MAT 1302 , Fall 2017

Professor: Catalin Rada

SOL

Surname ______________________ First Name ______________________

Student # ______________________

Instructions:

(a) You have 80 minutes to complete this exam.

(b) Show your work and justify your answers to receive full marks. Partial marks may be awarded for making sufficient progress towards a solution.

(c) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.

(d) **Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.**

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(e) No notes, books, calculators or scrap paper are allowed.

(f) The final page of the exam may be used for scrap work.

(g) Good luck!

1. **(2 points)** Show that the sets in (a) and (c) below are NOT subspaces (of the corresponding $\mathbf{R}^n$), and show that the set in (b) is a subspace.

(a) The set of solutions to the matrix-vector equation $Ax = b$, where A is an 3×67 matrix and b is a **nonzero** vector in R^3.

(b) $\{ \begin{bmatrix} 3x - 7y + 2z \\ z \\ x \\ y + z + x \end{bmatrix} | x, y, z \text{ in } \mathbf{R} \}$

(c) $\{ \begin{bmatrix} 112x + 3y + 4z \\ 2017 \\ x \\ y + z + x \end{bmatrix} | x, y, z \text{ in } \mathbf{R} \}$

a) The set DOES NOT contain the zero Vector. (so it is NOT a SUBSPACE)

b) The set is the span of

$\left\{ \begin{pmatrix} 3 \\ 0 \\ 1 \\ 1 \end{pmatrix}; \begin{pmatrix} -7 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ (So it is a SUBSPACE)

c) The set Does Not contain the zero Voctor. (So: it is NOT a subspace).

2. **(3 points)** Consider the following complex numbers: $z = -3 - 4i$ and $w = 1 - i$. Compute:

(a) $\bar{z}$ (b) $|z|^2$ (c) $\frac{w}{z}$ (d) $w + z$

Justify and show your work!

a) $\overline{z} = \overline{-3-4i} = -3+4i$

b) $|z|^2 = (-3)^2 + (-4)^2 = 9+16 = 25$

c) $\frac{w}{z} = w \cdot \frac{1}{z} = (1-i) \cdot \frac{\overline{z}}{|z|^2} =$

$= (1-i) \frac{(-3+4i)}{25} = \frac{(-3+4)+i(4+3)}{25}$

$= \frac{1}{25} + \frac{7}{25} i$

d) $w + z = -2 - 5i$

3. **(2 points)** Suppose A and B are 4×4 matrices with $\det A = -2$ and $\det B = -1$. Calculate $\det(-4A^T B^{-1} A)$. Justify and show your work!

$$(-4)^4 \cdot (-2) \cdot \frac{1}{-1} (-2) =$$

$$= -1024$$

4. **(2 points)** Consider the matrix

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and assume $\det A = -5$. Find $\det B$, where $B = \begin{bmatrix} -a & -b & -c \\ d-6a & e-6b & f-6c \\ 4g & 4h & 4i \end{bmatrix}$.

$$A \xrightarrow{(-1)R_1} C = \begin{pmatrix} -a & -b & -c \\ d & e & f \\ g & h & i \end{pmatrix} \xrightarrow{R_2 + 6R_1} D = \begin{pmatrix} -a & -b & -c \\ d-6a & e-6b & f-6c \\ g & h & i \end{pmatrix}$$

$$\xrightarrow{4R_3} B$$

Hence: $\det C = (-1)\det A = (-1)(-5) = 5$

$\det D = \det C = 5$

$\det B = 4 \cdot \det D = 4 \cdot 5 = 20$

Expand Across col 3:

5. **(2 points)** Compute the determinant of the following matrix $A = \begin{bmatrix} 1 & 1 & 0 & -1 & 2 \\ -1 & 1 & 0 & -1 & 1 \\ 2 & 1 & 1 & -1 & 3 \\ -1 & 2 & 0 & 1 & 1 \\ 0 & 3 & 0 & 0 & 0 \end{bmatrix}$

$$\det(A) = (-1)^{3+3} \cdot 1 \cdot \det\begin{pmatrix} 1 & 1 & -1 & 2 \\ -1 & 1 & -1 & 1 \\ -1 & 2 & 1 & 1 \\ 0 & 3 & 0 & 0 \end{pmatrix} =$$

Expand now across R_4

$$= \left(\det\begin{pmatrix} 1 & -1 & 2 \\ -1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}\right) \cdot 3 \cdot (-1)^{4+2} =$$

Use R_1

$$= 3\left[1 \cdot \det\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} - (-1)\det\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} + 2\det\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}\right] = 3\left[-2 + 0 + 2(-2)\right]$$

$$= -18$$

6. Consider a matrix $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 6 \end{bmatrix}$ whose reduced row echelon form is:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) [1 points] Find a basis of $NulA$.

(b) [1 point] What is the dimension of $NulA$?

(c) [1 points] Find a basis of $ColA$.

(d) [1 point] What is the rank of A?

a) x_1, x_3, x_4 are I.V, x_2, x_5 are free var.

