University of Ottawa
MAT1330 Midterm Exam

November 11, 2015. Duration: 80 minutes, Instructor: Catalin Rada

-SOL-

Family Name:_____

First Name:_____

DGD 1 DGD 2 DGD 3 DGD 4

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- Answer these questions in the space provided. Use the backs of pages for additional calculations if necessary. The correct answer requires justification, written legibly and logically: you must convince me that you know why your solution is correct.
- Where it is possible to check your work, do so. Please do not detach the pages.
- Only the Faculty approved calculators (TI-30X, TI-34X, Casio FX-260X and Casio FX-300X) are allowed.
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  By signing below, you acknowledge that you have ensured that you are complying with the above statement.

  _____

- You have 80 minutes to complete this exam. Good luck!

Student number:_____, Total marks: _____ out of 30

Problem	1	2	3	4	5	6
Marks						

Question 1. [3 points] Consider the function

$$f(x) = \begin{cases} 3(\frac{\sqrt{x}-2}{x-4}), & x \neq 4 \\ 2k-1, & x = 4. \end{cases}$$

Can you choose a value for k so that the function is continuous? Justify your answer and give the value of k if it exists.

To Be Continuous AT 4: $\lim_{x\to 4} f(x) = f(4)$

$$\lim_{x\to 4^-} 3\cdot\frac{\sqrt{x}-2}{x-4} = 3\cdot\lim_{x\to 4^-}\frac{\sqrt{x}-2}{(\sqrt{x})^2-2^2} = 3\lim_{x\to 4^-}\frac{\sqrt{x}-2}{(\sqrt{x}-2)}\cdot\frac{1}{\sqrt{x}+2}$$

$$= 3\cdot\lim_{x\to 4^-}\frac{1}{\sqrt{x}+2} = \frac{3}{\sqrt{4}+2} = \frac{3}{4}$$

Used: $A^2-B^2=(A-B)(A+B)$

$$f(4) = 2k-1 = \frac{3}{4} \Rightarrow 2k = 1+\frac{3}{4}$$

or $k = \frac{7}{8}$

Answer: $k = 7/8$

Question 2. [5 points] Find the following limits, using the rules from class. (Note: using a table of values will not give you any points.)

(a) $\lim_{x\to\infty}(2\ln(x)-\ln(7x^2-6)) =$ [ ]

$= \lim_{x\to\infty} \ln x^2 - \ln(7x^2-6) =$

$= \lim_{x\to\infty} \ln\left[\frac{x^2}{7x^2-6}\right] = \ln\left[\lim_{x\to\infty} \frac{x^2}{7x^2-6}\right] =$

$= \ln\left[\lim_{x\to\infty} \frac{1}{7-\frac{6}{x^2}}\right] = \ln\left(\frac{1}{7}\right) = -\ln 7$

We Used that $\ln$ is continuous ☺

(b) $\lim_{x\to\infty} \frac{\sqrt[3]{5x^6+10001}}{3x^2+111} =$ [ ]

$= \lim_{x\to\infty} \frac{\sqrt[3]{x^6\left[5+\frac{10001}{x^6}\right]}}{3x^2+111} =$

$= \lim_{x\to\infty} \frac{x^2\sqrt[3]{5+\frac{10001}{x^6}}}{x^2\left[3+\frac{111}{x^2}\right]} = \lim_{x\to\infty} \frac{\sqrt[3]{5+\frac{10001}{x^6}}}{3+\frac{111}{x^2}} =$

$= \frac{\sqrt[3]{5}}{3}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 3. [4 points] Write down the definition of the derivative of a function f in general. Then find the derivative of the function below from this definition (e.g., without using any of the differentiation rules from class).

$$f(x) = \frac{7x}{4x+9}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{7(x+h)}{4(x+h)+9} - \frac{7x}{4x+9}}{h} =$$

$$= \lim_{h \to 0} \frac{(7x+7h)(4x+9) - 7x[4x+4h+9]}{(4x+4h+9)\cdot(4x+9)} \cdot \frac{1}{h} =$$

$$= \lim_{h \to 0} \frac{28x^2 + 63x + 28hx + 63h - 28x^2 - 28hx - 63x}{h(4x+4h+9)(4x+9)}$$

$$= \lim_{h \to 0} \frac{63}{(4x+4h+9)(4x+9)} = \frac{63}{(4x+9)^2}$$

Answer: $f'(x) =$ $\boxed{\frac{63}{(4x+9)^2}}$

By Implicit Diff...

