

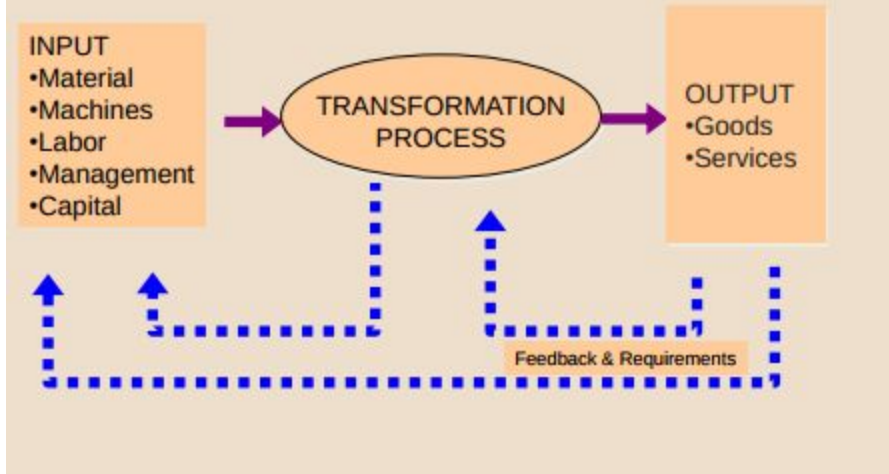
Comm 225 Notes

Chapter 1

Operations management: the design, operation, and improvement of manufacturing and service systems

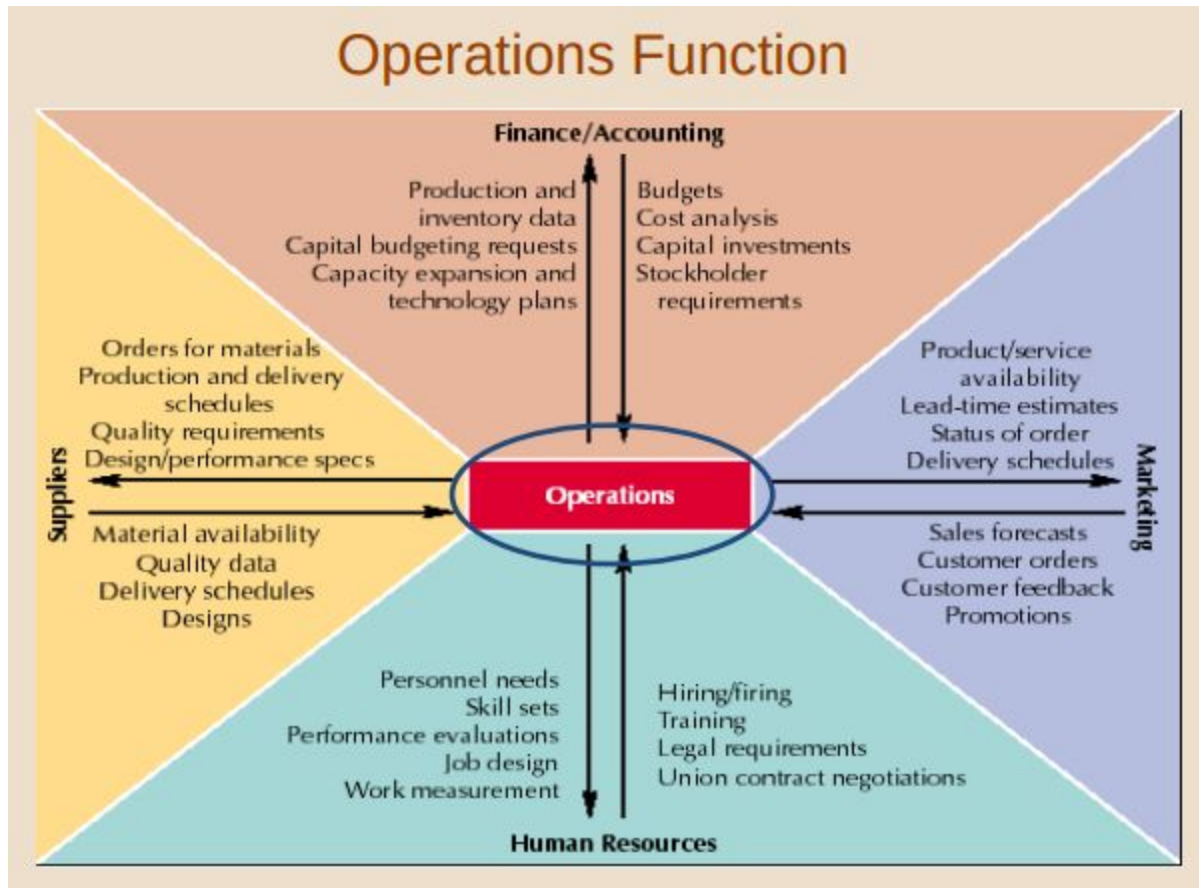
- Operations: a function or system that transforms inputs (material, machines, labour, management, capital) into outputs (goods & services) of greater value;
 - Transformation process: series of activities along a *value chain* extending from supplier to customer that add value to a product or service
 - Activities that don't add value are superfluous and should be eliminated → “non-value adding activities”
- Transformation process can be:
 - Physical: manufacturing operations
 - Locational: transportation/warehouse
 - Exchange: retail
 - Psychological: health care
 - Physiological: entertainment
 - Informational: communication
- Activities in operation management: organizing work, selecting processes, arranging layouts, locating facilities, designing jobs, measuring performance, controlling quality, schedules, inventory, planning productions

Operations as a Transformation Process



Four primary functional areas of a firm:

1. Marketing
2. Finance
3. Operations
4. Human resources



Evolution of operations and supply chain management:

- Craft production: the process of handcrafting products or services for individual customers
- Division of labour: dividing a job into a series of small tasks, each performed by a different worker
- Interchangeable parts: the standardization of parts, initially as replacement parts, which enabled mass production
- Scientific management: the systematic analysis of work methods
- Mass production: the high-volume production of a standardized product for a mass market
- Lean production: an adaptation of mass production that prizes quality and flexibility

Quality revolution: an emphasis on quality and the strategic role of operations

Historical events in operations management:

Era	Events/Concepts	Dates	Originator
Industrial Revolution	Steam engine	1769	James Watt
	Division of labor	1776	Adam Smith
	Interchangeable parts	1790	Eli Whitney
Scientific Management	Principles of scientific management	1911	Frederick W. Taylor
	Time and motion studies	1911	Frank and Lillian Gilbreth
	Activity scheduling chart	1912	Henry Gantt
	Moving assembly line	1913	Henry Ford

Era	Events/Concepts	Dates	Originator
Human Relations	Hawthorne studies	1930	Elton Mayo
	Motivation theories	1940s	Abraham Maslow
		1950s	Frederick Herzberg
		1960s	Douglas McGregor
Operations Research	Linear programming	1947	George Dantzig
	Digital computer	1951	Remington Rand
	Simulation, waiting line theory, decision theory, PERT/CPM	1950s	Operations research groups
	MRP, EDI, EFT, CIM	1960s, 1970s	Joseph Orlicky, IBM and others

Globalization:

- Why go “global”?
 - Favorable cost
 - Access to international markets
 - Response to changes in demand
 - Reliable sources of supply

- Keep up with trends/technologies
- Increased globalization results from internet and falling trade barriers

Supply chain management: managing the flow of information, products and services across a network of customers, enterprises and suppliers

Productivity and competitiveness:

Competitiveness: the degree to which a nation can produce goods and services that meet the test of international markets

Productivity: the ratio of output to input; where $P = \frac{OUTPUT}{INPUT}$

- Measures of productivity:
 - Single-factor productivity $\rightarrow \frac{output}{labor}, \frac{output}{materials}, \frac{output}{capital}$
 - Compares output to individual inputs
 - Multifactor productivity $\rightarrow \frac{output}{labor + materials + overhead}, \frac{output}{labor + energy + capital}$
 - Compares output to a combination of inputs
 - Total factor productivity: $\frac{goods\ and\ services\ produced}{all\ inputs\ used\ to\ produce\ them}$
 - Compares total quantity of goods and services produced with all the inputs used to produce them

Strategy and operations

Strategy: provides direction for achieving a mission

- Importance of well-defined strategy:
 - Provides direction for achieving mission
 - Unites the organization
 - Provides consistency in decisions
 - Keeps organization moving in the right direction

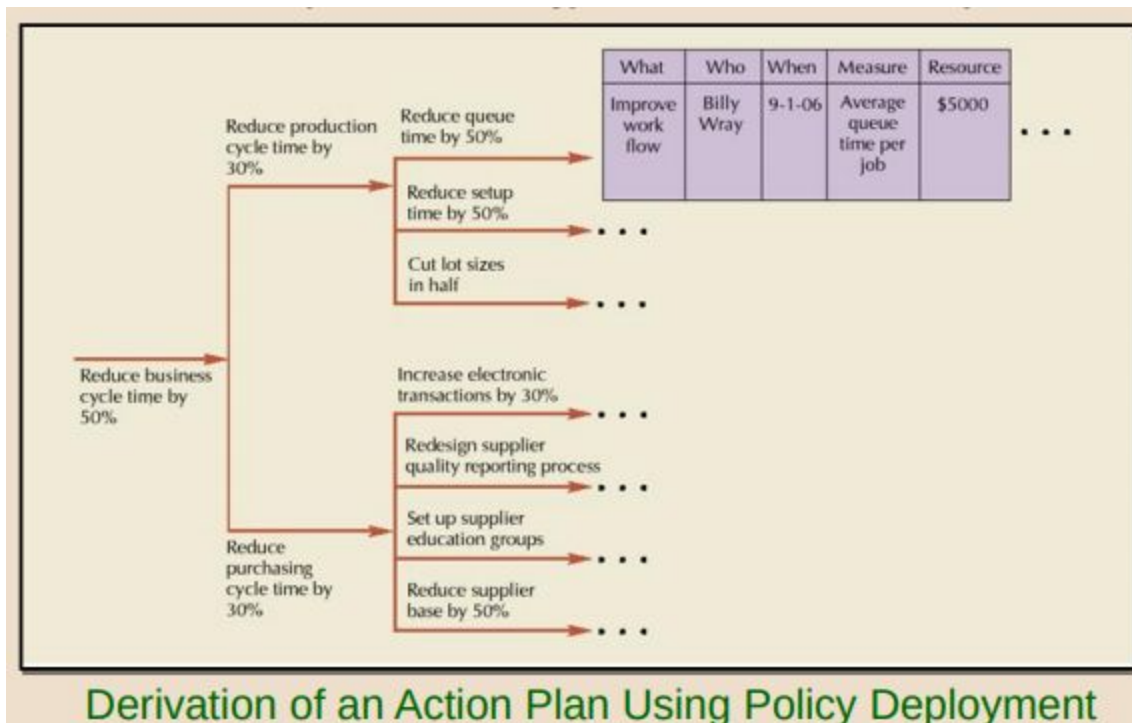


- Strategy formation consists of the 5 basic steps:
 - Defining a primary task: what the firm is in the business of doing
 - Assessing core competencies: what the firm does better than competitors
 - Determining order winners and qualifiers: what qualifies an item to be considered for purchase
 - Order qualifiers: what qualifies an item to be considered for purchase
 - Order winner: what wins the order
 - Positioning the firm: how the firm chooses to compete
 - Cost
 - Waste elimination
 - Examination of cost structure for reduction potential
 - Lean production: providing low costs through disciplined operations
 - Speed: fast moves, fast adaptations, tight linkages
 - Internet
 - Service organizations (McDonald's)
 - Manufacturers: time-based competition→build-to-order production and efficient supply chains

- Fashion industry
 - Quality
 - Minimizing defect rates or conforming to design specifications
 - Flexibility: ability to adjust to changes in product mix, production volume or design
 - *Mass customization*: the mass production of customized products
- Deploying the strategy
 - 2 types of planning system: policy deployment & balanced scorecard

