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Civil Engineering Department

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CVG2132 – FUNDAMENTALS OF ENVIRONMENTAL ENGINEERING

Homework 3: SOLUTIONS Professor: Rob Delatolla

Due Date: Oct. 13, 2017 (3:00pm) – Dropbox « CVG 2132 », Mezzanine A (0.5) CBY

Note: C_0 is the initial concentration or concentration entering the system,
C is the final concentration, Q is the flow rate, V is the reactor volume, k is the reaction kinetic constant and τ is the hydraulic retention time (HRT) defined as V/Q

Question 1. An industrial process produces a by-product compound 'C' which has been shown to be toxic to the environment and harmful to bacteria at high concentrations. Therefore, before the wastewater can be discharged to the public wastewater treatment plant, the concentration of 'C' must be reduced to below 30 mg C/L. The industrial wastewater, with a concentration of 2500 mg C/L, flows through a long series of channels before reaching the treatment reactor. The long series of channels can be modelled as a PFR with a volume of 25 m^3 and the treatment reactor can be modelled as a CSTR with a volume of 10 m^3. The compound 'C' naturally degrades in the series of channels before the treatment reactor with a reaction rate of 0.75 mg/(L*min). In the treatment reactor the addition of chemicals and catalysts cause compound 'C' to degrade according to second order kinetics described with the constant of 0.015 L/(mg*min).

a) Derive the appropriate HRT equations for the PFR channels and the CSTR treatment reactor (state all assumptions and show all work).
b) Does the final effluent meet the required concentration of compound 'C' if the flow rate is 245 m^3/d in the channel and through the treatment reactor? If not determine the maximum flow rate that can be accommodated to meet the discharge limit of 30 mg C/L.

Soluton: The objectives of this problem are to familiarise you with zero order PFRs, second order CSTRs and reactors in series.

Part A)

Step I) State assumptions

1. As there is no indication of change with respect to time we can assume steady state applies
2. Unless stated otherwise, assume dilute streams. Therefore the density of all streams is equivalent to the density of water i.e $\rho_W = \rho = 1000$ kg/m^3

Step II) Derive the equation for the PFR

Based on the units of k = 0.75 mg/(L*min) we can see that the reaction is zero order meaning that the concentration of chemical 'C' has no effect on consumption.

Therefore r = -k

Perform a constituent mass balance around a volume ΔV
As there is only a single stream entering and leaving the reactor and steady state conditions apply
$Q_{in} = Q_{eff} = Q$
Acc = In – Out + Reaction
Acc = 0 due to steady state

$$0 = QC_x - QC_{x+\Delta x} + rV$$
$$r = \frac{Q(C_{x+\Delta x} - C_x)}{V}$$
$$V = \Delta V = A\Delta X$$
$$C_{x+\Delta x} - C_x = \Delta C$$
$$r = \frac{(Q\Delta C)}{(A\Delta X)}$$
$$r = -k$$
$$-k = \frac{(Q\Delta C)}{(A\Delta X)}$$
$$-\frac{A\Delta X}{Q} = \frac{\Delta C}{k}$$

Integrate the length of the reactor

$$-\frac{A}{Q}\int_0^L dX = \frac{1}{k}\int_{C0}^{C} dC$$
$$-\tau = \frac{(C - C_0)}{k}$$
$$\tau = \frac{(C_0 - C)}{k}$$

<u>Step III) Derive the equation for the CSTR</u>
The problem statement states that due to the addition of chemicals and catalysts, the CSTR performs with second order kinetics. Therefore $r = -kC^2$
So, performing a constituent mass balance around the system
As there is only a single stream entering and leaving the reactor and steady state conditions apply
$Q_{in} = Q_{eff} = Q$
Acc = In – Out + reaction
Acc = 0 due to steady state

$$0 = QC_0 - QC + rV_{CSTR}$$
$$0 = QC_0 - QC - kC^2V_{CSTR}$$

Divide by Q

$$0 = C_0 - C - kC^2\tau$$
$$\tau = \frac{(C_0 - C)}{C^2k}$$
$$\tau = \frac{\left(\frac{C_0}{C} - 1\right)}{kC}$$

Part B)