Question 4. [6 points] The relation $ye^{9x/y} + xy^4 \ln(y) = 1$ defines a curve in the plane. Show that the point $(0, 1)$ is on this curve and calculate the tangent line to the curve at this point.

$$y' \cdot e^{\frac{9x}{y}} + y \cdot e^{\frac{9x}{y}} \cdot \left(\frac{9 \cdot y - 9x \cdot y'}{y^2}\right) + 1 \cdot y^4 \ln y +$$

$$+ x \left[4y^3 \cdot y' \ln y + y^4 \cdot \frac{y'}{y}\right] = 0$$

PLUG in $x=0$; $y=1$:

$$y'(0) \cdot 1 + 1 \cdot 1 \cdot \frac{9 - 0}{1^2} + 1 \cdot 1^4 \cdot \underbrace{\ln 1}_{=0} + 0 = 0$$

So: $\boxed{y'(0) = -9}$

So: $y = mx + b = -9x + b$

PLUG in $\begin{cases} x=0 \\ y=1 \end{cases}$ and get $\boxed{1 = b}$

$$y = -9x + 1$$

Answer: the tangent line is given by $y =$ $\boxed{y = -9x + 1}$

$f'(x) = 6x^2 - 30x + 24 = 6(x-1)(x-4)$

Question 5. [6 points] Where is the function $f(x) = 2x^3 - 15x^2 + 24x + 7$ increasing and where is it decreasing on the interval $x \in [0, 6]$. Find all the local and global maxima and minima of this function on this interval. Give their x and y values.

So, Critical #s are found: $f'(x) = 0 \Rightarrow$

$6(x-1)(x-4) = 0 \rightarrow x = 1, x = 4$

x	0		1		4		6
$f'(x)$		+ +	0	− −	0	+ +	
$f(x)$	7	↗	18	↘	−9	↗	43

Answer:

f is increasing for $(0,1) \cup (4,6)$

f is decreasing for $(1,4)$

f has local maxima at $x=1$; $y=18$

f has local minima at $x=4$; $y=-9$

f has global maxima at $x=6$; $y=43$

f has global minima at $x=4$; $y=-9$

Question 6. [6 points] Assume that a function f as well as its first and second derivatives are continuous for all $x \in (-\infty, \infty)$. Assume that the function has the following properties:

- $f'(x) < 0$ if $x < 2$ and $f'(x) > 0$ if $x > 2$.
- $f''(x) < 0$ if $|x| > 3$ and $f''(x) > 0$ if $|x| < 3$
- $\lim_{x\to\infty} f(x) = 2$ and $\lim_{x\to-\infty} f(x) = 1$
- $f(2) = 0$

Answer the following questions:

