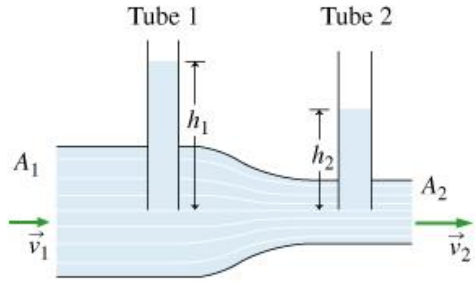
Problem 1 (3 points)

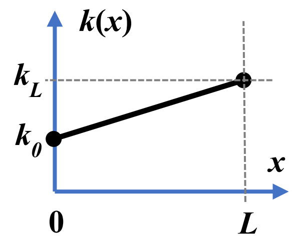
Consider a pair of vertical, open-ended glass tubes inserted into a horizontal pipe with a horizontal pipe carrying a steadily moving, incompressible fluid of density ρ (see figure) flowing with negligible dissipation. The cross-sectional area of the pipe is A_1 at the position of *tube 1*, and A_2 at the position of *tube 2*. The fluid rises to heights h_1 and h_2 in the two open-ended tubes.



a) With g the magnitude of the acceleration due to gravity (take g ~ 10 m s^{-2}), h_1 = 200 mm, h_2 = 100 mm, A_1 = 1 mm^2, and A_2 = 0.5 mm^2, calculate v_1, the speed of the fluid in the left end of the main pipe. Express the result in the SI units.

Problem 2 (5 points)

Find the thermal resistance $R = (T_H - T_C)/H$ of a rod of constant section A and total length L when the thermal conductivity of the material forming the rod changes linearly from $x = 0$ to $x = L$ according to the graph showed to the right. (T_H is the temperature of the hot end of the rod and T_C is the temperature of the cold end of the rod.)



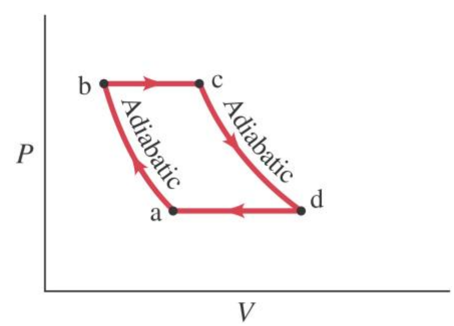
(*Hint*: You might find useful the following integral $\int x^{-1}dx = \ln x + constant$.)

Problem 3 (5 points)

The thermodynamic cycle showed in figure makes an ideal gas undergoing two adiabatic transformations (a → b and c → d) and two isobaric transformations (b → c and d → a). Show that the efficiency of this cycle is

$$e = 1 - \left(\frac{P_b}{P_a}\right)^{\frac{1-\gamma}{\gamma}}$$

where $\gamma = C_P/C_V$.



Problem 4 (5 points)

A cup of mass 200 g initially at temperature 20 ^{o}C is filled with 400 g of a liquid initially at temperature 100 ^{o}C. The cup has a specific heat c_{cup} = 1000 J/(Kg ^{o}C) and the liquid has a specific heat c_{liq} = 5000 J/(Kg ^{o}C).
Calculate the final temperature (T_{final}) at the tehrmal equilibrium and the total change in entropy (ΔS) as a result of the process leading to the themal equilibrium.

Problem 5 (1 points)

Which of the following is true about any system that undergoes a reversible thermodynamic process?

(A) There are no changes in the internal energy of the system.

(B) The temperature of the system remains constant during the process.

(C) The entropy of the system and its environment remains unchanged.

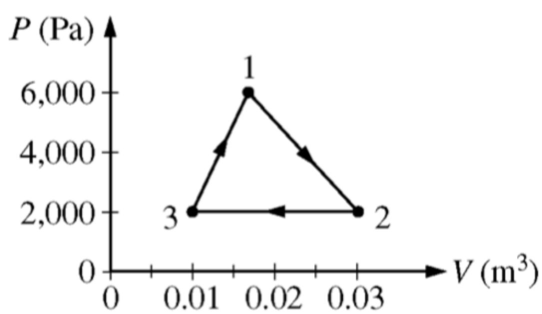
(D) The entropy of the system and its environment must increase.

(E) The net work done by the system is zero.

(F) None of the above

Problem 6 (1 points)

A sample of nitrogen gas (N_2) undergoes the cyclic thermodynamic process shown here. Which of the following gives the net heat transferred to the system in one complete cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$?



(A) - 80 J

(B) - 40 J

(C) + 40 J

(D) +80 J

(E) +180 J

(F) None of the above

Solutions

Problem 1 (3 points)

- This is the solved example in the book 12.9.
- It was also one of the homework problems that had 95% of correct answers.