1. **Policy deployment:** translates corporate strategy into measurable objectives

- *Hoshins*: the action plans generated from policy deployment process



2. **Balanced scorecard:** developed by Robert Kaplan & David Norton; a performance assessment tool that uses *key performance indicators (KPI)* a set of measures that help managers evaluate performance in critical areas such as:

- Finances: how should we look to our shareholders
- Customers: how should we look to our customers

- Processes: at which business processes should we excel
- Learning & growing: how will we sustain our ability to change and improve

Balanced Scorecard Worksheet							
Dimension	Objectives	Key Performance Indicator	Goal for 2008	KPI Results to Date	Score	Mean Performance	
Finances	Productivity	Become industry cost leader	% reduction in cost per unit	20%	10%	50%	65%
	Growth	Increase market share	Market share	50%	40%	80%	
Customers	Quality	Zero defects	% good quality first pass	100%	80%	80%	87%
	Timeliness	On-time delivery	% of on-time deliveries	95%	90%	95%	
Processes	Suppliers	Integrate into production	% orders delivered to assembly	50%	40%	80%	73%
		Reduce inspections	% suppliers ISO 9000 certified	90%	60%	67%	
	Products	Reduce time to produce	Cycle time	10 mins.	12 mins.	83%	52%
		Improve quality	# warranty claims	200	1000	20%	
	Distribution	Reduce transportation costs	% FTL shipments	75%	30%	40%	40%
	Post-sales Service	Improve response to customer inquiries	% queries satisfied on first pass	90%	60%	67%	67%
	Risk	Reduce inventory obsolescence	Inventory turnover	12	6	50%	50%
		Reduce customer backlog	% order backlogged	10%	20%	50%	
Learning & Growing	Human capital	Develop quality improvement skills	# of six sigma Black Belts	25	2	8%	35%
			% trained in SPC	80%	50%	63%	
	Information capital	Provide technology to improve processes	% customers who can track orders	100%	60%	60%	61%
			% suppliers who use EDI	80%	50%	63%	
	Organizational capital	Create innovative culture	# of employee suggestions	100	60	60%	55%
% of products new this year			20%	10%	50%		

Chapter 9

Activity: individual job or work effort requiring labour, resources, and time and that is subject to management control

Project: a unique, one-time operational activity or effort

Project management (PM): management of the work to develop and implement an innovation or change in an existing operation. 3 primary elements:

1. Planning:

- a. Objectives: detailed statement of what the project is to accomplish
 - Estimate of when the project can be completed
 - Cost and return
- b. Project scope: discussion of how to approach the project
 - Technological and resource feasibility
 - Justification of the project and what constitutes a success
- c. Requirements: general structure of managerial, reporting and performance responsibilities
 - Detailed list of staff, suppliers, subcontractors, managerial agreements, etc
- d. Schedules: list of major events, tasks and sub-schedules that form master schedule
- e. Resources: overall project budget
- f. Personnel: identification and recruitment of personnel required for the project team
- g. Control: procedures for monitoring and evaluating progress and performance including schedules and cost
- h. Risk and problem analysis: anticipating and assessing uncertainties, problems and potential difficulties that may arise

2. Scheduling

3. Controlling

Project return

ROI (return on investment): most popular measure of project benefit

$$\text{ROI} = \frac{\text{gain from project} - \text{cost of project}}{\text{cost of project}}$$

Project team and project manager

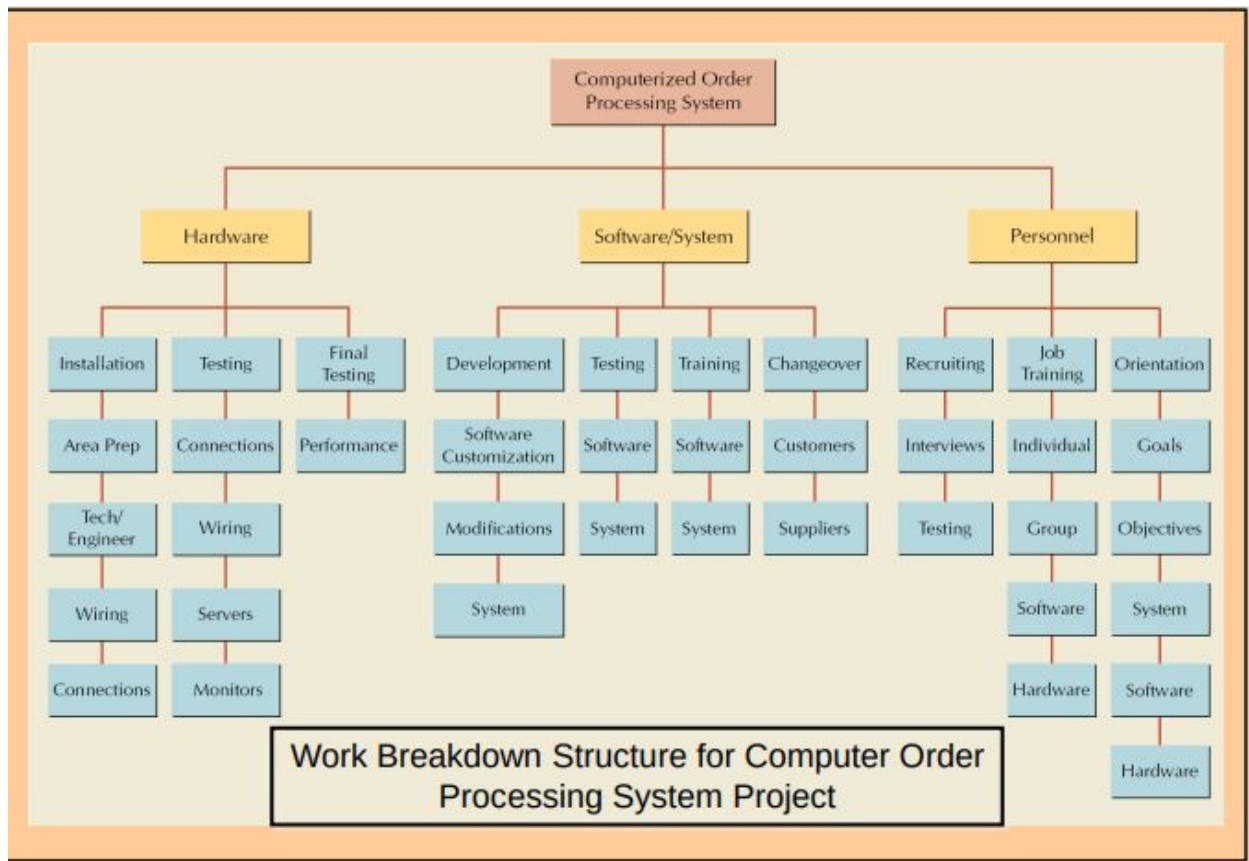
- Project team: individuals from various areas and department within company
- Matrix organization: a team structure with members from functional areas, depending on skills required

- Project manager: most important member of project team

Scope statement:

- Scope statement: a document that provides an understanding, justification and expected result of a project
- Statement of work: written description of objectives of a project

Work breakdown structure (WBS): Breaks a project into components, subcomponents, activities and tasks; tool used for project planning



Responsibility assignment matrix:

- Organizational breakdown structure (OBS): a chart that shows which organizational units are responsible for work items

- Responsibility assignment matrix (RAM): Shows who is responsible for work in a project

Global and diversity issues in project management:

- Global project teams are formed from different genders, cultures, ethnicities, etc
- Diversity among team members can add an extra dimension to project planning
- Cultural research and communication are important elements in the planning process

Project scheduling: It is the most critical element and the one that causes most problems. Why?

Because the most important criterion of a project is that it is finished on time

- Steps:
 - Define activities
 - Sequence activities
 - Estimate time
 - Develop schedule
- Techniques:
 - Gantt chart: bars represent time for each activity and status
 - Set of activities demonstrates how a *precedence relationship* works (the sequential relationship of project activities to each other)
 - *Slack*: amount of time an activity can be delayed without delaying entire project
 - $SLACK = LS - ES = LF - EF$
 - CPM/PERT: single technique that combines both
 - CPM: critical path method → deterministic task times, activity-on-node network construction. Difference: did not allow for variation; activity times were treated if they were known for certain, i.e “deterministic”
 - PERT: program evaluation and review technique → probabilistic task time estimates, activity-on-arrow network construction. Difference: allowed for variation; activity were treated as “probabilistic”
 - Advantage of CPM/PERT over the Gantt chart:

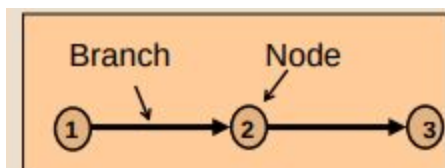
- Use of network to depict the precedence relationships between activities
- Visually easier to see precedence relationships especially for large projects with many activities
- Software: microsoft project

Project control

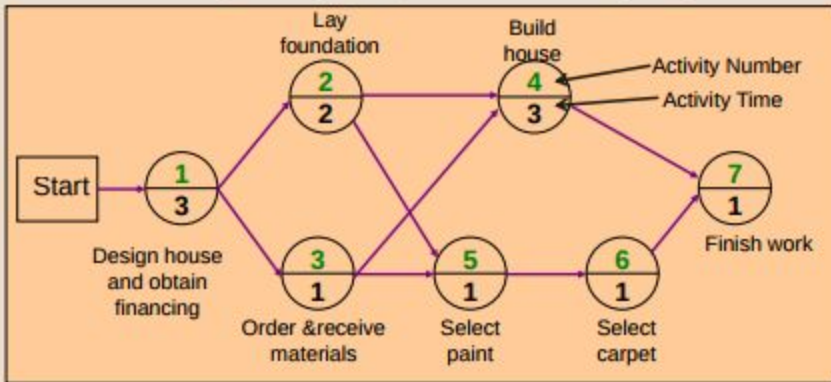
- Time management: process of ensuring the project schedule does not slip and is on time
 - *Time-cost tradeoff*: activities that may have slack time and resources can be shifted from them to activities that are not on schedule
- Cost management
- Performance management
 - *Earned value analysis (EVA)*: standard procedure to:
 - Numerically measure a project's progress
 - Forecast its completion date and cost
 - Measure schedule and budget variation
- Quality management
- Communication
- Enterprise project management: managing portfolios of projects

Project network

- Activity-on-node (AON): nodes represent activities, arrows show precedence relationships. Used with **CPM** (*book focuses on AON network*)

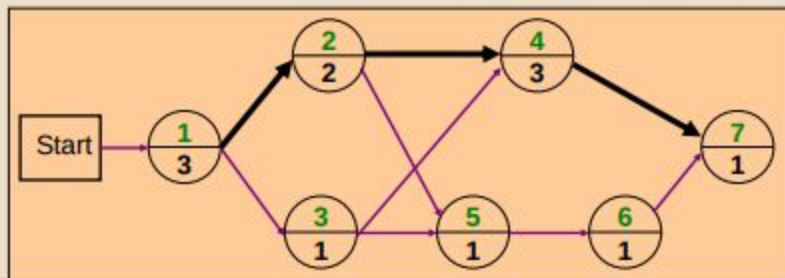


AON Network for House Building Project (Original example)



- Network path: a sequence of connected activities that runs from start to end of network
- Critical path: longest path through a network, minimum project completion time

Critical Path (Original Example)

