



Université d'Ottawa • University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Midterm 2 for MAT 1339B (Winter 2018)
Introduction to Calculus and Vectors
March 23, 2018

Duration : 80 minutes

Professor : Rachid Bentoumi

NAME : _____

STUDENT NUMBER : _____

You must sign below.

Version 1

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur : you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

- This is a closed book examination.
- Only Faculty standard calculators are permitted.
- There are 5 questions.
- The exam is out of 40 points.

1. Find the derivatives of the following functions :

(a) (3 points) $y = e^{2x} - e^{-2x}$

$$\begin{aligned} y' &= (e^{2x} - e^{-2x})' \\ &= (e^{2x})' - (e^{-2x})' \\ &= (2x)' e^{2x} - (-2x)' e^{-2x} \\ &= 2e^{2x} - (-2)e^{-2x} = 2e^{2x} + 2e^{-2x} \end{aligned}$$

(b) (3 points) $h(x) = 3e^{\tan x} + \ln(\cos x) - \sin(1)$

$$\begin{aligned} h'(x) &= 3(e^{\tan(x)})' + (\ln(\cos(x)))' - (\sin(1))' \\ &= 3 \tan'(x) e^{\tan(x)} + \frac{\cos'(x)}{\cos(x)} - 0 \\ &= 3 \sec^2(x) e^{\tan(x)} + \frac{-\sin(x)}{\cos(x)} \\ &= 3 \sec^2(x) e^{\tan(x)} - \frac{\sin(x)}{\cos(x)} \end{aligned}$$

(c) (3 points) $f(x) = 3 \sin^2(2x-4) - 2 \cos^2(3x+1)$

$$\begin{aligned} f'(x) &= 3 (\sin^2(2x-4))' - 2 (\cos^2(3x+1))' \\ &= 3 (2) \sin'(2x-4) \sin^{2-1}(2x-4) - 2 (2) \cos'(3x+1) \cos^{2-1}(3x+1) \\ &= 6 (2x-4)' \cos(2x-4) \sin(2x-4) - 4 (-1) (3x+1)' \sin(3x+1) \cos(3x+1) \\ &= 6 (2) \cos(2x-4) \sin(2x-4) + 4 (3) \sin(3x+1) \cos(3x+1) \\ &= 12 \cos(2x-4) \sin(2x-4) + 12 \sin(3x+1) \cos(3x+1) \end{aligned}$$

2. (4 points) Determine the equation of the tangent line to the curve of $y = 5^x$ at the point $x = -2$.

$$y = f'(x_0)(x - x_0) + f(x_0) \quad f(x) = 5^x, \quad x_0 = -2$$

We need to evaluate $f'(-2)$ and $f(-2)$

$$\text{First, } f(-2) = 5^{-2} = \frac{1}{25} = \frac{1}{25}$$

$$f'(x) = (5^x)' = 5^x \cdot \ln(5)$$

$$\Rightarrow f'(-2) = 5^{-2} \cdot \ln(5) = \frac{\ln(5)}{25}$$

$$\begin{aligned} \text{So, } y &= f'(-2)(x - (-2)) + f(-2) \\ &= \frac{\ln(5)}{25}(x + 2) + \frac{1}{25} \end{aligned}$$

3. For the following equations, determine the value of x

(a) (2 points) $9 + 4 \ln(x) = 17$

$$4 \ln(x) = 17 - 9 = 8$$

$$\Rightarrow \ln(x) = \frac{8}{4} = 2$$

$$\Rightarrow e^{\ln(x)} = e^2 \quad \Rightarrow x = e^2$$

(b) (2 points) $3^{x+2} = 4^{2-x}$

$$\ln(3^{x+2}) = \ln(4^{2-x})$$

$$\Rightarrow (x+2) \ln(3) = (2-x) \ln(4)$$

$$\Rightarrow x \ln(3) + 2 \ln(3) = 2 \ln(4) - x \ln(4)$$

$$\Rightarrow x \ln(3) + x \ln(4) = 2 \ln(4) - 2 \ln(3)$$

$$\Rightarrow x (\ln(3) + \ln(4)) = 2 \ln(4) - 2 \ln(3)$$

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$$\Rightarrow x = \frac{2 \ln(4) - 2 \ln(3)}{\ln(4) + \ln(3)}$$

