

MAST 217/2 2016 Introduction to Mathematical Thinking

Class test

The maximum mark for this test is 20. Each question is worth 4 marks.

Question 1

Describe the following set:

$$A = \{x \in \mathbb{R}: x^2 - (\sqrt{3} + \sqrt{15})x + 3\sqrt{5} = 0\} \cap \mathbb{Q}$$

Justify your answer.

Question 2

Prove: If A , B , and C are sets such that A is a proper subset of $P(B)$ and $P(B)$ is a proper subset of C and A has exactly 2 elements then the number of elements of C can be 5 but not 4.

Question 3

Conduct a logical analysis, in terms of conjunctions (AND) and disjunctions (OR), of the given condition $A(x)$ on the real variable x :

$$A(x): |x - 7| < 2|x + 1|$$

Question 4

Let $n \in \mathbb{Z}$. Prove:

$$5n + 2 \text{ is even} \Leftrightarrow 7n - 1 \text{ is odd}$$

Question 5

Let a, r be positive real numbers. We define the sequence $\{a_n\}$ by a recurrence relation as follows:

$$a_1 = a$$

$$a_{n+1} = a_n + r$$

Prove by induction on n that the sum S_n of the first n terms of this sequence can be calculated using the formula

$$S_n = na + \frac{n(n-1)}{2}r$$