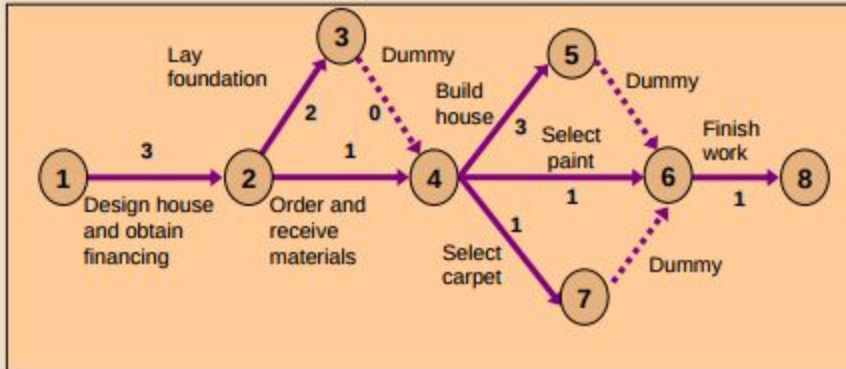


A:	1-2-4-7 $3 + 2 + 3 + 1 = 9$ months
B:	1-2-5-6-7 $3 + 2 + 1 + 1 + 1 = 8$ months
C:	1-3-4-7 $3 + 1 + 3 + 1 = 8$ months
D:	1-3-5-6-7 $3 + 1 + 1 + 1 + 1 = 7$ months

- ◆ **Critical path**
 - Longest path through a network
 - Minimum project completion time

- Activity-on-arrow (AOA): arrows represent activities, nodes are events or points in time.
Used with **PERT**

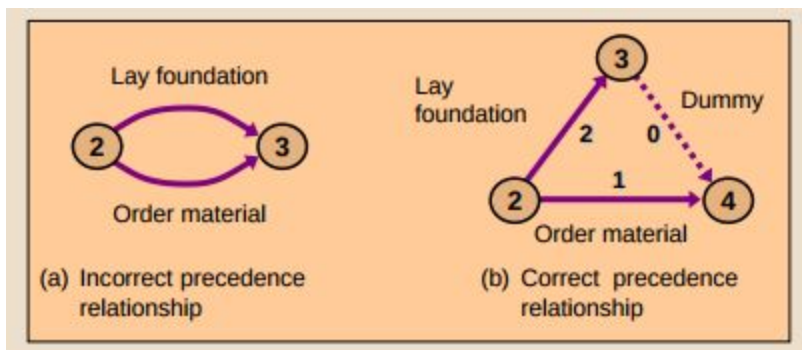
AOA Project Network for the modified example



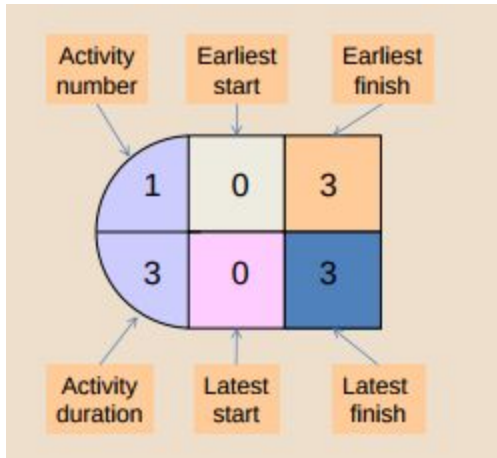
- Event: completion or beginning an activity in a project

Concurrent activities:

- Dummy activity: inserted into network to show a precedence relationship
 - Two or more activities cannot share the same start and end nodes
 - Indicates no actual passage of time (duration = 0)



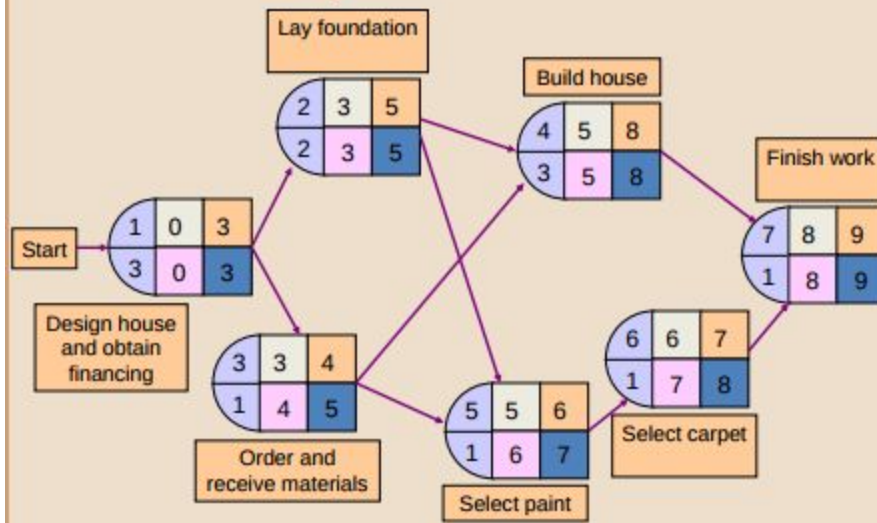
Node configuration



Activity scheduling

- Earliest state time (ES): Maximum time in which all preceding activities have been completed → the time when the activity start node is realized
 - **ES = Maximum (EF) of immediate predecessors**
 - ES for 1st activity is always = 0 + t
- Forward pass: start at beginning of CPM/PERT network to determine earliest activity times (start at first node and move forward through the network)
- Earliest finish time (EF): earliest time an activity can finish
 - **EF = ES + t (activity time)**
- Latest start time (LS): latest time an activity can start without delaying critical path time
 - **LS = LF - t**
- Latest finish time (LF): latest time an activity can be completed without delaying critical path time
 - **LF = min LS of immediate following activities**
- Backward pass: determines latest activity times by starting at the end of CPM/PERT network and working forward

Latest Activity Start and Finish Times



Activity Slack

Activity	LS	ES	LF	EF	Slack S
*1	0	0	3	3	0
*2	3	3	5	5	0
3	4	3	5	4	1
*4	5	5	8	8	0
5	6	5	7	6	1
6	7	6	8	7	1
*7	8	8	9	9	0

* Critical Path

$$\text{Slack} = \text{LS} - \text{ES} = \text{LF} - \text{EF}$$

Probabilistic time estimates

- Beta distribution: probability distribution traditionally used in CPM/PERT

Mean (expected time): $t = \frac{a + 4m + b}{6}$

Variance: $\sigma^2 = \left(\frac{b - a}{6} \right)^2$

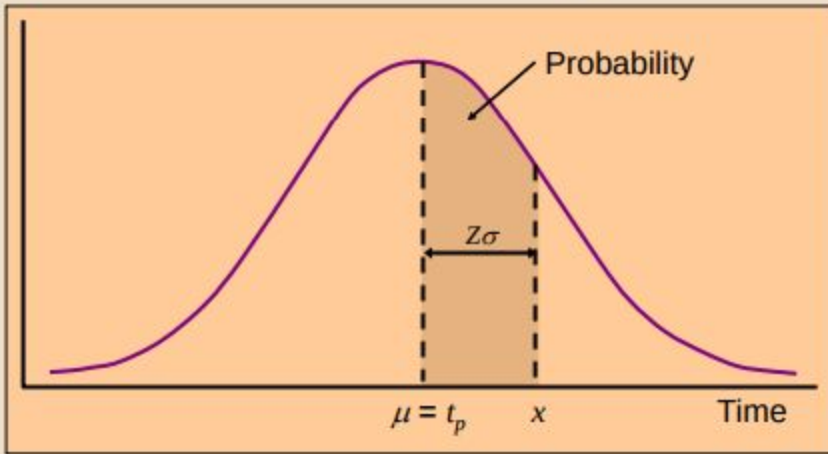
where

a = optimistic estimate

m = most likely time estimate

b = pessimistic time estimate

Normal Distribution of Project Time



Probabilistic Network Analysis

Determine probability that project is completed within specified time

where

$$Z = \frac{x - \mu}{\sigma}$$

$\mu = t_p$ = project mean completion time

σ = project standard deviation

x = proposed project completion time
(x is given in the question)

Z = number of standard deviations that x is from the mean

Project crashing:

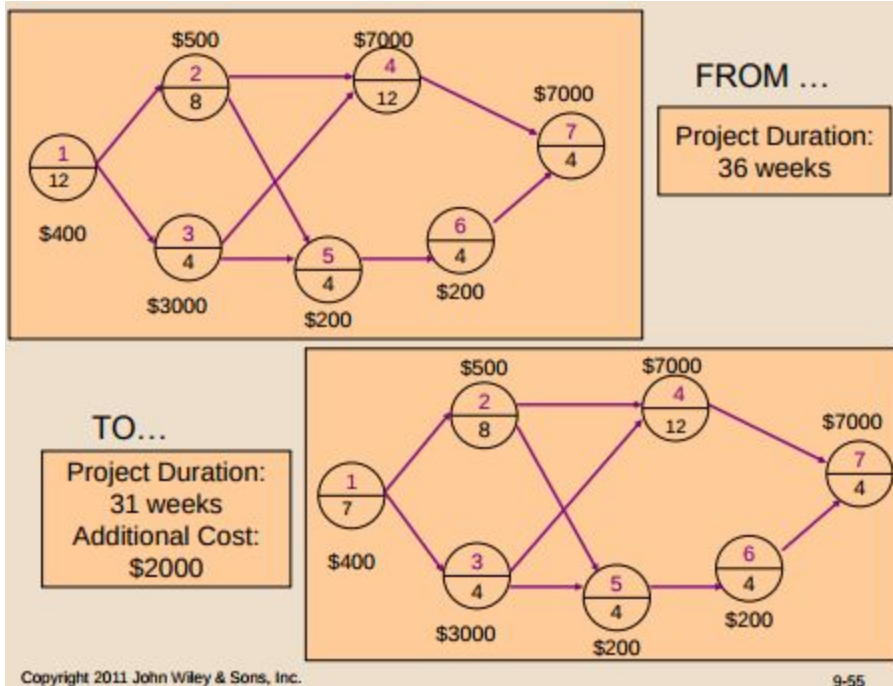
- Crashing: reducing project time by expending additional resources → goal is to reduce project duration at minimum cost
- Crash time: the reduced duration for performing an activity (achieved through the use of additional resources)
- Crash cost: cost of reducing activity time

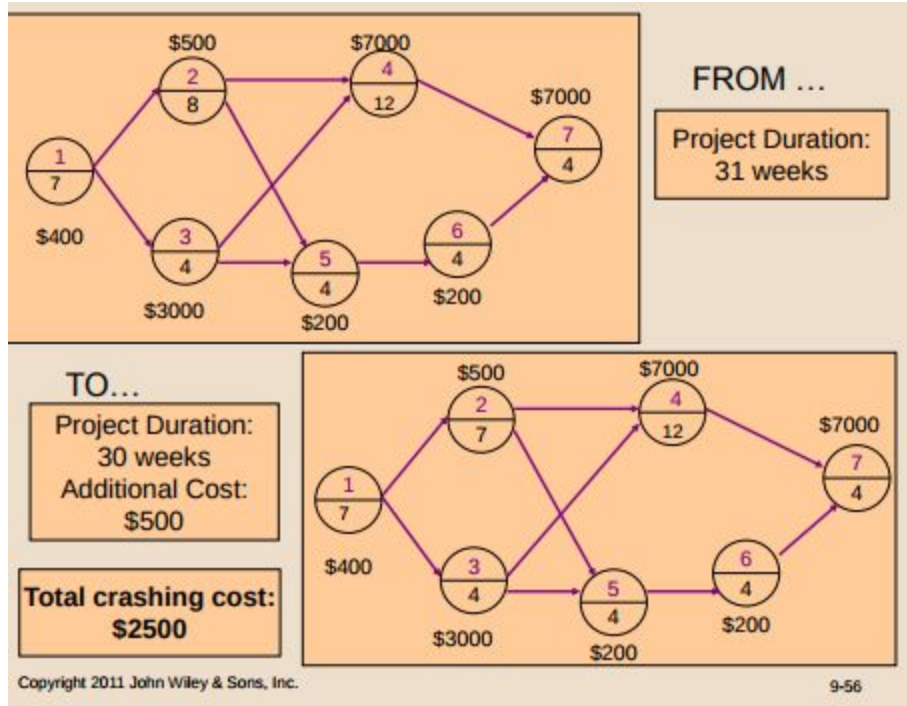
- Goal: reduce project duration at minimum cost

Project Crashing

ACTIVITY	NORMAL TIME (WEEKS)	CRASH TIME (WEEKS)	NORMAL COST	CRASH COST	TOTAL ALLOWABLE CRASH TIME (WEEKS)	CRASH COST PER WEEK
1	12	7	\$3,000	\$5,000	5	\$400
2	8	5	2,000	3,500	3	500
3	4	3	4,000	7,000	1	3,000
4	12	9	50,000	71,000	3	7,000
5	4	1	500	1,100	3	200
6	4	1	500	1,100	3	200
7	4	3	15,000	22,000	1	7,000
			<u>\$75,000</u>	<u>\$110,700</u>		

The home builder needs to have the house completed in 30 weeks. Which activities to crash and what is the associated extra cost?





