

- [1] 1. Find the domain of the function $g(x) = \frac{xe^{\sqrt{1-2x}}}{x^2 - 9}$.

solution: We need to make sure that the expression in the square root is non-negative, and we need to make sure that the denominator is not zero. So we must have $1 - 2x \geq 0$ which gives $x \leq 1/2$ and we also need to have $x^2 - 9 \neq 0$ which gives $x \neq \pm 3$. So the domain is $(-\infty, -3) \cup (-3, 1/2]$.

- [2] 2. Using the definition of the derivative, find $\frac{d}{dx}\sqrt{x+7}$.

solution:

$$\begin{aligned}
 & \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)+7} - \sqrt{x+7}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)+7} - \sqrt{x+7}}{h} \times \frac{\sqrt{(x+h)+7} + \sqrt{x+7}}{\sqrt{(x+h)+7} + \sqrt{x+7}} \\
 &= \lim_{h \rightarrow 0} \frac{\left(\sqrt{(x+h)+7}\right)^2 - \left(\sqrt{x+7}\right)^2}{h \left(\sqrt{(x+h)+7} + \sqrt{x+7}\right)} \\
 &= \lim_{h \rightarrow 0} \frac{((x+h)+7) - (x+7)}{h \left(\sqrt{(x+h)+7} + \sqrt{x+7}\right)} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h \left(\sqrt{(x+h)+7} + \sqrt{x+7}\right)} \\
 &= \lim_{h \rightarrow 0} \frac{1}{\left(\sqrt{(x+h)+7} + \sqrt{x+7}\right)} \\
 &= \frac{1}{\left(\sqrt{x+0+7} + \sqrt{x+7}\right)} \\
 &= \frac{1}{(2\sqrt{x+7})}
 \end{aligned}$$

- [4] 3. Evaluate each of the limits. Show your work!

a) $\lim_{x \rightarrow 2} \frac{x-2}{2x^2-4x}$

solution:

$$\begin{aligned}
 \lim_{x \rightarrow 2} \frac{x-2}{2x^2-4x} &= \lim_{x \rightarrow 2} \frac{x-2}{2x(x-2)} \\
 &= \lim_{x \rightarrow 2} \frac{1}{2x} \\
 &= \frac{1}{2(2)} \\
 &= \frac{1}{4}
 \end{aligned}$$

b) $\lim_{x \rightarrow \infty} \frac{x^3 - x - 7}{2x^3 + \sqrt{x}}$

solution:

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \frac{x^3 - x - 7}{2x^3 + \sqrt{x}} &= \lim_{x \rightarrow \infty} \frac{x^3 - x - 7}{2x^3 + \sqrt{x}} \times \frac{1/x^3}{1/x^3} \\
 &= \lim_{x \rightarrow \infty} \frac{1 - 1/x^2 - 7/x^3}{2 + 1/x^{5/2}} \\
 &= \frac{1 - 0 - 0}{2 + 0} \\
 &= \frac{1}{2}
 \end{aligned}$$

- [2] 4. Suppose that $x^3 + xy - e^y = 5$. Find $y' = \frac{dy}{dx}$ in terms of x and y .

solution:

$$\begin{aligned}
 x^3 + xy - e^y &= 5 \\
 3x^2 + y + xy' - e^y y' &= 0 \\
 xy' - e^y y' &= -3x^2 - y \\
 y' &= \frac{-3x^2 - y}{x - e^y}
 \end{aligned}$$

- [2] 5. Let $f(x) = -2x^3 + x$. Find the equation of the line tangent to f at the point $x = 1$.

solution: First we find the derivative: $f'(x) = -6x^2 + 1$. The slope is $f'(1) = -5$. The value of the function is $f(1) = -1$. Then we have the line.

$$\begin{aligned}
 (y - y_0) &= m(x - x_0) \\
 (y - (-1)) &= (-5)(x - 1) \\
 y &= -5x + 4
 \end{aligned}$$

- [6] 6. Differentiate each of the following. Show your work! You do not need to simplify your answer.

a) $u(t) = (t^2 + \pi)^7(e^t + 1)$

solution:

$$u'(t) = 7(t^2 + \pi)^6(2t)(e^t + 1) + (t^2 + \pi)^7(e^t)$$

b) $w(r) = \ln(\sqrt{1 - r^2})$.

solution:

$$w'(r) = \frac{1}{\sqrt{1 - r^2}} \frac{1}{2} (1 - r^2)^{-1/2} (-2r)$$

c) $g(x) = \frac{\tan(x)}{x^2 + 2^x}$.

solution:

$$g'(x) = \frac{\sec^2(x)(x^2 + 2^x) - \tan(x)(2x + 2^x \ln(2))}{(x^2 + 2^x)^2}$$

- [2] 7. Use logarithmic differentiation to find the derivative of $f(x) = (x^2 + 2)^{\sin(x)}$. Give your answer in terms of x (not f), but you do not need to simplify.

solution: First take $\ln$ of both sides.

$$\begin{aligned} \ln(f) &= \ln\left((x^2 + 2)^{\sin(x)}\right) \\ &= (\sin(x)) \ln(x^2 + 2) \end{aligned}$$

Now differentiate implicitly, using product rule and chain rule. Remember to substitute f at the end.

$$\begin{aligned} \frac{1}{f} f' &= (\cos(x)) \ln(x^2 + 2) + (\sin(x)) \frac{1}{x^2 + 2} (2x) \\ f' &= (x^2 + 2)^{\sin(x)} (\cos(x)) \ln(x^2 + 2) + (\sin(x)) \frac{1}{x^2 + 2} (2x) \end{aligned}$$

- [3] 8. A rectangle is changing width and height continuously. When the height is 20cm and the area is 100cm^2 , the width is increasing at a rate of 1cm/s and the area is increasing at a rate of $10\text{cm}^2/\text{s}$. At this moment, what is the rate of change of the height?

solution: Consider a rectangle of width w , height h and area A . We know that $A = wh$. Differentiating we get $\frac{dA}{dt} = \frac{dw}{dt}h + w\frac{dh}{dt}$. When the height is 20 and the area is 100 the width

must be $100/20 = 5$. At this moment we know $\frac{dw}{dt} = 1$ and $\frac{dA}{dt} = 10$.

$$\begin{aligned}\frac{dA}{dt} &= \frac{dw}{dt}h + w\frac{dh}{dt} \\ 10 &= (1)(20) + (5)\frac{dh}{dt} \\ \frac{dh}{dt} &= \frac{10 - 20}{5} \\ &= -2\end{aligned}$$

Note that the height is decreasing, since its derivative is negative.