Step IV) Given Information

Q_{in} = 245 m^3/d

C_0 = 2500 mg/L

V_{PFR} = 25 m^3

V_{CSTR} = 10 m^3

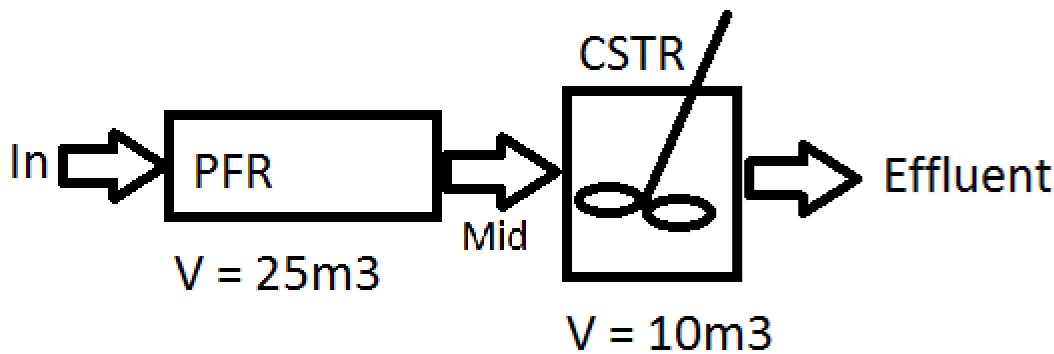
K_0 = 0.75 mg/(L*min) = 0.75 g/(m^3*min) * 1440 min/d = 1080 g/(m^3*d)

K_2 = 0.015 L/(mg*min) = 0.015 m^3/(g*min) * 1440 min/d = 21.6 m^3/(g*d)

Goal = 30 mg/L = 30 g/m^3

Unknowns: Q_{mid}, Q_{Eff}, C_{mid}, C_{eff}

Step V) Draw system diagram



Step VI) Total Mass Balance around the PFR

Acc = In – Out

Acc = 0 b/c steady state

$0 = Q_{in}\rho_{in} - Q_{min}\rho_{mid}$

Dilute streams therefore $\rho_{in} = \rho_{mid} = \rho$ so density can be divided out

$Q_{in} = Q_{mid}$

Step VII) Determine the PFR effluent concentration of 'C'

Using the HRT equation derived in step II

$\tau = (C_0 - C)/k_0$

rearrange to isolate for C

$C = C_0 - \tau k_0$

$\tau = V_{PFR}/Q$ = 25m^3/245 m^3/d = 0.102d

C = 2500 g/m^3 – 0.102d*1080 m^3/(g*d)

C = 2500 g/m^3 – 110.16 g/m^3

C = 2389.84 g/m^3 = 2390 g/m^3

Step VIII) Determine the flowrate exiting the CSTR

Total Mass Balance around the CSTR

Acc = In – Out

Acc = 0 b/c steady state

$0 = Q_{mid}\rho_{mid} - Q_{Eff}\rho_{Eff}$

Dilute streams therefore $\rho_{mid} = \rho_{Eff}$ so density can be divided out

$Q_{mid} = Q_{Eff}$

<u>Step IX) Determine the effluent concentration of ‘C’</u>
Using the HRT equation derived in step III
$\tau = (C_0/C - 1)/KC$
rearranging to isolate C we get
$C^2\tau k + C - C_0 = 0$
$\tau = V_{CSTR}/Q = 10m^3/245m^3/d$, $k = 21.6\ m^3/(g*d)$ and $C_0 = C_{mid} = 2390\ g/m^3$
Using the quadratic equation we find that $C = C_{eff} = 51.5\ g/m^3$

This value is higher than the target of 30 g/m^3 therefore we need to find the required flowrate

<u>Step X) Determine the required flowrate</u>
As C_0 for the CSTR is C from the PFR we can combine the total equations to get
$\tau_{CSTR} = ((C_{in} - \tau_{PFR}k_0)/C_{eff} - 1)/C_{eff}k_2$
$V_{CSTR}/Q = ((C_{in} - V_{PFR}k_0/Q)/C_{eff} - 1)/C_{eff}k_2$
$(C_{eff}k_2V_{CSTR})/Q = C_{in}/C_{eff} - V_{PFR}k_0/QC_{eff} - 1$
$C_{eff}k_2V_{CSTR} = C_{in}Q/C_{eff} - V_{PFR}k_0/C_{eff} - Q$ multiplying the Q term by C_{eff}/C_{eff}
$C_{eff}k_2V_{CSTR} = C_{in}Q/C_{eff} - VPF_Rk_0/C_{eff} - C_{eff}Q/C_{eff}$ Now we can group the right hand side to get
$C_{eff}k_2V_{CSTR} = (QC_{in} - QC_{eff} - V_{PFR}k_0)/C_{eff}$
$C_{eff}^2k_2V_{CSTR} = Q(C_{in} - C_{eff}) - V_{PFR}k_0$
$C_{eff}^2k_2V_{CSTR} + V_{PFR}k_0 = Q(C_{in} - C_{eff})$
$Q = (C_{eff}^2k_2V_{CSTR} + V_{PFR}k_0)/(C_{in} - C_{eff})$
Now we want a C_{eff} value of 30 g/m^3 and we have C_{in}= 2500 mg/L
$Q = ((30\ g/m^3)^2(21.6\ m^3/(g*d))(10m^3) + 25m^3*1080g/(m^3*d))/(2500\ g/m^3 - 30\ g/m^3)$
$Q = 89.6\ m^3/d$
Therefore to reach the desired effluent concentration a flowrate of less than 89.6 m^3/d is required.