Time-cost relationship:

- Crashing costs increase as project duration decreases
- Indirect costs increase as project duration increases
- Reduce project length as long as crashing costs are less than indirect costs

Chapter 12

Forecasting: a prediction of what will happen in the future. Forecasting of product demand is the basis for most important planning decisions.

- Uncertain process → long-term forecasts are usually less accurate than short-term forecasts

Qualitative forecast methods: subjective (based on judgement opinion, past experience)

Quantitative forecast methods: based on mathematical formulas

Quality management: accurately forecasting customer demand is a key to providing good quality service (e.g.: mcdonalds)

Strategic planning:

- Successful strategic planning requires accurate forecasts of future products and markets.

Forecasts include:

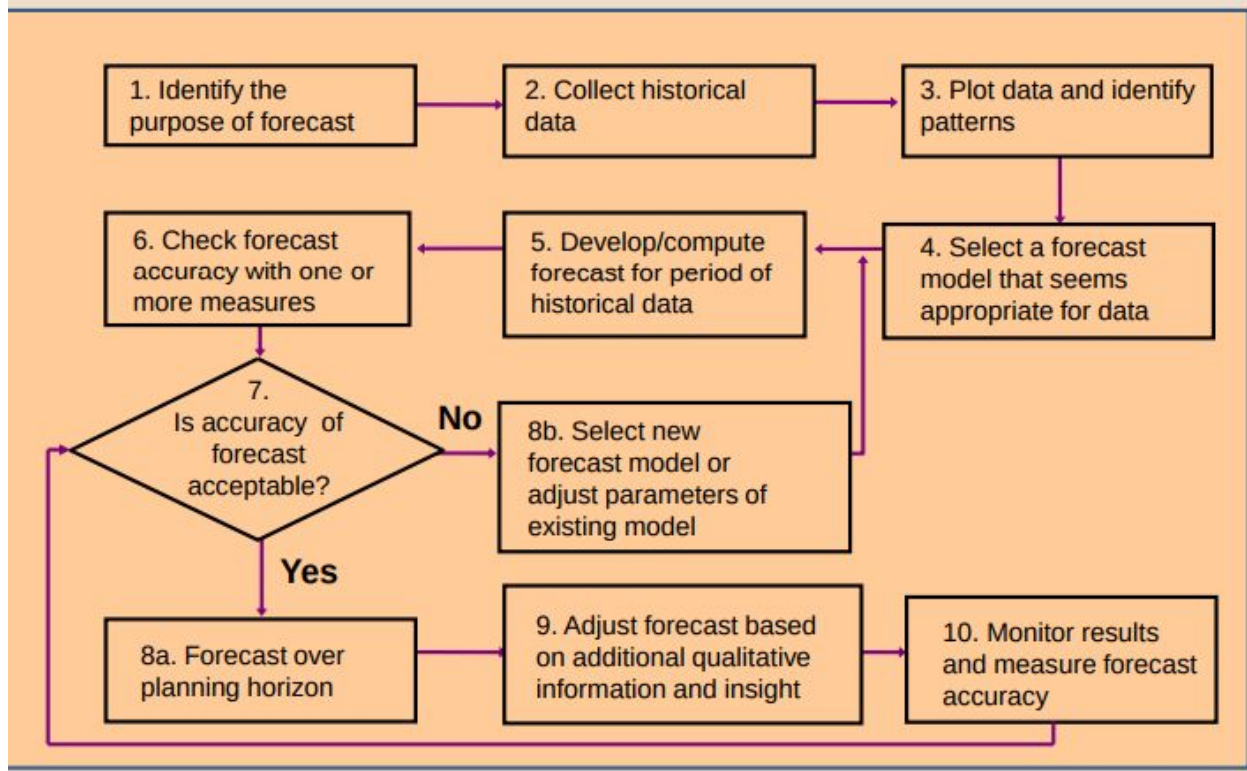
- What new products customers want
- How much of these new products the customers will want
- The level of quality and other features expected in these products

Types of forecasting methods:

1. Time series method: statistical techniques that use historical demand data to predict future demand
 - Time series: a collection of observations taken at successive points in time spaced at uniform time intervals (daily, weekly, monthly, yearly)
 - Most popular for short-term forecasting
 - Assume that what happened in the past will continue in the future
 - Relate forecast to only 1 factor, time
 - Include: moving average, exponential smoothing, linear trend line
2. Regression methods (casual): attempt to develop a mathematical relationship between demand and variables that cause its behavior
 - Linear regression: mathematical technique that relates a dependent variable to an independent variable in the form of a linear equation → $y = a + bx$
 - Correlation: a measure of the strength of the relationship between independent and dependent variable → “**r**”
 - $r \pm 1 = \text{strong relationship}$
 - $- = \text{strong negative}$ & $+ = \text{strong positive}$
 - $0.5 \leq r < 1 = \text{medium relationship}$
 - $r < 0.5 = \text{weak relationship}$

- Coefficient of determination: the % of variation in the dependent variable that results from the independent variable → “ r^2 ”
 - Multiple regression: a relationship of demand to two or more independent variables → $y = B_0 + B_1X_1 + B_2X_2\dots$
3. Qualitative: use mgmt judgement, expertise and opinion to predict future demand
- Most common type for the long-term strategic planning process
 - Management, marketing, purchasing and engineering are sources for internal qualitative forecasts
 - Delphi method:
 - Developed at the Rand Corporation after WWII to forecast the impact of a hypothetical nuclear attack on the US
 - Procedure for acquiring informed judgment and opinions from knowledgeable individuals using a series of questionnaires to develop a consensus about what will happen in the future. Involves soliciting forecasts about technological advances from experts

Forecasting Process



Types of forecasting methods depends on:

- Time frame (how far in the future is being forecasted)
 - Short to mid-range forecast
 - Encompasses immediate future
 - Daily up to 2 years
 - Determines production and delivery schedule and inventory levels
 - Long-range forecast
 - Encompasses a period of time longer than 2 years
 - Used for planning new products, entering new markets, constructing new facilities, developing technologies
- Demand behavior and possible existence of patterns:
 - The various types of demand behavior are:

- Levels: a horizontal pattern of time series, where the data can fluctuate (randomly) around a constant average value (e.g.: gas)
 - Trends: a gradual, long-term up or down movement of demand
 - Random variations: movements in demand that do not follow a pattern
 - Cycles: an up and down movement in demand. E.g.: new housing starts and construction related products tend to follow cycles in the economy
 - Seasonal patterns: an up and down repetitive movement in demand occurring periodically. E.g.: during the winter, snow blowers increase in price
 - Autocorrelation: the correlation among values of observed data separated by a fixed number of periods.
 - Irregular movements: instances when demand behavior exhibits no pattern. E.g.: local flood will cause a momentary increase in carpet demand
- Causes of demand behavior

Supply chain management: encompasses all of the facilities, functions, and activities involved in producing a product/service from suppliers to customers

- Includes:
 - Purchasing, inventory, production, scheduling, facility location, transportation and distribution which are all affected by product demand (short-term) & new products and processes, technology advances, changing markets (long-term)
- Forecasts of product demands determine:
 - How much inventory is needed
 - How much product to make
 - How much material to purchase from suppliers
- Bad forecasting affects supply chain by:
 - Over-forecast: over-production & excess inventory → Spoilage, waste, obsolescence, high inventory costs

- Under-forecast: under-production & inventory shortages → Late deliveries, stock out and shortage costs, high shipping costs
- *Bullwhip effect*: the distortion of information about product demand (including forecasts) as it is transmitted up the supply chain back toward suppliers, caused when slight demand variability is magnified as information moves back upstream in the supply chain
- Continuous replenishment:
 - Supplier and customer share continuously updated data
 - Typically managed by supplier
 - Reduces inventory for company
 - Speeds customer delivery
- Variations of continuous replenishment
 - Quick response
 - JIT (Just-in-time)
 - VMI (vendor-managed inventory)
 - Stockless inventory

Demand behavior:

- Trend: a gradual, long-term up or down movement of demand
- Random variations: movements in demand that do not follow a pattern
- Cycle: up and down movement in demand that repeats itself over a lengthy time period (more than 1 year)
- Seasonal pattern: an oscillating movement in demand that occurs periodically (in short run) and is repetitive

Moving average: uses average demand for a fixed sequence of periods (3 or 5 month)

depending on how much the forecaster wants to “smooth” the demand data → longer moving average period = smoother

- Naive (intuitive) forecast: demand in current period is used as next period’s forecast

- Does not take into account historical demand behavior, it relies on demand in the current period
- Simple moving average: uses average demand for a fixed sequence of periods. Stable demand with no pronounced behavioral patterns
 - dampen/smooth out: the random increases and decreases of a forecast that uses only one period →the longer the moving average is, the smoother it will be and vice versa
 - Useful for forecasting demand that is stable and does not display any pronounced demand behavior (trend and seasonal pattern)
- Pros/cons:
 - Disadvantage of using moving average method: it does not react to variations that occur for a reason (cycles, seasonal effects)
 - Advantage of using moving average method: simple to use, inexpensive, quick, generally useful for short-run (but not for long-run)

5-month Simple Moving Average

MONTH	ORDERS PER MONTH	MOVING AVERAGE
Jan	120	-
Feb	90	-
Mar	100	-
Apr	75	-
May	110	-
June	50	99.0
July	75	85.0
Aug	130	82.0
Sept	110	88.0
Oct	90	95.0
Nov	-	91.0

$$MA_5 = \frac{\sum_{i=1}^5 D_i}{5}$$

$$= \frac{90 + 110 + 130 + 75 + 50}{5}$$

= 91 orders for Nov

3-month Simple Moving Average

MONTH	ORDERS PER MONTH	MOVING AVERAGE
Jan	120	-
Feb	90	-
Mar	100	-
Apr	75	103.3
May	110	88.3
June	50	95.0
July	75	78.3
Aug	130	78.3
Sept	110	85.0
Oct	90	105.0
Nov	-	110.0

$$MA_3 = \frac{\sum_{i=1}^3 D_i}{3}$$

$$= \frac{90 + 110 + 130}{3}$$

= 110 orders for Nov

Simple Moving Average

$$MA_n = \frac{\sum_{i=1}^n D_i}{n}$$

where

n = number of periods in
the moving average

D_i = demand in period i

Moving Average: Naive Approach

MONTH	ORDERS	
	PER MONTH	FORECAST
Jan	120	-
Feb	90	120
Mar	100	90
Apr	75	100
May	110	75
June	50	110
July	75	50
Aug	130	75
Sept	110	130
Oct	90	110
Nov	-	90

- Weighted moving average: weights are assigned to most recent data. Usually higher weights are assigned to the more recent data points

$$WMA_n = \sum_{i=1}^n W_i D_i$$

where

W_i = the weight for period i ,
between 0 and 100
percent

$$\sum W_i = 1.00$$

Weighted Moving Average Example

<i>MONTH</i>	<i>WEIGHT</i>	<i>DATA</i>
<i>August</i>	17%	130
<i>September</i>	33%	110
<i>October</i>	50%	90
		3
November Forecast	$WMA_3 = \sum_{i=1}^3 W_i D_i$	
	$= (0.50)(90) + (0.33)(110) + (0.17)(130)$	
	$= 103.4$ orders	