4. Consider the following function $f(x) = \frac{3x+1}{2-x}$

(a) (1 point) Determine the domain of f .

$$2-x \neq 0 \Rightarrow x \neq 2. \text{ So that, } D_f = (-\infty, 2) \cup (2, \infty)$$

(b) (2 points) Find, if it is possible, the coordinates of the intersections of f with x-axis and y-axis.

$$\text{x-intercept: we solve } f(x) = 0. \text{ So, } 3x+1=0 \text{ if } x = -\frac{1}{3}$$

$$\text{y-intercept: set } x=0 \text{ and compute } f(0) = \frac{3(0)+1}{2-0} = \frac{1}{2}$$

$$f(x) \text{ crosses x-axis at } \left(-\frac{1}{3}, 0\right)$$

$$f(x) \text{ crosses y-axis at } \left(0, \frac{1}{2}\right)$$

(c) (2 points) Determine, if it is possible, the vertical and horizontal asymptotes.

$$\text{H.A} \quad \lim_{x \rightarrow \pm\infty} \frac{3x+1}{2-x} = \frac{3}{-1} = -3$$

Hence, $y = -3$ is a H.A

$$\text{V.A} \quad \lim_{x \rightarrow 2^-} \frac{3x+1}{2-x} = \frac{3(2)+1}{0^+} = \frac{7}{0^+} = +\infty$$

$$\lim_{x \rightarrow 2^+} \frac{3x+1}{2-x} = \frac{3(2)+1}{0^-} = \frac{7}{0^-} = -\infty$$

Hence, $x = 2$ is a v.A

(d) (3 points) Calculate the first derivative of f and find all possible critical points.

$$f'(x) = \left(\frac{3x+1}{2-x} \right)' = \frac{3(2-x) - (3x+1)(-1)}{(2-x)^2}$$

$$= \frac{6-3x+3x+1}{(2-x)^2} = \frac{7}{(2-x)^2} > 0$$

There is no x such that $f'(x) = 0$ since $f'(x) > 0$.
 $f'(2)$ does not exist. However $2 \notin Df \Rightarrow$ No c.p for $f(x)$

(e) (3 points) Calculate the second derivative of f and find all possible inflection points.





$$f''(x) = \left(\frac{7}{(2-x)^2} \right)' = \left(7(2-x)^{-2} \right)' = 7(-2)(2-x)^{-3}$$