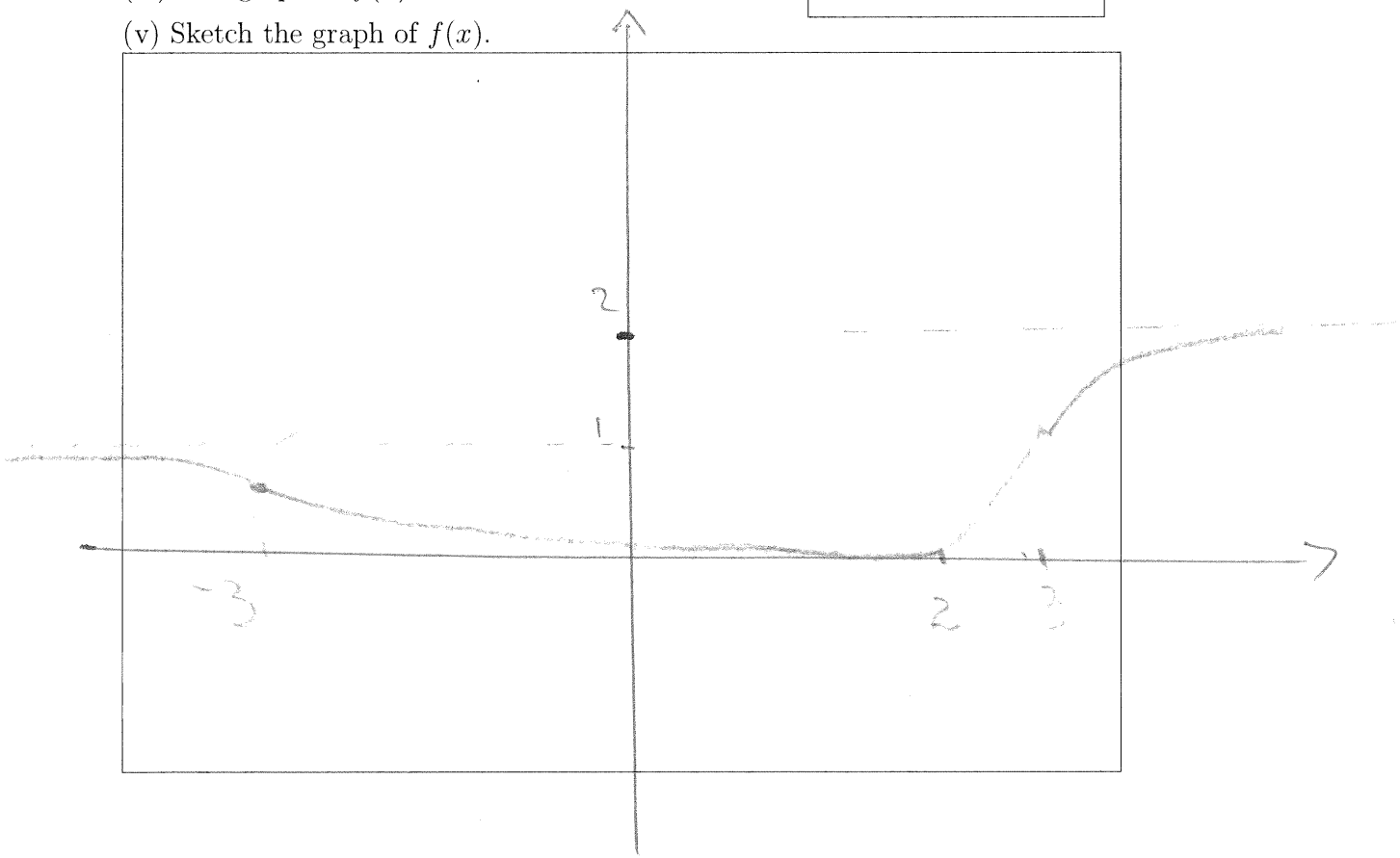
(i) The graph of $f(x)$ increasing for $(2, \infty)$

(ii) The graph of $f(x)$ concave up for $(-3, 3)$ or $|x|<3$

(iii) The graph of $f(x)$ attains a relative maximum at none

(iv) The graph of $f(x)$ attains a relative minimum at $x=2$

(v) Sketch the graph of $f(x)$.



Rough work!

University of Ottawa
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November 12, 2015. Duration: 80 minutes. Instructor: Catalin Rada.

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Marks						

Question 1. [3 points] Consider the function

$$f(x) = \begin{cases} 2(\frac{\sqrt{x}-1}{x-1}), & x \neq 1 \\ 2k+1, & x = 1. \end{cases}$$

Can you choose a value for k so that the function is continuous? Justify your answer and give the value of k if it exists.

To be continuous at 1: $\lim_{x\to 1} f(x) = f(1)$

$$\lim_{x\to 1^-} 2\cdot\frac{\sqrt{x}-1}{x-1} = 2\cdot\lim_{x\to 1^-}\frac{\sqrt{x}-1}{\sqrt{x}-1}\cdot\frac{1}{\sqrt{x}+1}$$

$A^2-B^2=(A-B)\cdot(A+B)$

$$= 2\cdot\lim_{x\to 1^-}\frac{1}{\sqrt{x}+1} = 2\cdot\frac{1}{1+1} = \frac{2}{2} = 1$$

$f(1) = 2k+1 = 1 \Rightarrow k = 0$

Answer: $k=0$

Question 2. [5 points] Find the following limits, using the rules from class. (Note: using a table of values will not give you any points.)

(a) $\lim_{x\to\infty} (3\ln(x) - \ln(6x^3 - 3)) =$ $\boxed{}$

$= \lim_{x\to\infty} \ln x^3 - \ln(6x^3-3) = \lim_{x\to\infty} \ln\left[\frac{x^3}{6x^3-3}\right] =$

$= \ln\left[\lim_{x\to\infty} \frac{x^3}{6x^3-3}\right] = \ln\left[\lim_{x\to\infty} \frac{1}{6-\frac{3}{x^3}}\right] = \ln\left(\frac{1}{6}\right) =$

$= -\ln 6$

We Used that $\ln$ is CONTINUOUS!