$x_4 = -3\cancel{x}_5$; $x_3 = -x_5$; $x_1 = -x_2 - 2x_5$.

Hence $\begin{pmatrix} -x_2 - 2x_5 \\ x_2 \\ -x_5 \\ -3x_5 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -2 \\ 0 \\ -1 \\ -3 \\ 1 \end{pmatrix} \leftarrow$ P.V.F.

BASIS: $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} -2 \\ 0 \\ -1 \\ -3 \\ 1 \end{pmatrix} \right\}$

b) dim Nul $A = 2$

c) BASIS : $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 3 \end{pmatrix}; \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

d) rank $A = 3$

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1. **(2 points)** Show that the sets in (a) and (c) below are NOT subspaces (of the corresponding $\mathbf{R}^n$), and show that the set in (b) is a subspace.

(a) The set of solutions to the matrix-vector equation $Ax = b$, where A is an 4×67 matrix and b is a **nonzero** vector in R^4.

(b) $\{ \begin{bmatrix} 8x - 7y + 6z \\ 2z \\ y \\ y + z + x \end{bmatrix} | x, y, z \text{ in } \mathbf{R} \}$

(c) $\{ \begin{bmatrix} 2x + 3y + 9z \\ -17 \\ 2x \\ y + z + x \end{bmatrix} | x, y, z \text{ in } \mathbf{R} \}$

a) The set does NOT contain the zero Vector. (So: it is NOT a SUBSPACE)

b) The set is the span of

$\left\{ \begin{pmatrix} 8 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -7 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}$ (So: it is a SUBSPACE)

c) The set Does NOT contain the zero-vector. (So: it is NOT a SUBSPACE)

2. **(3 points)** Consider the following complex numbers: $z = -4 - 3i$ and $w = 1 - i$. Compute:

(a) $\bar{z}$ (b) $|z|^2$ (c) $\frac{w}{z}$ (d) $w - z$

Justify and show your work!

a) $\overline{z} = -4 + 3i$

b) $|z|^2 = (-4)^2 + (-3)^2 = 16 + 9 = 25$

c) $\frac{w}{z} = (1-i)\,\frac{1}{z} = (1-i) \cdot \frac{\overline{z}}{|z|^2} =$

$= (1-i) \cdot \frac{-4+3i}{25} = \frac{(-4+3) + i\,(3+4)}{25}$

$= -\frac{1}{25} + \frac{7}{25}\,i$

d) $5 + 2i$

3. **(2 points)** Suppose A and B are 4×4 matrices with $\det A = -1$ and $\det B = -2$. Calculate $\det(-8A^T B^{-1} A)$. Justify and show your work!

$$(-8)^4 \cdot (-1) \frac{1}{-2} \cdot (-1) =$$

$$= -2048$$

4. **(2 points)** Consider the matrix

$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and assume $\det A = -5$. Find $\det B$, where $B = \begin{bmatrix} -a & -b & -c \\ d-5a & e-5b & f-5c \\ 3g & 3h & 3i \end{bmatrix}$.

$$A \xrightarrow{(-1)R_1} C = \begin{pmatrix} -a & -b & -c \\ d & e & f \\ g & h & i \end{pmatrix} \xrightarrow{R_2 + 5R_1}$$

$$D = \begin{pmatrix} -a & -b & -c \\ d-5a & e-5b & f-5c \\ g & h & i \end{pmatrix} \xrightarrow{3 \cdot R_3} B$$

So:

$\det C = (-1) \det A = (-1)(-5) = 5$

$\det D = \det C = 5$

$\det B = 3 \cdot \det D = 3 \cdot 5 = 15$

see the other

5. **(2 points)** Compute the determinant of the following matrix $A = \begin{bmatrix} 1 & 1 & 0 & -1 & 2 \\ -1 & 1 & 0 & -1 & 1 \\ 2 & 1 & 1 & -1 & 3 \\ -1 & 2 & 0 & 1 & 1 \\ 0 & 3 & 0 & 0 & 0 \end{bmatrix}$

version

6. Consider a matrix $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 6 \end{bmatrix}$ whose reduced row echelon form is:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) [1 points] Find a basis of $ColA$.

(b) [1 point] What is the rank of A?

(c) [1 points] Find a basis of $NulA$.

(d) [1 point] What is the dimension of $NulA$?

a) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 3 \end{pmatrix}; \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 6 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

b) rank $A = 3$

c) x_1, x_3, x_4 are IND var; x_2, x_5 free.

$x_4 = -3x_5$; $x_3 = -x_5$; $x_1 = -x_2 - 2x_5$

So: $\begin{pmatrix} -x_2 - 2x_5 \\ x_2 \\ x_5 \\ -3x_5 \\ x_5 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -2 \\ 0 \\ -1 \\ -3 \\ 1 \end{pmatrix}$ ← P.V.F.

basis: $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} -2 \\ 0 \\ -1 \\ -3 \\ 1 \end{pmatrix} \right\}$

d) $\dim NulA = 2$