$$= (7)(-2)(-1)(2-x)^{-3} = \frac{14}{(2-x)^3}$$

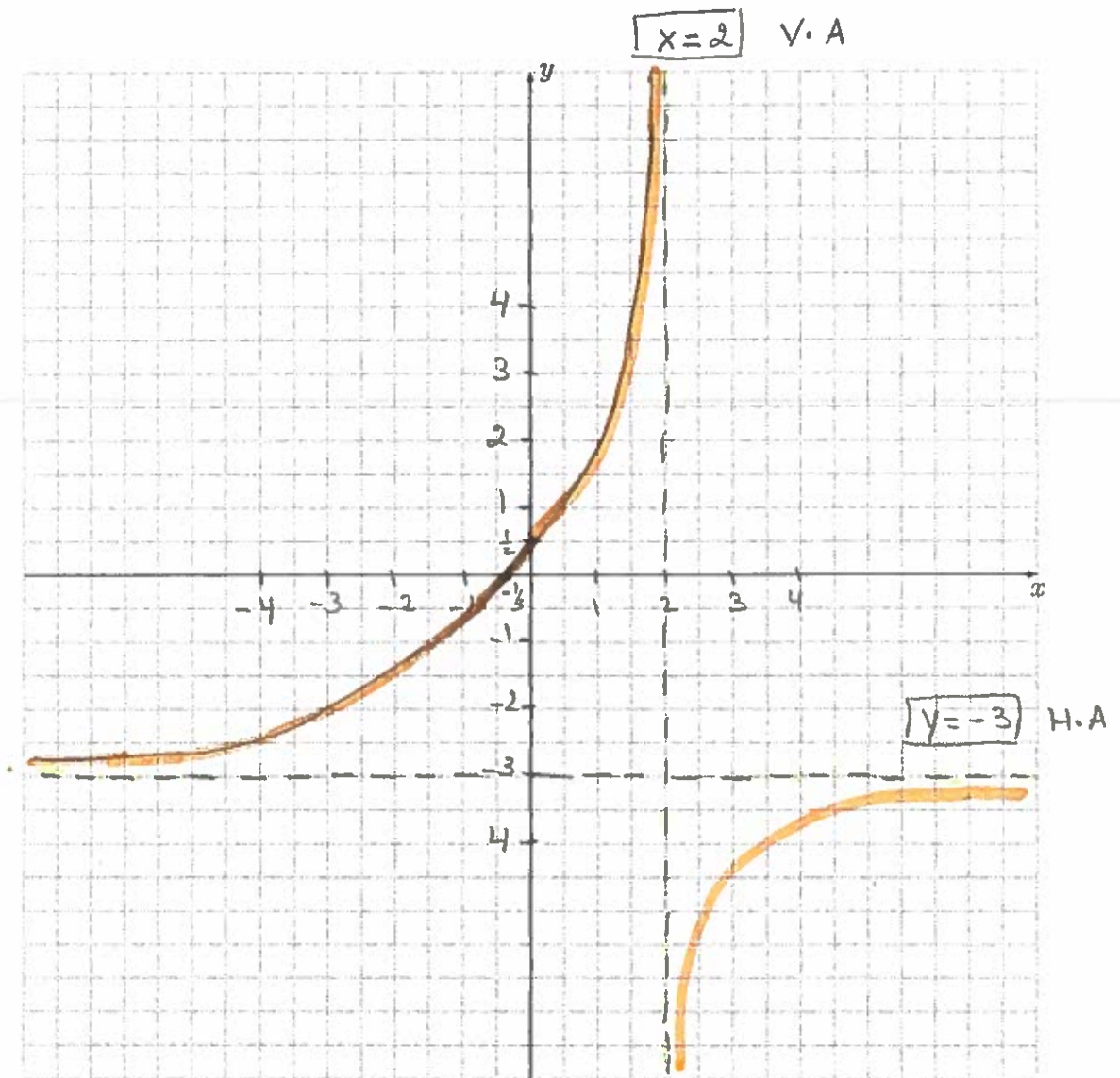
there is no x such that $f''(x) = 0$.

there is no inflection point for $f(x)$.

(f) (3 points) Build the variation table.

x	$-\infty$	2	∞
$f'(x)$	+	A	+
f		V.A	
$f''(x)$	+	A	-
Concavity of f		V.A	

(g) (3 points) Sketch the graph of the function f



5. A bacterial colony's population is modelled by the function

$$P(t) = 60e^{0.8t}$$

where P is the number of bacteria after t days.

(a) (1 point) What is the initial population of the bacterial colony's?

$$P_0 = P(0) = 60e^{0.8(0)} = 60e^0 = 60$$

(b) (1 point) Determine the bacterial population after 3 days?

$$\text{at } t=3, \text{ we have } P(3) = 60e^{0.8(3)} = 661.39 \\ \approx 662$$

(c) (2 points) How long will it take for the population to reach 10 times its initial population?

$$\text{We need to solve } P(t) = 10P_0 \Rightarrow P_0 e^{0.8t} = 10P_0$$

$$\Rightarrow e^{0.8t} = 10$$

$$\Rightarrow \ln(e^{0.8t}) = \ln(10)$$

$$\Rightarrow 0.8t = \ln(10)$$

$$\Rightarrow t = \frac{\ln(10)}{0.8} = 2.879 \text{ days}$$

(d) (2 points) Compute the rate of change of this population at the third day?

$$P'(t) = (60e^{0.8t})' = 60(0.8)e^{0.8t} = 48e^{0.8t}$$

at $t=3$ days, we have

$$P'(3) = 48e^{0.8(3)} \\ = 529.1125$$