Question 2. The Ottawa wastewater treatment facility is investigating the potential of ammonia treatment at their facility. The aeration basin of the Ottawa facility is a baffled tank that can be modelled as an ideal PFR. The wastewater facility’s influent flow rate is 545 000 m^3/d with an assumed influent ammonia concentration of 30 mg NH_3-N/L. The baffled tank has a volume of 25 000 m^3 and an ammonia removal reaction kinetics constant of 8.68 L/(mg*d). Note that the ammonia removal reaction is dependent on both the ammonia concentration and dissolved oxygen concentration, $r = kC_{NH3}C_{O2}$. The Ottawa wastewater treatment facility is considering providing a constant dissolved oxygen concentration of 4 mg O_2/L or 8 mg O_2/L in the aeration basin to attain an effluent ammonia target of 1.25 mg NH_3-N/L.

a) Derive the pseudo-first order HRT equation for the baffled tank (state all assumptions and show all work).
b) Determine which of the oxygen concentrations (4 mg O_2/L or 8 mg O_2/L) are required to meet the effluent ammonia requirement. As all options must be explored, please calculate both options.

Solution: The objective of this problem is to familiarise you with pseudo first order and first order PFRs

Part A)
<u>Step I) State assumptions</u>
1. As there is no indication of change with respect to time we can assume steady state applies
2. Unless stated otherwise, assume dilute streams. Therefore the density of all streams is equivalent to the density of water i.e $\rho_W = \rho = 1000\ kg/m^3$

Step II) Derive the pseudo-first order PFR equation

The problem statement describes that the concentration of oxygen therefore we can combine the C_{O2} and k_2 terms to get a first order k' term. Therefore r = -k'C_{NH4} = -$k_2 C_{NH4} C_{O2}$

Acc = In – Out ± Generation

Acc = 0 due to steady state

As there is only a single stream entering and leaving the reactor and steady state conditions apply

$Q_{in} = Q_{eff} = Q$

$0 = QC_0 - QC_{eff} + rV$

As this is a pfr set $C_0 = C_x$ and C_{eff} as $C_{x+\Delta x}$

$$0 = QC_x - QC_{x+\Delta x} + rV$$

$$r = \frac{Q(C_{x+\Delta x} - C_x)}{V}$$

$$V = \Delta V = A\Delta X$$

$$C_{x+\Delta x} - C_x = \Delta C$$

$$r = \frac{(Q\Delta C)}{(A\Delta X)}$$

$$r = -kC$$

$$-kC = \frac{(Q\Delta C)}{(A\Delta X)}$$

$$-\frac{A\Delta X}{Q} = \frac{\Delta C}{kC}$$

Integrate the length of the reactor

$$-\frac{A}{Q}\int_0^L dX = \frac{1}{k}\int_{C0}^{C}\frac{1}{C}dC$$

$$-\tau = \frac{(lnC - lnC_0)}{k}$$

$$\tau = \frac{(lnC_0 - lnC)}{k}$$

Part B)

Step III) Given Information

Q_{in} = 545,000 m^3/d

V_{PFR} = 25000 m^3

K_2 = 8.68 L/(mg*d) = 8.68 m^3/(g*d)