Exponential smoothing: averaging method that reacts more strongly to recent changes in demand

- Averaging method
- Weighs most recent data more strongly
- Forecasts reacts more the recent changes in demand
- Widely used, accurate method
- Requires minimal data:
 - Forecast for the current period
 - Actual demand for the current period
 - A weighting factor called “smoothing constant” (α) has a value between 0-1
 - The closer α is to 0, the less it reflects most recent demand, smoothing & dampening will be more
 - The closer α is to 1, the more is reflects most recent demand, smoothing & dampening will be less

Exponential Smoothing

$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t$$

where:

F_{t+1} = Forecast for next period

D_t = Actual demand for present period

F_t = Previously determined forecast for present period

α = Weighting factor, smoothing constant

Effect of Smoothing Constant

$$0.0 \leq \alpha \leq 1.0$$

$$\text{If } \alpha = 0.20, \text{ then } F_{t+1} = 0.20 D_t + 0.80 F_t$$

$$\text{If } \alpha = 0, \text{ then } F_{t+1} = 0 D_t + 1 F_t = F_t$$

Forecast does not reflect recent data

$$\text{If } \alpha = 1, \text{ then } F_{t+1} = 1 D_t + 0 F_t = D_t$$

Forecast based only on most recent data

The higher α is, the more responsive the forecast is to recent changes in demand & less smoothing

Exponential Smoothing ($\alpha=0.30$)

<u>PERIOD</u>	<u>MONTH</u>	<u>DEMAND</u>
---------------	--------------	---------------

1	Jan	37
2	Feb	40
3	Mar	41
4	Apr	37
5	May	45
6	Jun	50
7	Jul	43
8	Aug	47
9	Sep	56
10	Oct	52
11	Nov	55
12	Dec	54

Assume: $F_1 = D_1 = 37$

$$\begin{aligned}F_2 &= \alpha D_1 + (1 - \alpha)F_1 \\ &= (0.30)(37) + (0.70)(37) \\ &= 37\end{aligned}$$

$$\begin{aligned}F_3 &= \alpha D_2 + (1 - \alpha)F_2 \\ &= (0.30)(40) + (0.70)(37) \\ &= 37.9\end{aligned}$$

$$\begin{aligned}F_{13} &= \alpha D_{12} + (1 - \alpha)F_{12} \\ &= (0.30)(54) + (0.70)(50.84) \\ &= 51.79\end{aligned}$$

Exponential Smoothing

PERIOD	MONTH	DEMAND	FORECAST, F_{t+1}	
			($\alpha = 0.3$)	($\alpha = 0.5$)
1	Jan	37	–	–
2	Feb	40	37.00	37.00
3	Mar	41	37.90	38.50
4	Apr	37	38.83	39.75
5	May	45	38.28	38.37
6	Jun	50	40.29	41.68
7	Jul	43	43.21	45.84
8	Aug	47	43.14	44.42
9	Sep	56	44.30	45.71
10	Oct	52	47.81	50.85
11	Nov	55	49.06	51.42
12	Dec	54	50.84	53.21
13	Jan	–	51.79	53.61

Adjusted Exponential Smoothing

Used when there exists an upward or downward trend in the data

$$AF_{t+1} = F_{t+1} + T_{t+1}$$



Obtained previously from exponential smoothing

where

T = an exponentially smoothed trend factor

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta) T_t$$

where

T_t = the last period trend factor

β = a smoothing constant for trend

$$0 \leq \beta \leq 1$$

Higher β values better reflect changes in trend

Adjusted Exponential Smoothing ($\beta=0.30$)

PERIOD	MONTH	DEMAND	FORECAST	<i>Assume: $T_2 = 0$</i>
1	Jan	37	37.00	
2	Feb	40	37.00	
3	Mar	41	38.50	$T_3 = \beta(F_3 - F_2) + (1 - \beta) T_2$
4	Apr	37	39.75	$= (0.30)(38.5 - 37.0) +$
5	May	45	38.37	$(0.70)(0)$
6	Jun	50	41.68	$= 0.45$
7	Jul	43	45.84	$AF_3 = F_3 + T_3 = 38.5 + 0.45$
8	Aug	47	44.42	$= 38.95$
9	Sep	56	45.71	$T_{13} = \beta(F_{13} - F_{12}) + (1 - \beta) T_{12}$
10	Oct	52	50.85	$= (0.30)(53.61 - 53.21) +$
11	Nov	55	51.42	$(0.70)(1.77)$
12	Dec	54	53.21	$= 1.36$
13	Jan	-	53.61	$AF_{13} = F_{13} + T_{13} = 53.61 +$
				$1.36 = 54.97$

Adjusted Exponential Smoothing

PERIOD	MONTH	DEMAND	FORECAST F_{t+1}	TREND T_{t+1}	ADJUSTED FORECAST AF_{t+1}
1	Jan	37	37.00	–	–
2	Feb	40	37.00	0.00	37.00
3	Mar	41	38.50	0.45	38.95
4	Apr	37	39.75	0.69	40.44
5	May	45	38.37	0.07	38.44
6	Jun	50	41.68	1.04	42.72
7	Jul	43	45.84	1.97	47.81
8	Aug	47	44.42	0.95	45.37
9	Sep	56	45.71	1.05	46.76
10	Oct	52	50.85	2.28	53.13
11	Nov	55	51.42	1.76	53.18
12	Dec	54	53.21	1.77	54.98
13	Jan	–	53.61	1.36	54.97

Linear Trend Line

Also used when there exists an upward or downward trend in the data

$$y = a + bx$$

where

a = intercept

b = slope of the line

x = time period

y = forecast for demand for period x

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

where

n = number of periods

$$\bar{x} = \frac{\sum x}{n} = \text{mean of the } x \text{ values}$$

$$\bar{y} = \frac{\sum y}{n} = \text{mean of the } y \text{ values}$$

Least Squares Example

x (PERIOD)	y (DEMAND)	xy	x^2
1	37	37	1
2	40	80	4
3	41	123	9
4	37	148	16
5	45	225	25
6	50	300	36
7	43	301	49
8	47	376	64
9	56	504	81
10	52	520	100
11	55	605	121
12	54	648	144
Total	78	557	650

Least Squares Example

$$\bar{x} = \frac{78}{12} = 6.5$$

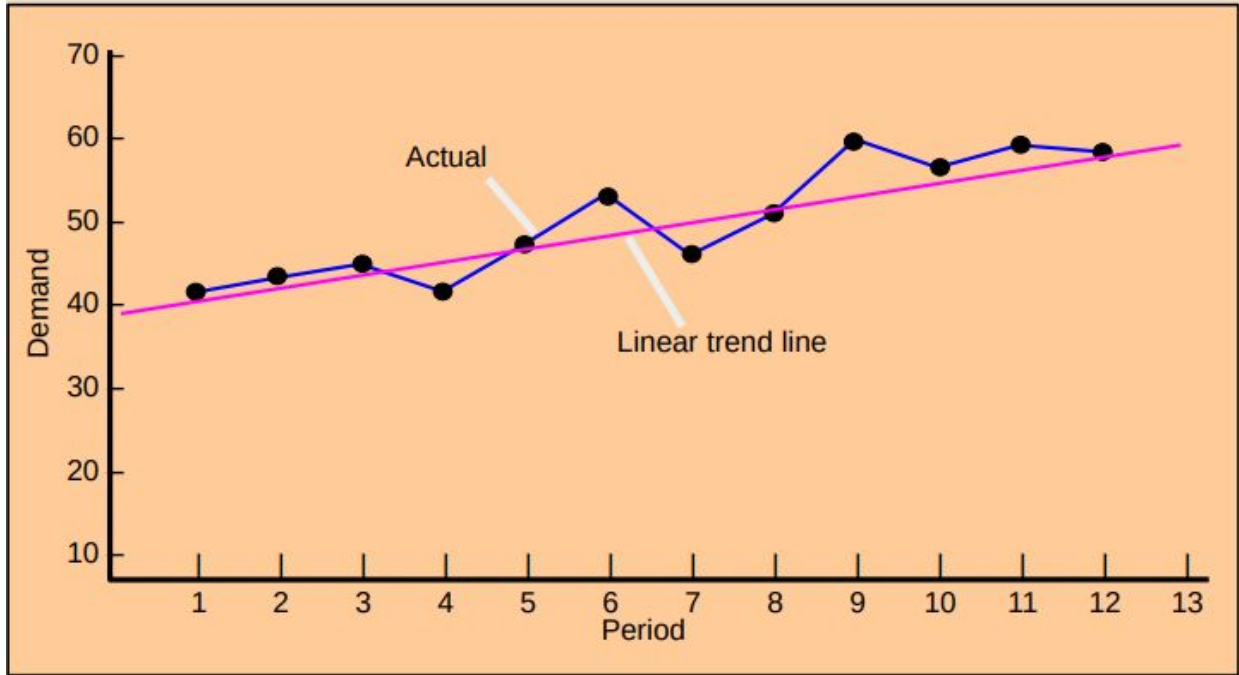
$$\bar{y} = \frac{557}{12} = 46.42$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3867 - (12)(6.5)(46.42)}{650 - 12(6.5)^2} = 1.72$$

$$\begin{aligned} a &= \bar{y} - b\bar{x} \\ &= 46.42 - (1.72)(6.5) = 35.2 \end{aligned}$$

Linear trend line $y = 35.2 + 1.72x$

Forecast for period 13: $y = 35.2 + 1.72(13) = 57.56$ units



Seasonal adjustments:

- Repetitive increase/decrease in demand
- Examples: clothing, toys, sports equipment, electronic appliances
- Seasonal patterns can occur on a daily, weekly, monthly basis
- *Seasonal factor*: adjust for seasonality by multiplying the normal forecast by a seasonal factor
- Use it to get seasonally adjusted forecasts from normal forecasts

$$\text{Seasonal factor} = S_i = \frac{D_i}{\sum D}$$

Seasonal Adjustment

DEMAND (1000'S PER QUARTER)					
YEAR	1	2	3	4	Total
2002	12.6	8.6	6.3	17.5	45.0
2003	14.1	10.3	7.5	18.2	50.1
2004	15.3	10.6	8.1	19.6	53.6
Total	42.0	29.5	21.9	55.3	148.7

$$S_1 = \frac{D_1}{\sum D} = \frac{42.0}{148.7} = 0.28$$

$$S_3 = \frac{D_3}{\sum D} = \frac{21.9}{148.7} = 0.15$$

$$S_2 = \frac{D_2}{\sum D} = \frac{29.5}{148.7} = 0.20$$

$$S_4 = \frac{D_4}{\sum D} = \frac{55.3}{148.7} = 0.37$$

Seasonal Adjustment

For 2005

$$y = 40.97 + 4.30x = 40.97 + 4.30(4) = 58.17$$

$$SF_1 = (S_1) (F_5) = (0.28)(58.17) = 16.28$$

$$SF_2 = (S_2) (F_5) = (0.20)(58.17) = 11.63$$

$$SF_3 = (S_3) (F_5) = (0.15)(58.17) = 8.73$$

$$SF_4 = (S_4) (F_5) = (0.37)(58.17) = 21.53$$

Forecast accuracy:

- Forecast error: difference between forecast and actual demand
 - F.E = $e_t = D_t - F_t$
- MAD: mean absolute deviation
 - Smaller MAD (relative to data) = more accurate forecast
 - Larger MAD = less accurate forecast
- MAPD: mean absolute percent deviation → measures the absolute error as a percentage of demand rather than per period
 - Lower MAPD = more accurate forecast
- MSE: mean squared deviation → square of the forecast error → $MSE = \frac{\sum(D_t - F_t)^2}{n}$
 - Lower MSE = more accurate forecast
- Cumulative error: the sum of forecast errors → $E = \sum e_t$

- Large, positive value = forecast is likely lower than actual demand or is biased low
- Large, negative value = forecast is likely higher than actual demand or is biased high
- Average error or bias: the per-period average of cumulative error; computed by averaging

cumulative error over the number of time periods: $\bar{E} = \frac{\sum e_t}{n}$

- Positive value = low bias
- Negative value = high bias
- Value close to 0 = lack of bias

Mean Absolute Deviation (MAD)

$$\text{MAD} = \frac{\sum |D_t - F_t|}{n}$$

where

t = period number

D_t = demand in period t

F_t = forecast for period t

n = total number of periods

$| |$ = absolute value

MAD Example

PERIOD	DEMAND, D_t	F_t ($\alpha = 0.3$)	$(D_t - F_t)$	$ D_t - F_t $
1	37	37.00	-	-
2	40	37.00	3.00	3.00
3	41	37.90	3.10	3.10
4	37	38.83	-1.83	1.83
5	45	38.28	6.72	6.72
6	50	40.29	9.69	9.69
7	43	43.20	-0.20	0.20
8	47	43.14	3.86	3.86
9	56	44.30	11.70	11.70
10	52	47.81	4.19	4.19
11	55	49.06	5.94	5.94
12	54	50.84	3.15	3.15
	<u>557</u>		<u>49.31</u>	<u>53.39</u>

MAD Calculation

$$\begin{aligned} \text{MAD} &= \frac{\sum |D_t - F_t|}{n} \\ &= \frac{53.39}{11} \\ &= 4.85 \end{aligned}$$

The lower the value of MAD, relative to the magnitude of the data, the more accurate the forecast.

