MAT 1302, Fall 2017

Test 2

Professor: Catalin Rada

Sol

Surname ______________________ First Name ______________________

Student # ______________________

Instructions:

(a) You have 80 minutes to complete this exam.

(b) Show your work and justify your answers to receive full marks. Partial marks may be awarded for making sufficient progress towards a solution.

(c) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.

(d) **Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.**

(Signature): ______________________

(e) The final page of the exam may be used for scrap work.

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	1.5	2	2.5	4	1	4	15
Score							

1. **(1.5 points)** Compute $A(B - C)$ where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 1 & 3 & 5 \\ 3 & -1 & 1 \end{bmatrix}$$

,

$$B = \begin{bmatrix} -1 & 0 & -2 \\ 2 & 0 & 0 \\ -1 & 3 & 5 \end{bmatrix}$$

,

$$C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 0 \\ -1 & 3 & 1 \end{bmatrix}$$

$$B - C = \begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$A(B-C) = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 1 & 3 & 5 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -8 \\ 2 & -2 & 0 \\ 4 & -1 & 20 \\ 2 & -3 & 4 \end{pmatrix}$$

$\underline{\underline{4 \times 3}}$ $\qquad$ $\underline{\underline{3 \times 3}}$

2. (a) **(1 points)** Let $A = \begin{bmatrix} -2 & -1 \\ 2 & 4 \end{bmatrix}$. Find its inverse.

(b) **(1 points)** Suppose that $b = \begin{bmatrix} -6 \\ 6 \end{bmatrix}$. Use an inverse of a matrix (and not Row reduction) to solve $Ax = b$.

a) $A^{-1} = \dfrac{1}{(-2)\cdot 4 - 2(-1)} \begin{pmatrix} 4 & 1 \\ -2 & -2 \end{pmatrix} =$

$= \dfrac{1}{-8+2} \begin{pmatrix} 4 & 1 \\ -2 & -2 \end{pmatrix} = \dfrac{1}{-6} \begin{pmatrix} 4 & 1 \\ -2 & -2 \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{-6} & -\frac{1}{6} \\ \frac{2}{6} & \frac{2}{6} \end{pmatrix}$

b) $Ax=b \Longrightarrow x = A^{-1}b$. So

$x = \begin{pmatrix} \frac{4}{-6} & -\frac{1}{6} \\ \frac{2}{6} & \frac{2}{6} \end{pmatrix} \begin{pmatrix} -6 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

3. (a) **(2 points)** Are the following vectors linearly independent? Justify your answer.

$$v_1 = \begin{bmatrix} -1 \\ 4 \\ -2 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ -13 \\ 7 \\ 7 \end{bmatrix}, v_3 = \begin{bmatrix} -2 \\ 1 \\ 9 \\ -5 \end{bmatrix}$$

SOL:

$$\left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 4 & -13 & 1 & 0 \\ -2 & 7 & 9 & 0 \\ -3 & 7 & -5 & 0 \end{array}\right] \xrightarrow[\substack{R_3-2R_1 \\ R_4-3R_1}]{R_2+4R_1} \left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 1 & 13 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow[R_4-2R_2]{R_3+R_2} \left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 15 & 0 \end{array}\right) \xrightarrow[\frac{1}{15}R_4]{\frac{1}{6}R_3} \left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow{R_4-R_3}$$

$$\left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Since there is a Unique SOLUTION (Trivial solution), They are LIN. INDEP

(b) **(0.5 points)** Is the set $\{v_1, v_2, v_3, 0\}$ linearly independent? Justify your answer.

SOLUTION one : NO ; This set includes the 0 vector.

or

SOLUTION TWO:

NO: LOOK AT: $0 \cdot v_1 + 0 \cdot v_2 + 0 \cdot v_3 + 1 \cdot 0 = 0$

(NOT all coeff. are 0)

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & -3 & 1 & 0 & 1 & 0 & 0 \\ 1 & -2 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & -4 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_4-2R_1]{R_2-R_1,\ R_3-R_1} \left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_4+R_3}$$

4. **(4 points)** Determine if the following matrix is invertible. If it is invertible, then find its inverse.