Solutions:

The pressure in zone-1 is $p_1 = \rho g h_1$ and using the Bernoulli equation plus the continuity equation we obtain:

$$\sqrt{\frac{2g\left(h_1 - h_2\right)}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

Then

$$v_1 = \sqrt{(2/3)} \ \ \mathrm{m/s}$$

Problem 2 (5 points)

Solutions:

Given the linear dependence in the graph, $k(x)$ follows the law

$$k(x) = k_0 + (k_L - k_0)\frac{x}{L}.$$

The heat current is given by

$$H = -k(x)\, A \frac{dT}{dx}$$

then

$$-\frac{1}{H}\int dT = \frac{1}{A}\int \frac{dx}{k(x)}$$

$$-\frac{1}{H}\int_{T_0 = T_H}^{T_L = T_C} dT = \frac{T_H - T_C}{H} = R \; ; \quad \frac{1}{A}\int_0^L \frac{dx}{k_0 + (k_L - k_0)\frac{x}{L}}$$

Introducing the variable $k = k_0 + (k_L - k_0)\frac{x}{L}$ and then $dk = \frac{k_L - k_0}{L}\, dx$

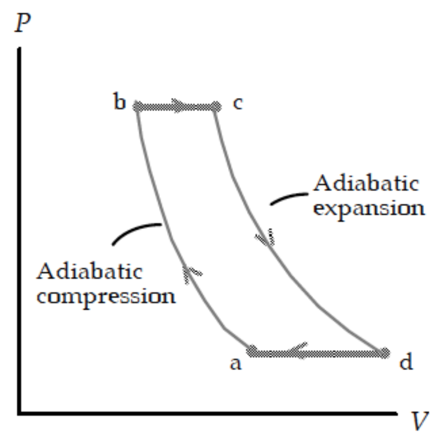
$$R = \frac{1}{A}\frac{L}{k_L - k_0}\int_{k_0}^{k_L}\frac{dk}{k} = \frac{1}{A}\frac{L}{k_L - k_0}\ln k\Big|_{k_0}^{k_L}$$

$$\boxed{R = \frac{L}{A(k_L - k_0)}\ln\frac{k_L}{k_0}}$$

Problem 3 (5 points)

- DGD#4 Problem 19.42.
- Book example 19.7.
- Close to Problem 19.27, which had 97% correct answers.

Solutions:



Since two of the processes are adiabatic, no heat transfer occurs in those processes. Thus the heat transfer must occur along the isobaric processes.

$$Q_H = Q_{bc} = nC_P(T_c - T_b) \;;\; Q_L = Q_{da} = nC_P(T_d - T_a)$$

$$e = 1 - \frac{Q_L}{Q_H} = 1 - \frac{nC_P(T_d - T_a)}{nC_P(T_c - T_b)} = 1 - \frac{(T_d - T_a)}{(T_c - T_b)}$$

Use the ideal gas relationship, which says that $PV = nRT$.

$$e = 1 - \frac{(T_d - T_a)}{(T_c - T_b)} = 1 - \frac{\left(\frac{P_d V_d}{nR} - \frac{P_a V_a}{nR}\right)}{\left(\frac{P_c V_c}{nR} - \frac{P_b V_b}{nR}\right)} = 1 - \frac{(P_d V_d - P_a V_a)}{(P_c V_c - P_b V_b)}$$

$$= 1 - \frac{P_a(V_d - V_a)}{P_b(V_c - V_b)}$$

Because process ab is adiabatic, we have $P_a V_a^\gamma = P_b V_b^\gamma \;\rightarrow\; V_a = V_b \left(\frac{P_b}{P_a}\right)^{1/\gamma}$. Because process cd is adiabatic, we have $P_b V_c^\gamma = P_a V_d^\gamma \;\rightarrow\; V_d = V_c \left(\frac{P_b}{P_a}\right)^{1/\gamma}$. Substitute these into the efficiency expression.

$$e = 1 - \frac{P_a(V_d - V_a)}{P_b(V_c - V_b)} = 1 - \frac{P_a\left(V_c\left(\frac{P_b}{P_a}\right)^{1/\gamma} - V_b\left(\frac{P_b}{P_a}\right)^{1/\gamma}\right)}{P_b(V_c - V_b)} = 1 - \frac{P_a\left(\frac{P_b}{P_a}\right)^{1/\gamma}(V_c - V_b)}{P_b(V_c - V_b)}$$

$$= 1 - \left(\frac{P_b}{P_a}\right)^{\frac{1}{\gamma} - 1} = \boxed{1 - \left(\frac{P_b}{P_a}\right)^{\frac{1-\gamma}{\gamma}}}$$

Problem 4 (5 points)

- DGD#4 Problem 20.53.
- Book example 20.6.

Solutions:

The equilibrium temperature is found using calorimetry arguments: The heat lost by the liquid is equal to the heat gained by the cup

$$m_{liq} c_{liq} (T_{liq} - T_{final}) = m_{cup} c_{cup} (T_{cup} - T_{final})$$

$T_{final} < 100$ °C (~ 90 °C)

$$\Delta S = \Delta S_{cup} + \Delta S_{liq} =$$

$$= \int_{T_{cup}}^{T_{final}} \frac{dQ_{cup}}{T} + \int_{T_{liq}}^{T_{final}} \frac{dQ_{liq}}{T} = m_{cup} c_{cup} \int_{T_{cup}}^{T_{final}} \frac{dT}{T} + m_{liq} c_{liq} \int_{T_{liq}}^{T_{final}} \frac{dT}{T}$$

$$= m_{cup} c_{cup} ln \frac{T_{final}}{T_{cup}} + m_{liq} c_{liq} ln \frac{T_{final}}{T_{liq}}$$

Problem 5 **(1 points)**

(C)

This the equation 20.21 of the book stating that the total entropy change during any reversible cycle is zero.

Problem 6 **(1 points)**

(C)

See example 19.3.

In any cycle Q = W, then the work done by the system is to the heat added to the system, which in this case is given by the area of the triangle: + 0.5 (0.02) (4,000) J