Other Accuracy Measures

Mean absolute percent deviation (MAPD)

$$MAPD = \frac{\sum |D_t - F_t|}{\sum D_t}$$

Cumulative error

$$E = \sum e_t$$

Average error

$$\bar{E} = \frac{\sum e_t}{n}$$

Comparison of Forecasts

FORECAST	MAD	MAPD	E	\bar{E}
Exponential smoothing ($\alpha = 0.30$)	4.85	9.6%	49.31	4.48
Exponential smoothing ($\alpha = 0.50$)	4.04	8.5%	33.21	3.02
Adjusted exponential smoothing ($\alpha = 0.50, \beta = 0.30$)	3.81	7.5%	21.14	1.92
Linear trend line	2.29	4.9%	–	–

Forecast control:

- Tracking signal: monitors the forecast to see if it is biased high or low
- Statistical control charts

- 1 MAD \approx 0.8 σ
- Control limits of 2 to 5 MADs are used most frequently

$$\text{Tracking signal} = \frac{\sum(D_t - F_t)}{\text{MAD}} = \frac{E}{\text{MAD}}$$

Note: Tracking signal is recomputed each period, with updated “running” values of E and MAD .

Tracking Signal Values

PERIOD	DEMAND D_t	FORECAST, F_t	ERROR $D_t - F_t$	$\sum E =$ $\sum(D_t - F_t)$	MAD	TRACKING SIGNAL
1	37	37.00	-	-	-	-
2	40	37.00	3.00	3.00	3.00	1.00
3	41	37.90	3.10	6.10	3.05	2.00
4	37	38.83	-1.83	4.27	2.64	1.62
5	45	38.28	6.72	10.99	3.66	3.00
6	50	40.29	9.69	20.68	4.87	4.25
7	43	43.20	-0.20	20.48	4.09	5.01
8	47	43.14	3.86	24.34	4.06	6.00
9	56	44.30	11.70	36.04	5.01	7.19
10	52	47.81	4.19	40.23	4.92	8.18
11	55	49.06	5.94	46.17	5.02	9.20
12	54	50.84	3.15	49.32	4.85	10.17

$$TS_3 = \frac{6.10}{3.05} = 2.00$$

Statistical Control Charts

$$\sigma = \sqrt{\frac{\sum(D_t - F_t)^2}{n - 1}}$$

- Using σ we can calculate statistical control limits for the forecast error
- Control limits are typically set at $\pm 3\sigma$

Time series forecasting using excel:

- Excel can be used to develop forecast:
 - Moving average
 - Exponential smoothing
 - Adjusted exponential smoothing: an exponential smoothing forecast adjusted for trend
 - Beta (α): is a value between 0-1; reflects weight given to the most recent trend data. Usually determined subjectively based on the judgement of the forecaster
 - High α = more trend changes than low α
 - Linear trend line: a linear regression model relating demand (dependent variable) to time (independent variable) → used when demand displays an obvious trend over time: $y = a + bx$

SUMMARY OF KEY FORMULAS

Moving Average

$$MA_n = \frac{\sum_{i=1}^n D_i}{n}$$

Weighted Moving Average

$$WMA_n = \sum_{i=1}^n W_i D_i$$

Exponential Smoothing

$$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$$

Adjusted Exponential Smoothing

$$AF_{t+1} = F_{t+1} + T_{t+1}$$

Trend Factor

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)T_t$$

Mean Absolute Deviation

$$MAD = \frac{\sum |D_t - F_t|}{n}$$

Mean Absolute Percent Deviation

$$MAPD = \frac{\sum |D_t - F_t|}{\sum D_t}$$

Mean Squared Error

$$MSE = \frac{\sum (D_t - F_t)^2}{n - 1}$$

Cumulative Error

$$E = \sum e_t$$

Average Error (Bias)

$$\bar{E} = \frac{\sum e_t}{n}$$

Trend Factor

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)T_t$$

Linear Trend Line

$$y = a + bx$$

Least Squares

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

Seasonal Factor

$$S_i = \frac{D_i}{\sum D}$$

Seasonally Adjusted Forecast

$$SF_t = (S_t)(F_t)$$

Average Error (Bias)

$$E = \sum e_t$$

$$\bar{E} = \frac{\sum e_t}{n}$$

Tracking Signal

$$TS = \frac{\sum (D_t - F_t)}{MAD} = \frac{E}{MAD}$$

Linear Regression Equation

$$y = a + bx$$

Correlation Coefficient

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

Coefficient of Determination

$$\text{Coefficient of determination} = r^2$$

Chapter 13

Role of inventory in Supply Chain Management

- U.S companies carry \$1.1 trillion dollar in inventory spread out along the supply chain
- Cost of holding inventory includes
 - Insurance
 - Obsolescence
 - Depreciation
 - Interest
 - Opportunity cost
 - Storage cost
- Why to hold inventories?
 - Meet customer demand for a product (safety or buffer stock)
 - Meet demand that is seasonal/cyclical. E.g.: toy manufacturers produce large inventories during summer and fall in anticipation for the holidays
 - Stocks of parts and raw materials is kept to meet variations in supplier deliveries
 - Take advantage of price discount or as a hedge against anticipated price increase. E.g.: Walmart will purchase an entire supplier's soap stock because of low pricing
 - The cost of ordering might be so high
 - Within a facility, to provide independence between production stages and to avoid work stoppages
- Many emerging practices in inventory management is due in large part to advances in information technology (IT), all of which enable companies to track and locate inventory throughout supply chain. Examples of IT advances:
 - Enterprise resource planning systems (ERP)
 - Bar codes
 - Radio frequency identification (RFID)
 - Point of sales data

- Examples of successful SCM Practices where inventory can be located at supplier's facility, at the buyer's or anywhere in between:
 - Vendor managed inventory (VMI)
 - Continuous replenishment program (CRP)
 - Supplier hubs
 - Third party logistics (3PL)
 - Supplier hubs used by: Dell, Apple, Fiat, Hewlett-Packard, Nokia, Cisco, sam's club, samsung, volkswagen
- Advantages of modern Supply chain management practices:
 - Supplier owns inventory until buyer needs it, relieving buyer of inventory costs
 - Order sizes are reduced
 - Deliveries (supplier pays for) are increased
 - Buyer avoids maintaining storage facilities
- Bullwhip effect:
 - Demand information is distorted as it moves away from the end-use customer
 - Higher safety stock inventories are stores to compensate
- Level of customer service: ability to effectively meet internal organizational demand or external customer demand in a timely, efficient manner
 - As level of inventory **increases** (to provide better customer service) so does inventory cost **increase** and loss of sales & customers **decreases**
 - *Zero defects*: philosophy of quality management that states long-term benefits of quality in terms of larger market share outweigh lower short-term production-related costs (inventory costs). Hard to apply this principle because goal of business is to reduce prices through reduced inventory costs

What is inventory?

- Inventory: Stock of items kept to meet future demand
- Purpose of inventory management:
 - How many units to order

- When to order (when to replenish)
- Types of inventories:
 - Raw materials
 - Purchases parts and suppliers
 - Work-in-process (partially completed) products (WIP)
 - Items being transported
 - Tools and equipment

Demand:

Two forms of demand:

1. Dependent:
 - Demand for items used to produce final products
 - Tires for autos
 - Demand of one item depends on demand for another item
2. Independent:
 - Demand for items used by external customers
 - Cars, appliances, computers and houses
 - Demand depends on external market conditions

Costs:

Inventory costs: carrying, ordering and shortage costs

- Carrying (holding cost)
 - Cost of holding an item in inventory
 - As inventory **increases**, carrying costs **increases** (positive relationship)
 - Expressed as:
 - Dollar value/unit basis per time period
 - % of the value of an item (usually 10-40%)
 - Carrying cost and order cost have an inverse relationship & **total cost combines both of them**

- Includes:
 - Facility storage (rent, depreciation, power, heat, cooling, lighting, security, refrigeration, taxes, insurance)
 - Material handling (equipment)
 - Labour
 - Record keeping
 - Borrowing to purchase inventory (interest on loans, taxes)
 - Product deterioration, spoilage, breakage, obsolescence, pilferage
- Ordering cost
 - Cost of replenishing inventory
 - Expressed as a dollar amount/order (independent of order size)
 - Includes:
 - Requisition and purchase orders
 - Transportation and shipping
 - Receiving
 - Inspection
 - Handling
 - Accounting and auditing costs
 - Reacts inversely to carrying costs: as order size **increases**, ordering costs **decrease** and carrying costs **increase**
- Shortage (stockout) cost:
 - Temporary or permanent loss of sales when demand cannot be met
 - Possible backorder costs, loss of reputations, loss of future sales
 - Shortage costs have an inverse relationship to carrying costs: as the amount of inventory **increases**, the carrying cost **increases**, and shortage costs **decrease**

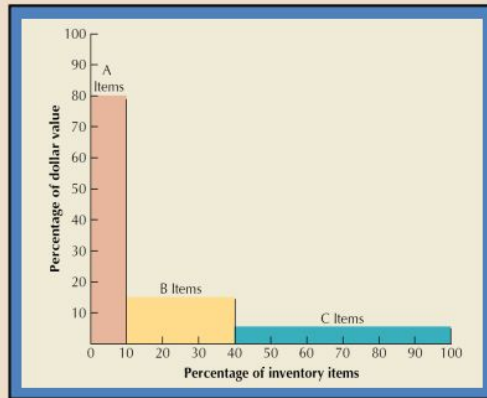
Inventory control systems: controls the level of inventory by determining how much to order (level of replenishment_ and when to order. 2 basic types: (1) continuous & (2) periodic

- Continuous system (fixed-order quantity): constant amount ordered when inventory declines to predetermined level (known as the *reorder point*), a new order is placed to replenish stock of inventory. E.g.: checkbooks
 - The amount ordered is known as the *economic order quantity (EOQ)*. Also known as *perpetual system* and *fixed-order quantity system*
 - Pros: inventory level is continuously monitored so management always knows the inventory status, good for critical items such as replacement parts, raw materials, supplies
 - Cons: maintaining a continuous record of amount of inventory can be costly
- Periodic system (fixed-time period): order placed for variable amount after fixed passage of time. E.g.: ordering textbooks according to course enrollments
 - Also known as *fixed-time period system* or *periodic review system*)
 - Pros: no required record keeping
 - Cons: less direct control (larger inventory levels vs. continuous system)
- *Safety stock*: use it when demand exceeds inventory

