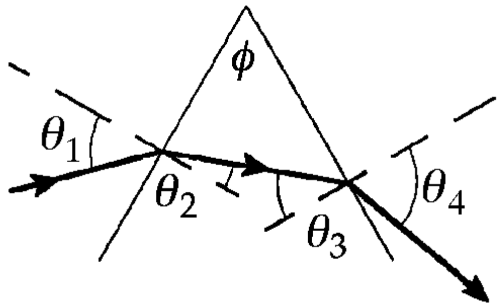
$$\frac{\mathrm{d}(\Delta t)}{dx} = \frac{1}{v}\left(\frac{2x}{2\sqrt{a^2 + x^2}} + \frac{-2(d - x)}{2\sqrt{b^2 + (d - x)^2}}\right) = 0$$

$$\frac{\mathrm{d}(\Delta t)}{dx} = 0 \Longrightarrow sin\theta_i - sin\theta_r = 0 \Longrightarrow \theta_i = \theta_r$$

ANS 6

the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$.

Using the figure to the right,



$$(\theta_2)_{\text{violet}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.66}\right) = 27.48°$$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.62}\right) = 28.22° \; .$$

For the outgoing ray, $\theta_3 = 60.0° - \theta_2$ and $\sin\theta_4 = n\sin\theta_3$:

$$(\theta_4)_{\text{violet}} = \sin^{-1}\left[1.66\sin 32.52°\right] = 63.17°$$

$$(\theta_4)_{\text{red}} = \sin^{-1}\left[1.62\sin 31.78°\right] = 58.56° \; .$$

The angular dispersion is the difference

$$\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$$