(b) $\lim_{x\to\infty} \frac{\sqrt[3]{7x^6+20x+1}}{4x^2+10001} =$ $\boxed{}$

$= \lim_{x\to\infty} \frac{\sqrt[3]{x^6\left[7+\frac{20}{x^5}+\frac{1}{x^6}\right]}}{4x^2} = \lim_{x\to\infty} \frac{x^2\sqrt[3]{7+\frac{20}{x^5}+\frac{1}{x^6}}}{4x^2}$

$= \lim_{x\to\infty} \frac{\sqrt[3]{7+\frac{20}{x^5}+\frac{1}{x^6}}}{4} = \boxed{\frac{\sqrt[3]{7}}{4}}$

Question 3. [4 points] Write down the definition of the derivative of a function f in general. Then find the derivative of the function below from this definition (e.g., without using any of the ~~differentiation rules from class~~).

$$f(x) = \frac{3x}{2x+40}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{3(x+h)}{2(x+h)+40} - \frac{3x}{2x+40}}{h} = \lim_{h\to 0} \frac{(3x+3h)(2x+40) - 3x[2x+2h+40]}{[2x+2h+40]\cdot[2x+40]} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{6x^2 + 120x + 6xh + 120h - 6x^2 - 6xh - 120x}{h[2x+2h+40]\cdot[2x+40]}$$

$$= \lim_{h\to 0} \frac{120h}{h[2x+2h+40]\cdot[2x+40]} = \lim_{h\to 0} \frac{120}{(2x+2h+40)(2x+40)}$$

$$= \frac{120}{(2x+40)^2}$$

Answer: $f'(x) =$ $\frac{120}{(2x+40)^2}$

By Implicit Diff:

Question 4. [6 points] The relation $ye^{8x/y} + xy^3 \ln(y) = 1$ defines a curve in the plane. Show that the point $(0, 1)$ is on this curve and calculate the tangent line to the curve at this point.

$$y' \cdot e^{\frac{8x}{y}} + y e^{\frac{8x}{y}} \cdot \left(\frac{8 \cdot y - 8x \cdot y'}{y^2} \right) + 1 \cdot y^3 \ln y +$$

$$+ x \cdot \left[3y^2 \cdot y' \cdot \ln y + y^3 \cdot \frac{y'}{y} \right] = 0$$

PLUG IN $\boxed{x=0;\ y=1}$

$$y'(0) + \frac{8 - 0}{1^2} + 1 \cdot 1^3 \underbrace{\ln 1}_{=0} + 0 = 0$$

So: $\boxed{y'(0) = -8}$

So: $y = mx + b = -8x + b$ $\Big\}$ $\Rightarrow$ $1 = b \Rightarrow$

PLUG IN $x=0;\ y=1$

$$y = \boxed{-8x + 1}$$

Answer: the tangent line is given by $y =$ $\boxed{-8x + 1}$

Question 5. [6 points] Where is the function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 4$ increasing and where is it decreasing on the interval $x \in [-3, 6]$. Find all the local and global maxima and minima of this function on this interval. Give their x and y values.

$f'(x) = x^2 - x - 6 = (x-3)(x+2)$; So Critical #s are found: $f'(x) = 0 \Rightarrow (x-3)(x+2) = 0 \rightarrow \begin{cases} x = 3 \\ x = -2 \end{cases}$

x	-3		-2		3		6
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	8.5	↗	$11\frac{1}{3}$	↘	-9.5	↗	22

Answer:

f is increasing for $x \in (-3, -2) \cup (3, 6)$

f is decreasing for $x \in (-2, 3)$

f has local maxima at $x = -2$; $y = 11\frac{1}{3}$

f has local minima at $x = 3$; $y = -9.5$

f has global maxima at $x = 6$; $y = 22$

f has global minima at $x = 3$; $y = -9.5$

Question 6. [6 points] Assume that a function f as well as its first and second derivatives are continuous for all $x \in (-\infty, \infty)$. Assume that the function has the following properties:

