

Name _____ Student N° _____

Relationships between w , V , M

$$\frac{dV}{dx} = -w(x), \quad \frac{dM}{dx} = V$$

Centroid

$$\bar{x} = \frac{Q_y}{A} = \frac{\sum \bar{x}_i A_i}{\sum A_i}, \quad \bar{y} = \frac{Q_x}{A} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$

Moments of inertia

$$I_x = \int y^2 dA, \quad I_y = \int x^2 dA$$

Polar moment of inertia

$$J_o = \int r^2 dA = I_x + I_y$$

Product of moment of inertia

$$I_{xy} = \int xy dA$$

Parallel axis theorem

$$I_x = I_{x_c} + Ad_1^2 \quad I_y = I_{y_c} + Ad_2^2$$

$$J_o = J_c + Ad^2 \quad I_{xy} = I_{x_c y_c} + Ad_1 d_2$$

Principal moments of inertia (max/min)

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$\tan(2\theta) = \frac{-2I_{xy}}{(I_x - I_y)}$$

Note: the geometric properties table for simple shapes (rectangle, circle and triangle) will be provided with the exam.

Axial members**Normal stress & strain**

$$\sigma = \frac{P}{A}, \quad \varepsilon = \frac{\delta}{L}$$

Axial deformation

$$\delta = \frac{PL}{AE}, \quad \delta = \sum \frac{N_i L_i}{A_i E_i}, \quad \delta = \int_0^L \frac{N}{AE} dx$$

Space for notes

- You may include notes in this **BOXED** area **ONLY**
- The notes must be **hand written** (no photocopies or computer print)
- You may include explanations/ diagrams
- You are **not permitted** to include **sample problems**

TO BE RETURNED WITH THE EXAM

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Torsion*Shear stress (circular section)*

$$\tau = \frac{T\rho}{J}$$

Angle of twist

$$\phi = \frac{TL}{JG}, \quad \phi = \sum \frac{T_i L_i}{J_i G_i}, \quad \phi = \int_0^L \frac{T}{JG} dx$$

Flexure*Normal stress*

$$\sigma = -\left(\frac{My}{I}\right) \quad |\sigma_{\max}| = \left|\frac{My_{\max}}{I}\right|$$

Shear stress

$$\tau = \frac{VQ}{Ib} \quad \tau_{\max} = \frac{VQ_{\max}}{Ib} \quad Q = \sum (A_i')(\bar{y}_i')$$

Deflections

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

Discontinuity functions for $n \geq 0$

$$\langle x-a \rangle^n = \begin{cases} 0 & \text{for } x < a \\ (x-a)^n & \text{for } x \geq a \end{cases}$$

$$\int_{-\infty}^x \langle x-a \rangle^n dx = \frac{\langle x-a \rangle^{n+1}}{n+1}$$

Note: the table with the $M\langle x \rangle$ functions is provided on page 4.

Buckling

| | B.C.'s | K |
|------------------------------------|---------------|----------|
| $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$ | Pinned-pinned | 1 |
| | Fixed-free | 2 |
| | Fixed-fixed | 0.5 |
| | Fixed-pinned | 0.7 |

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Stress transformation equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}, \quad \tau_{12} = 0$$

Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}, \quad \theta_s = \theta_p \pm 45^\circ$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Mohr's circle

$$C = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = C \pm R$$

$$\tau_{\max} = R, \quad \sigma_{\text{avg}} = C$$

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DISCONTINUITY FUNCTIONS

| | |
|--|--|
| | $w(x) = M_o \langle x-a \rangle^{-2}$ $V(x) = -M_o \langle x-a \rangle^{-1}$ $M(x) = -M_o \langle x-a \rangle^0$ |
| | $w(x) = P \langle x-a \rangle^{-1}$ $V(x) = -P \langle x-a \rangle^0$ $M(x) = -P \langle x-a \rangle$ |
| | $w(x) = w_o \langle x-a \rangle^0$ $V(x) = -w_o \langle x-a \rangle$ $M(x) = -\frac{1}{2} w_o \langle x-a \rangle^2$ |
| | $w(x) = w_o \langle x-a_1 \rangle^0 - w_o \langle x-a_2 \rangle^0$ $V(x) = -w_o \langle x-a_1 \rangle + w_o \langle x-a_2 \rangle$ $M(x) = -\frac{1}{2} w_o \langle x-a_1 \rangle^2 + \frac{1}{2} w_o \langle x-a_2 \rangle^2$ |
| | $w(x) = \frac{w_o}{b} \langle x-a \rangle$ $V(x) = -\frac{1}{2} \frac{w_o}{b} \langle x-a \rangle^2$ $M(x) = -\frac{1}{6} \frac{w_o}{b} \langle x-a \rangle^3$ |
| | $w(x) = \frac{w_o}{b} \langle x-a_1 \rangle - \frac{w_o}{b} \langle x-a_2 \rangle - w_o \langle x-a_2 \rangle^0$ $V(x) = -\frac{1}{2} \frac{w_o}{b} \langle x-a_1 \rangle^2 + \frac{1}{2} \frac{w_o}{b} \langle x-a_2 \rangle^2 + w_o \langle x-a_2 \rangle$ $M(x) = -\frac{1}{6} \frac{w_o}{b} \langle x-a_1 \rangle^3 + \frac{1}{6} \frac{w_o}{b} \langle x-a_2 \rangle^3 + \frac{1}{2} w_o \langle x-a_2 \rangle^2$ |