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 1 & -1 & -3 & 1 \\ 1 & -2 & 1 & 1 \\ 2 & -4 & -1 & 1 \end{bmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -3 & 0 & 1 & 1 \end{array}\right) \xrightarrow{(-1)R_4} \left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & -1 & -1 \end{array}\right)$$

$$\xrightarrow{R_1-R_4} \left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & -2 & 0 & 1 & 1 \\ 0 & 1 & -3 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & -1 & -1 \end{array}\right) \xrightarrow{R_2+3R_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & -2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & -1 & -1 \end{array}\right) \xrightarrow{R_1+2R_2} \left(\begin{array}{c|cccc} \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{I_4} & -10 & 2 & 7 & 1 \\ & -4 & 1 & 3 & 0 \\ & -1 & 0 & 1 & 0 \\ & 3 & 0 & -1 & -1 \end{array}\right)$$

Since I obtained I_4 in the left side of the super augmented Mx, the Mx is invertible

and its inverse is:

$$\begin{pmatrix} -10 & 2 & 7 & 1 \\ -4 & 1 & 3 & 0 \\ -1 & 0 & 1 & 0 \\ 3 & 0 & -1 & -1 \end{pmatrix}$$

5. **(1 points)** If $2A^T + I_2 = \begin{bmatrix} -9 & 2 \\ -4 & 1 \end{bmatrix}$ find A.

$$2 \cdot A^T = \begin{pmatrix} -9 & 2 \\ -4 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -10 & 2 \\ -4 & 0 \end{pmatrix}$$

$$A^T = \begin{pmatrix} -5 & 1 \\ -2 & 0 \end{pmatrix}$$

$$A = (A^T)^T = \boxed{\begin{pmatrix} -5 & -2 \\ 1 & 0 \end{pmatrix}}$$

M , S

6. An economy has two sectors: Manufacturing and Service. In order to produce one unit of output, Manufacturing requires .3 units from its own sector and .6 units from Service. On the other hand, Service requires .4 units from its own sector and .2 units from Manufacturing to produce one unit of output.

(a) **(1 point)** Write down the consumption matrix C for this economy.

$$C = \begin{pmatrix} 0.3 & 0.2 \\ 0.6 & 0.4 \end{pmatrix}$$

(b) **(1 point)** Determine the intermediate demands if Service decides to produce 15 units and Manufacturing decides to produce 20 units.

$$15\begin{pmatrix} 0.2 \\ 0.4 \end{pmatrix} + 20\begin{pmatrix} 0.3 \\ 0.6 \end{pmatrix} = \begin{pmatrix} 9 \\ 18 \end{pmatrix}$$

(c) **(2 points)** Determine the production levels required to meet a final demand of 15 units from Manufacturing and 9 units from Service.

Leontief MODEL: $(I-C)x = d$

$$I - C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.3 & 0.2 \\ 0.6 & 0.4 \end{pmatrix} = \begin{pmatrix} 0.7 & -0.2 \\ -0.6 & 0.6 \end{pmatrix}$$

$$(I-C)^{-1} = \frac{1}{(0.7)(0.6) - (-0.6)\cdot(-0.2)} \begin{pmatrix} 0.6 & 0.2 \\ 0.6 & 0.7 \end{pmatrix} =$$

$$= \frac{1}{0.3}\begin{pmatrix} 0.6 & 0.2 \\ 0.6 & 0.7 \end{pmatrix} = \frac{1}{\frac{3}{10}}\begin{pmatrix} \frac{6}{10} & \frac{2}{10} \\ \frac{6}{10} & \frac{7}{10} \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{3} \\ 2 & \frac{7}{3} \end{pmatrix}$$

$$x = (I-C)^{-1}d = \begin{pmatrix} 2 & \frac{2}{3} \\ 2 & \frac{7}{3} \end{pmatrix}\begin{pmatrix} 15 \\ 9 \end{pmatrix} = \begin{pmatrix} 36 \\ 51 \end{pmatrix}.$$

Space for rough work

MAT 1302, Fall 2017

Test 2

Professor: Catalin Rada

SOL

Surname ______________________ First Name ______________________

Student # ______________________

Instructions:

(a) You have 80 minutes to complete this exam.
(b) Show your work and justify your answers to receive full marks. Partial marks may be awarded for making sufficient progress towards a solution.
(c) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(d) **Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.**

(Signature): ______________________

(e) The final page of the exam may be used for scrap work.