- $f'(x) < 0$ if $x < 1$ and $f'(x) > 0$ if $x > 1$.
- $f''(x) < 0$ if $|x| > 2$ and $f''(x) > 0$ if $|x| < 2$
- $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 2$
- $f(1) = 0$

Answer the following questions:

(i) The graph of $f(x)$ increasing for $x \in (1, \infty)$

(ii) The graph of $f(x)$ concave up for $x \in (-2, 2)$ or $|x| < 2$

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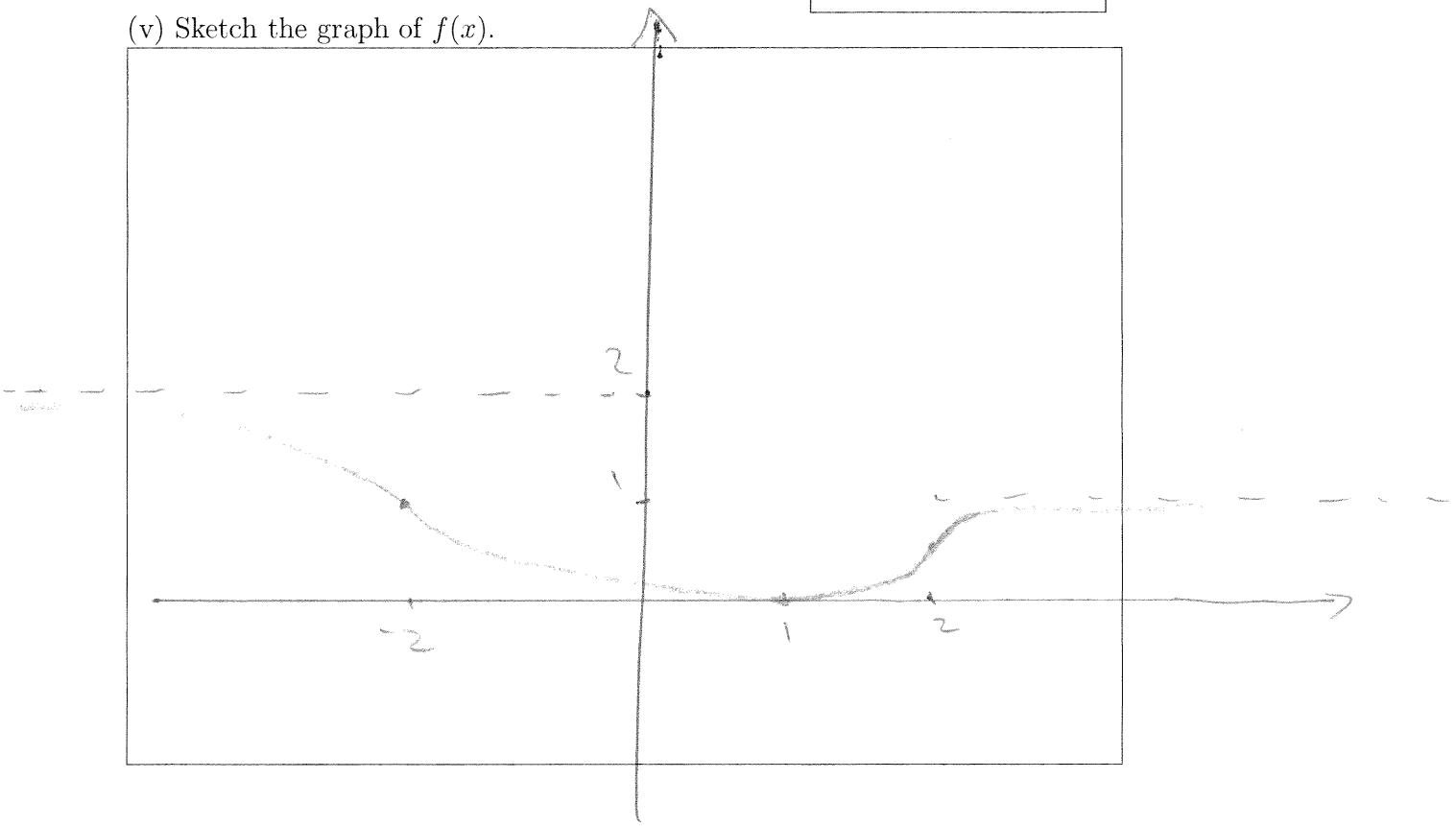
(v) Sketch the graph of $f(x)$.



Rough work!