ABC classification: inventory classification system in which a small % of A items accounts for most on the inventory value

- The higher the value of the inventory, the tighter the control
- Original rationale for ABC analysis: continuous inventory monitoring was expensive and not justified for many items
- Steps in ABC analysis:
 - Classify all inventory items as either A, B or C
 - Determine level of inventory control for each classification

- **Class A**
 - 5 – 15 % of units
 - 70 – 80 % of value
- **Class B**
 - 30 % of units
 - 15 % of value
- **Class C**
 - 50 – 60 % of units
 - 5 – 10 % of value



PART	UNIT COST	ANNUAL USAGE
1	\$ 60	90
2	350	40
3	30	130
4	80	60
5	30	100
6	20	180
7	10	170
8	320	50
9	510	60
10	20	120
		1050

PART	TOTAL VALUE	% OF TOTAL VALUE	% OF TOTAL QUANTITY	% CUMMULATIVE
9	\$30,600	35.9	6.0	6.0
8	16,000	18.7	5.0	11.0
2	14,000	16.4	4.0	15.0
1	5,400	6.3	9.0	24.0
4	4,800	5.6	6.0	30.0
3	3,900	4.6	13.0	43.0
6	3,600	4.2	18.0	61.0
5	3,000	3.5	10.0	71.0
10	2,400	2.8	12.0	83.0
7	1,700	2.0	17.0	100.0
	\$85,400			

CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
A	9, 8, 2	71.0	15.0
B	1, 4, 3	16.5	28.0
C	6, 5, 10, 7	12.5	57.0

Economic order quantity (EOQ) models

- Also referred to as the *economic lot-size model*
- Ford Harris, 1915

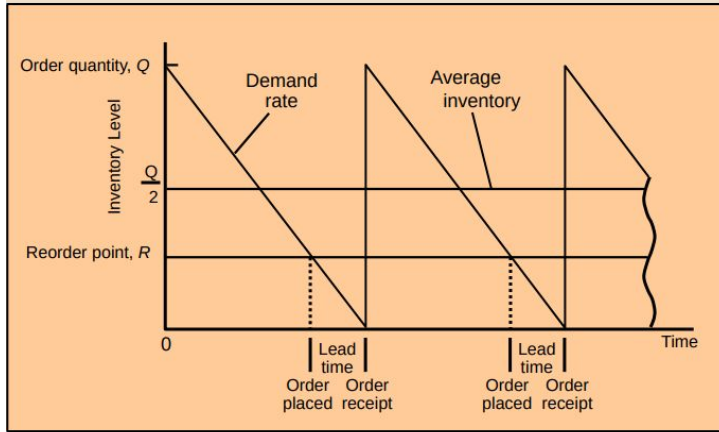
- EOQ: determine the optimal order quantity that will minimize total inventory costs. 2 variants of this model:
- 1. Basic EOQ model: formula for determining the optimal order size that minimizes the sum of carrying costs and ordering costs, derived under a set of simplifying and restrictive assumptions:
 - Demand is known with certainty and is constant over time
 - No shortages allowed
 - Lead time for the receipt of orders is constant
 - The order quantity is received all at once
- Order cycle: the time between receipt of orders in an inventory cycle
- Reorder point has a constant demand and constant lead time

$$R = dL$$

Where: d = demand rate/period (e.g.: daily) & L = lead time

- Optimal Q : Lowest point on the total cost curve
- EOQ model is *robust* because Q is a square root, errors in the estimation of D , C_c & C_o are dampened

Inventory Order Cycle



EOQ Cost Model

C_o - cost of placing order

D - annual demand

C_c - annual per-unit carrying cost

Q - order quantity

$$\text{Annual ordering cost} = \frac{C_o D}{Q}$$

$$\text{Annual carrying cost} = \frac{C_c Q}{2}$$

$$\text{Total cost} = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

EOQ Cost Model

Deriving Q_{opt}

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$\frac{\partial TC}{\partial Q} = -\frac{C_o D}{Q^2} + \frac{C_c}{2}$$

$$0 = -\frac{C_o D}{Q^2} + \frac{C_c}{2}$$

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c}}$$

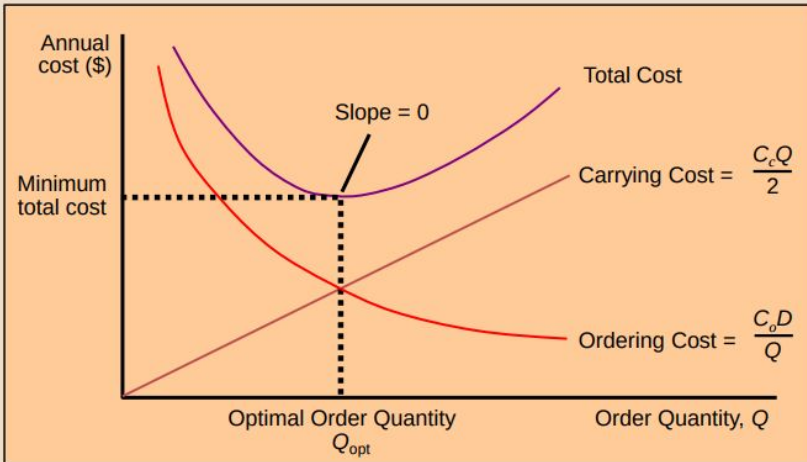
Proving equality of costs at optimal point

$$\frac{C_o D}{Q} = \frac{C_c Q}{2}$$

$$Q^2 = \frac{2C_o D}{C_c}$$

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c}}$$

EOQ Cost Model



EOQ Example (page 566)

$C_c = \$0.75$ per gallon $C_o = \$150$ $D = 10,000$ gallons

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}} \qquad TC_{min} = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$Q_{opt} = \sqrt{\frac{2(150)(10,000)}{(0.75)}} \qquad TC_{min} = \frac{(150)(10,000)}{2,000} + \frac{(0.75)(2,000)}{2}$$

$$Q_{opt} = 2,000 \text{ gallons} \qquad TC_{min} = \$750 + \$750 = \$1,500$$

Orders per year = D/Q_{opt}	Order cycle time = 311 days / (D/Q_{opt})
= $10,000/2,000$	= $311/5$
= 5 orders/year	= 62.2 store days

$$R = dL$$

where

d = demand rate per period

L = lead time

Note: d and L must have the same time unit

Reorder Point (Example)

Demand = 10,000 gallons/year

Store open 311 days/year

Daily demand = $10,000 / 311 = 32.154$
gallons/day

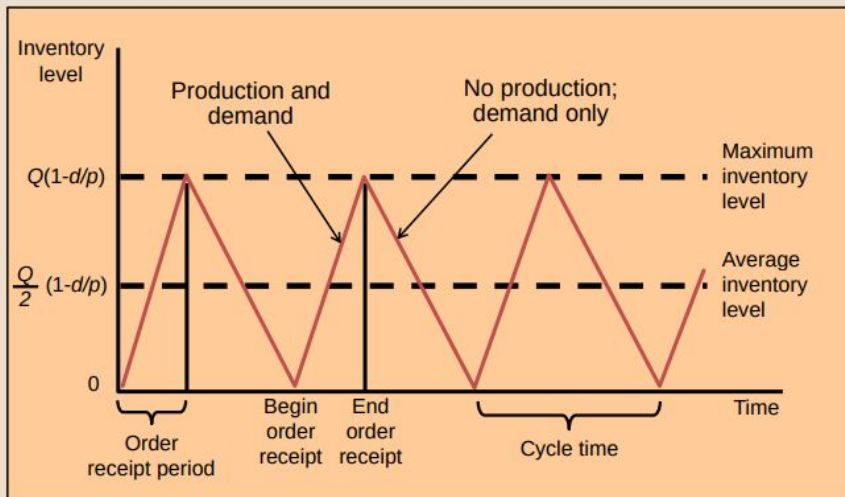
Lead time = $L = 10$ days

$R = dL = (32.154)(10) = 321.54$ gallons

- Economic Production quantity model (EPQ): and inventory system in which an order is received gradually, as inventory is simultaneously being depleted
 - Known as *gradual usage/non-instantaneous receipt model*
 - The assumption that orders are received all at once is **relaxed**
 - EPQ usually used when:
 - Inventory user is also producer
 - Retailer is also producer
 - Orders are delivered continuously overtime
 - To determine average inventory level we need; where d cannot exceed p since no shortages nor can $p = d$ (no order size) → we want $p \geq d$:

- p : daily rate at which the order is received over time, also known as the *production rate*
- d : daily rate at which inventory is demanded

Production Quantity Model



Production Quantity Model

p = production rate

d = demand rate

$$\text{Maximum inventory level} = Q - \frac{Q}{p} d$$

$$= Q \left(1 - \frac{d}{p} \right)$$

$$\text{Average inventory level} = \frac{Q}{2} \left(1 - \frac{d}{p} \right)$$

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c \left(1 - \frac{d}{p} \right)}}$$

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} \left(1 - \frac{d}{p} \right)$$

Production Quantity Model (page 568)

$$\begin{array}{lll} C_c = \$0.75 \text{ per gallon} & C_o = \$150 & D = 10,000 \text{ gallons} \\ d = 10,000/311 = 32.2 \text{ gallons per day} & & p = 150 \text{ gallons per day} \end{array}$$

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c \left(1 - \frac{d}{p}\right)}} = \sqrt{\frac{2(150)(10,000)}{0.75 \left(1 - \frac{32.2}{150}\right)}} = 2,256.8 \text{ gallons}$$

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} \left(1 - \frac{d}{p}\right) = \$1,329$$

$$\text{Production run} = \frac{Q}{p} = \frac{2,256.8}{150} = 15.05 \text{ days per order}$$

Production Quantity Model

$$\text{Number of production runs} = \frac{D}{Q} = \frac{10,000}{2,256.8} = 4.43 \text{ runs/year}$$

$$\begin{aligned} \text{Maximum inventory level} &= Q \left(1 - \frac{d}{p}\right) = 2,256.8 \left(1 - \frac{32.2}{150}\right) \\ &= 1,772 \text{ gallons} \end{aligned}$$

Quantity Discounts

Price per unit decreases as order quantity increases

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

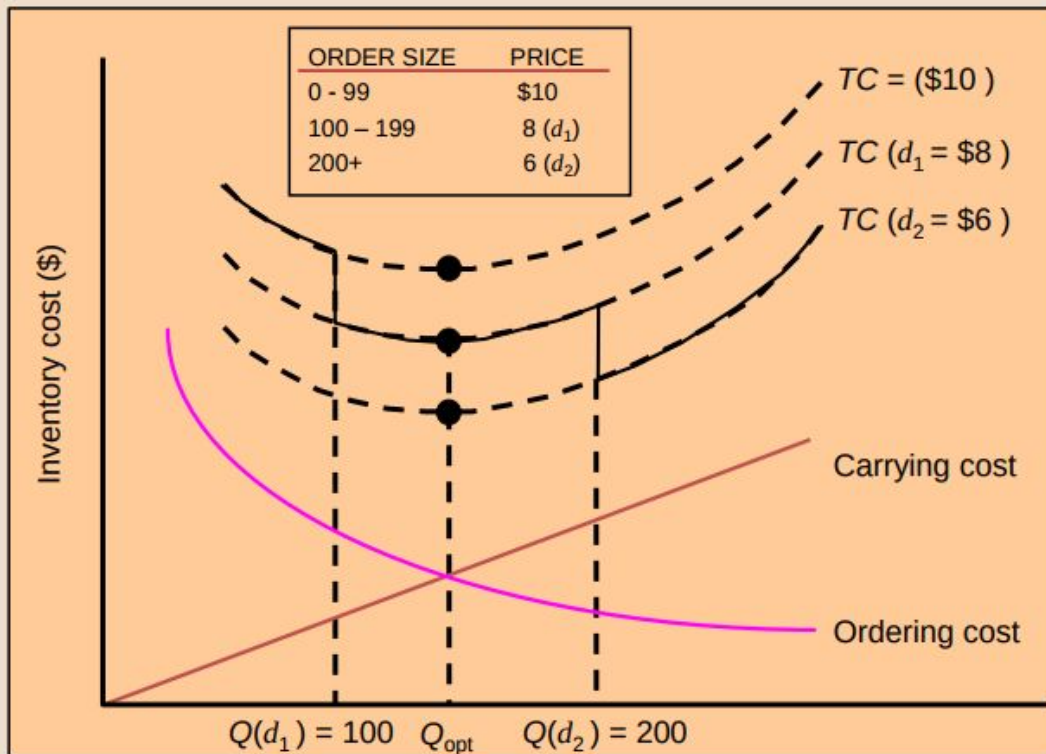
where

P = per unit price of the item

D = annual demand

The total cost function must now include the purchase price of the item being ordered