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	1.5	2	2.5	4	1	4	15
Score							

1. **(1.5 points)** Compute $A(B-C)$ where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 1 & 3 & 5 \\ 3 & -1 & 1 \end{bmatrix}$$

,

$$B = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

,

$$C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 0 \\ -1 & 3 & 1 \end{bmatrix}$$

$$B-C = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 2 & -6 & 1 \end{pmatrix}$$

$$A(B-C) = \underbrace{\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 1 & 3 & 5 \\ 3 & -1 & 1 \end{pmatrix}}_{4\times 3} \cdot \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 2 & -6 & 1 \end{pmatrix}}_{3\times 3} =$$

$$= \begin{pmatrix} -3 & 11 & -2 \\ 2 & -2 & 0 \\ 14 & -31 & 5 \\ 4 & -9 & 1 \end{pmatrix}$$

2. (a) **(1 points)** Let $A = \begin{bmatrix} 2 & 1 \\ -2 & -4 \end{bmatrix}$. Find its inverse.

(b) **(1 points)** Suppose that $b = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$. Use an inverse of a matrix (and not Row reduction) to solve $Ax = b$.

a) $A^{-1} = \dfrac{1}{2(-4) - 1(-2)} \begin{pmatrix} -4 & -1 \\ 2 & 2 \end{pmatrix} =$

$= \dfrac{1}{-8+2} \begin{pmatrix} -4 & -1 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} \frac{-4}{-6} & \frac{-1}{-6} \\ \frac{2}{-6} & \frac{2}{-6} \end{pmatrix} =$

$= \begin{pmatrix} \frac{4}{6} & \frac{1}{6} \\ -\frac{2}{6} & -\frac{2}{6} \end{pmatrix}$

!!

b) $Ax = b \Rightarrow x = A^{-1}b = \begin{pmatrix} +\frac{4}{6} & +\frac{1}{6} \\ -\frac{2}{6} & -\frac{2}{6} \end{pmatrix} \begin{pmatrix} 6 \\ -6 \end{pmatrix} =$

$= \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

SOL:

3. (a) **(2 points)** Are the following vectors linearly independent? Justify your answer.

$$v_1 = \begin{bmatrix} -1 \\ 4 \\ -2 \\ -3 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ -13 \\ 7 \\ 7 \end{bmatrix}, v_3 = \begin{bmatrix} -2 \\ 1 \\ 9 \\ -5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 4 & -13 & 1 & 0 \\ -2 & 7 & 9 & 0 \\ -3 & 7 & -5 & 0 \end{array}\right] \xrightarrow[\substack{R_4-3R_1 \\ R_3-2R_1}]{R_2+4R_1} \left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 1 & 13 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow[R_3+R_2]{R_4-2R_2} \left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 15 & 0 \end{array}\right) \xrightarrow[\frac{1}{6}R_3]{\frac{1}{15}R_4} \left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_4-R_3}$$

$$\left(\begin{array}{ccc|c} -1 & 3 & -2 & 0 \\ 0 & -1 & -7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Since there is a unique solution (= trivial SOL.), they are LIN. INDEP.

(b) **(0.5 points)** Is the set $\{0, v_1, v_2, v_3\}$ linearly independent? Justify your answer.

Solution one: NO, because this set includes the 0-vector.

or

Solution two: NO, LOOK AT:

$$\underline{1} \cdot 0 + \underline{0} \cdot V_1 + \underline{0} \cdot V_2 + \underline{0} \cdot V_3 = 0$$

(NOT all coeff are 0)

4. **(4 points)** Determine if the following matrix is invertible. If it is invertible, then find its inverse.

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 1 & -1 & -3 & 1 \\ 1 & -2 & 1 & 1 \\ 2 & -4 & -1 & 1 \end{bmatrix}$$

5. **(1 points)** If $2A^T + I_2 = \begin{bmatrix} -11 & 4 \\ -2 & -1 \end{bmatrix}$ find A.

$$2 \cdot A^T = \begin{pmatrix} -11 & 4 \\ -2 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -12 & 4 \\ -2 & -2 \end{pmatrix}$$

$$A^T = \begin{pmatrix} -6 & 2 \\ -1 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} -6 & -1 \\ 2 & -1 \end{pmatrix}$$

$$\parallel$$

$$(A^T)^T$$

6. An economy has two sectors: Manufacturing and Service. In order to produce one unit of output, Manufacturing requires .3 units from its own sector and .6 units from Service. On the other hand, Service requires .4 units from its own sector and .2 units from Manufacturing to produce one unit of output.

(a) **(1 point)** Write down the consumption matrix C for this economy.

(b) **(1 point)** Determine the intermediate demands if Service decides to produce 15 units and Manufacturing decides to produce 20 units.

(c) **(2 points)** Determine the production levels required to meet a final demand of 15 units from Manufacturing and 9 units from Service.

Space for rough work