Quantity Discount Model



Safety stock

- Safety stock: buffer added to on hand inventory during lead time

$$SS = Z \text{ score} \times \sigma \times \sqrt{\text{lead time}}$$

- Service level:
 - Probability that inventory available during lead time will meet demand (probability that a stock-out will **not** occur)
 - P(demand during lead time \leftarrow reorder point)
 - E.g.: service level of 90% means that 0.90 probability that demand will be met during the lead time and 0.10 probability that a stock-out will occur
 - **SL = Probability (product availability) = 1 - P (stockout)**
- Standard deviation of demand
- Lead time

Stockout: inventory shortage

- 50% of the time you will be in a stockout situation

Quantity discount:

Quantity discount: give for specific higher order quantities; price discount on item if predetermined numbers of units are ordered

- Basic EOQ model can be used to determine optimal order size with quantity discounts but model is slightly altered. Total inventory cost function now includes purchase price of item being ordered:

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

- Where P= per-unit price of the item & D= annual demand
- PD = constant value that does not alter shape of total cost curve; minimum point on the cost curve would still be at same location corresponding to same value of Q \rightarrow thus optimal order size is the same no matter what the purchase price is
- However purchase price does affect order-size decision when discount is available

Reorder point with variable demand:

- To compute a reorder point with a safety stock that will meet a specific service level, assume the demand during each day of lead time is uncertain, independent, and can be described with a normal distribution.

Reorder Point With Variable Demand

$$R = \bar{d}L + z\sigma_d\sqrt{L}$$

where

\bar{d} = average daily demand

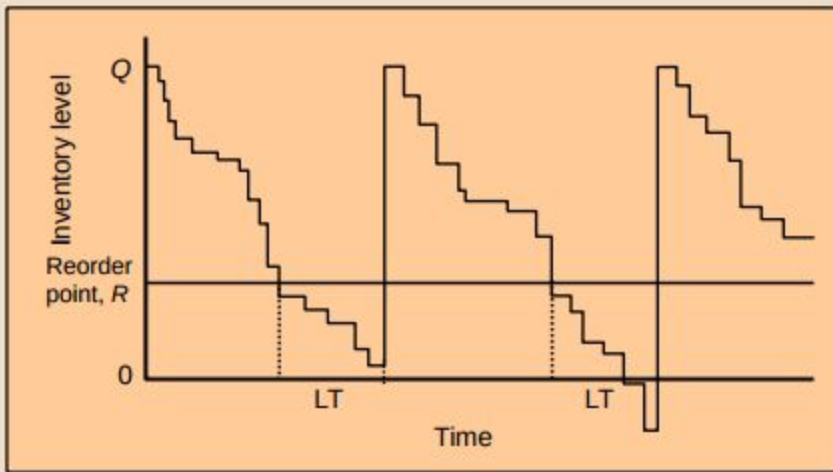
L = lead time

σ_d = the standard deviation of daily demand

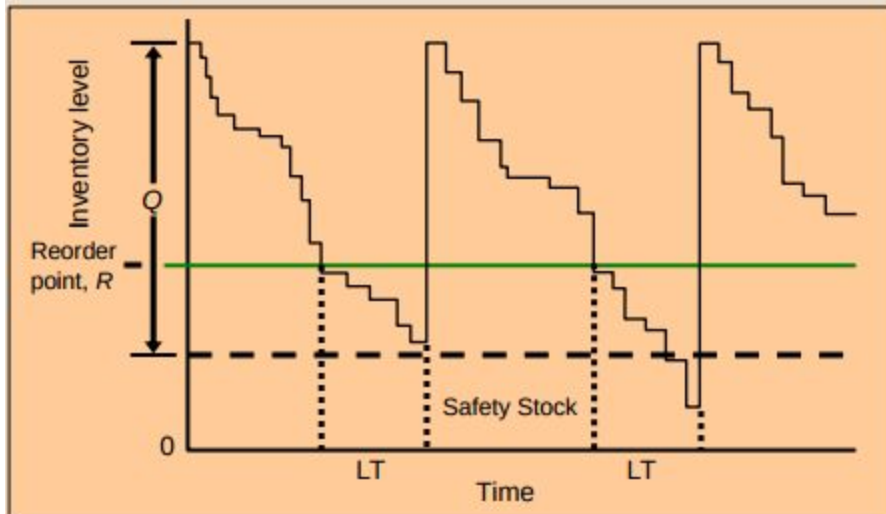
z = number of standard deviations
corresponding to the service level
probability

$z\sigma_d\sqrt{L}$ = safety stock

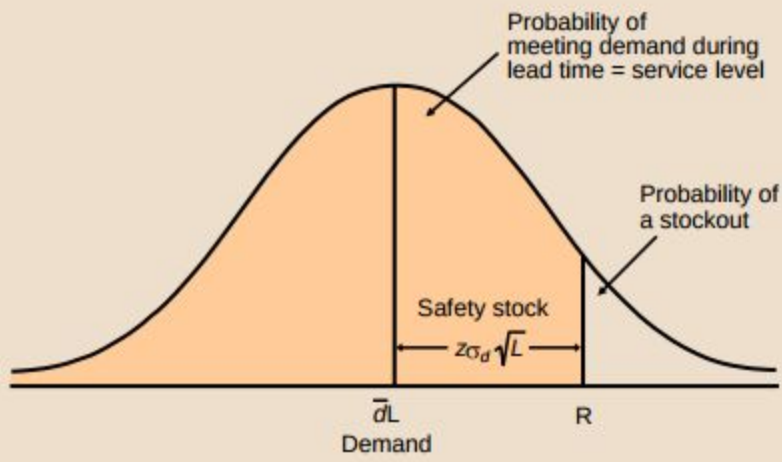
Variable Demand With Reorder Point



Reorder Point With Safety Stock



Reorder Point For a Service Level



Reorder Point For Variable Demand

The paint store wants a reorder point with a 95% service level and a 5% stockout probability

$$\bar{d} = 30 \text{ gallons per day}$$

$$L = 10 \text{ days}$$

$$\sigma_d = 5 \text{ gallons per day}$$

For a 95% service level, $z = 1.65$

$$R = \bar{d}L + z \sigma_d \sqrt{L}$$

$$= 30(10) + (1.65)(5)(\sqrt{10})$$

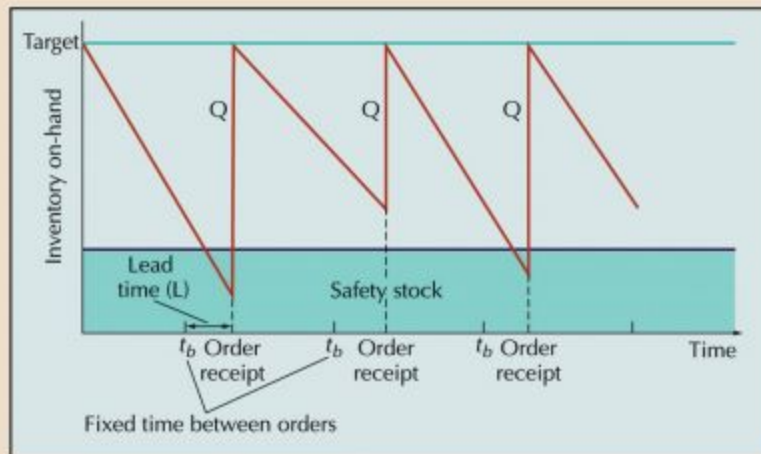
$$= 326.1 \text{ gallons}$$

$$\text{Safety stock} = z \sigma_d \sqrt{L}$$

$$= (1.65)(5)(\sqrt{10})$$

$$= 26.1 \text{ gallons}$$

Periodic Inventory System



Order quantity with variable demand:

- If the demand rate and lead time are constant, then the fixed-period model will have a fixed-order quantity that will be made at specific time intervals, which is the same as the fixed-quantity model (EOQ) model under similar conditions. However, fixed-period model reacts differently than the fixed-order model when demand is variable

Order Quantity for a Periodic Inventory System

$$Q = \bar{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I$$

where

\bar{d} = average demand rate

t_b = the fixed time between orders

L = lead time

σ_d = standard deviation of demand

$z\sigma_d \sqrt{t_b + L}$ = safety stock

I = inventory level

Fixed-Period Model With Variable Demand

$d = 6$ packages per day

$\sigma_d = 1.2$ packages

$t_b = 60$ days

$L = 5$ days

$I = 8$ packages

$z = 1.65$ (for a 95% service level)

$$\begin{aligned} Q &= \bar{d}(t_b + L) + z\sigma_d \sqrt{t_b + L} - I \\ &= (6)(60 + 5) + (1.65)(1.2) \sqrt{60 + 5} - 8 \\ &= 397.96 \text{ packages} \end{aligned}$$

Order quantity for the single-period inventory model

- Single-period model: applies to situations in which only one order can be placed for a product in anticipation of a future selling season where demand is uncertain. At the end of selling season: product is either sold or there is surplus.
- Objective of model:

- To find optimal order quantity or stocking level that minimized the total long-run shortage cost and the excess cost
- Examples of these products:
 - Perishable goods (newspapers)
 - Magazines
 - Christmas trees, cards, halloween costumes
 - Wedding bouquets
 - Grocery items (milk bananas)
- In such a single-period selling season, decision is how much of the product to order at the beginning of the selling season. Costs involved are:
 - Shortage cost or underage cost (C_s): includes opportunity cost of lost sales and charge of loss of customer goodwill, marginal profit (computed as difference between sales revenue/unit and production cost/unit)
 - Excess cost or overage cost (C_o): cost of overestimating demand. Includes loss of ordering one additional item and finding that it cannot be sold (has to be salvaged) which is computed as difference between production cost/unit and salvage value

fixed-
mally

$$\text{Service Level} = P(D \leq Q^*) = \frac{C_u}{C_u + C_o}$$

FORMULAS

Reorder Point with Constant Demand and Lead Time

$$R = dL$$

Reorder Point with Variable Demand

$$R = \bar{d}L + z\sigma_d\sqrt{L}$$

Fixed-Time-Period Order Quantity with Variable Demand

$$Q = \bar{d}(t_k + L) + z\sigma_d\sqrt{t_k + L} - I$$

Single-Period Inventory Model

$$\text{Service Level} = P(D \leq Q^*) = \frac{C_u}{C_u + C_o}$$

$$Q^* = \bar{d} + z\sigma_d \text{ (For normally distributed demand)}$$

SUMMARY OF KEY FORMU

Basic EOQ Model

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}}$$

EOQ Model with Non-instantaneous Receipt

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} \left(1 - \frac{d}{p}\right)$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c \left(1 - \frac{d}{p}\right)}}$$

Inventory Cost for Quantity Discounts

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} + PD$$

SUMMARY OF KEY TERM