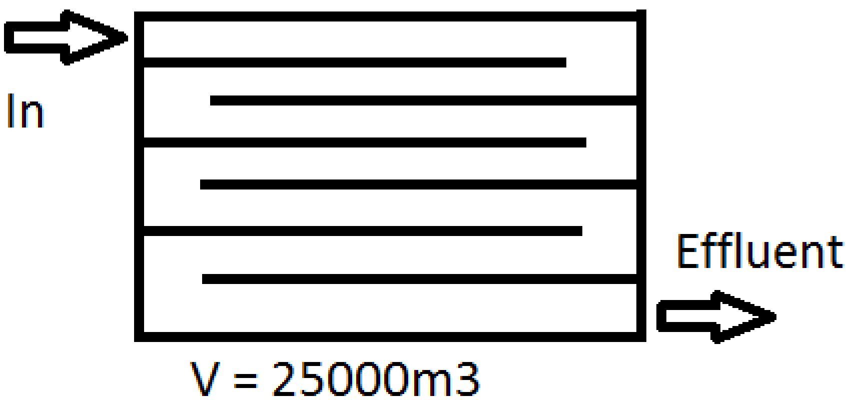
DO_1 = 4 g O_2/m^3

DO_2 = 8 g O_2/m^3

C_{in} = 30 mg NH_4^+-N/L = 30 g N/m^3

Unknowns: C_{eff} at 4 g O_2/L and C_{eff} at 8 g O_2/L

Step IV) Draw the system diagram



Step V) Determine the effluent concentration with a DO of 4 g O_2/m^3

Using the equation previously defined we get

$\tau = (\ln C_0 - \ln C)/k$

Where $C_0 = C_{in}$, $C = C_{eff}$, $\tau = V_{PFR}/Q$ and $k = k' = k_2 C_{O2}$

For C_{O2} = 4 g O_2/m^3 k' = 8.68 $m^3/(g*d)$*4 gO_2/m^3 = 34.72 1/d

Rearranging the equation gets us

$C_{eff} = C_{in}e^{-\tau k'}$

If $\tau = V_{PFR}/Q$ = 25000 m^3/545,000 m^3/d = 0.0459 d then

$C_{eff} = (30\ g/m^3)e^{-(0.0459\ d)(34.72\ 1/d)}$

$C_{eff} = 6.1\ g/m^3$

Step VI) Determine the effluent concentration with a DO of 8 g O_2/m^3

Using the equation previously defined we get

$\tau = (\ln C_0 - \ln C)/k$

Where $C_0 = C_{in}$, $C = C_{eff}$, $\tau = V_{PFR}/Q$ and $k = k' = k_2 C_{O2}$

For C_{O2} = 8 g O_2/m^3 k' = 8.68 $m^3/(g*d)$*8 gO_2/m^3 = 69.44 1/d

Rearranging the equation gets us

$C_{eff} = C_{in}e^{-\tau k'}$

If $\tau = V_{PFR}/Q$ = 25000 m^3/545,000 m^3/d = 0.0459 d then

$C_{eff} = (30\ g\ N/m^3)e^{-(0.0459\ d)(69.44\ 1/d)}$

$C_{eff} = 1.24\ g\ N/m^3$

Therefore a dissolved oxygen concentration of 8 g O_2/m^3 is required to attain a final effluent concentration lower than 1.25 mg NH_4^+-N/L

Question 3. The documentary 'Water Life' discusses the great lakes and how they have experienced anthropogenic effects over the last decades.

a) Describe two effects on the great lakes that have been observed in the last 50 years.
b) Explain how these effects were initiated.
c) Comment on how or if these effects can be mediated or resolved.

Some possible answers from the movie clips shown in class are: (other answers are acceptable)

Part A)

1. Lamprey eel: Lamprey has no natural predators and they feed on the fish species.

2. Asian Carp: Asian Carp eats 40% of its body weight each day and are not edible fish.
3. Zebra Mussels: Have taken over the water systems completely in a few years and remove most of the nutrients from the water and the fish are starving to death.
4. Caspian shrimp: Observed in 2006, ½" long bright red shrimp from the Black and Caspian Sea, effects are still unknown

Part B) Most of these species were introduced by large tankers that travel between many waters.

Part C) Electric fences are used for Asian Carp to prevent them from moving between lakes, others the effects are extremely hard to mediate if not impossible. At this point try to preserve waters that have yet to be affected.

Question 4. A small, well-mixed lagoon, with a length of 20m, a width of 10m and a depth of 6.25m, is used to treat a diluted wastewater stream with a concentration of 47 mg BOD/L (BOD: biochemical oxygen demand). The influent wastewater stream flow rate is 280 m^3/d and the reaction kinetics can be modelled by the reaction constant of 0.007 mg/(L*min).

One day, a surge of undiluted wastewater entered the small lagoon after which the lagoon was closed until the concentration inside the lagoon returned to normal concentrations (i.e. concentrations prior to the surge). The concentration inside the lagoon at the moment of closing was 167 mg BOD/L.

a) Derive the appropriate HRT equation for lagoon if it is modelled by a CSTR (state all assumptions and show all work).
b) Determine the lagoon effluent BOD concentration prior to the surge.
c) Determine the amount of time it will take for the concentration inside the lagoon to return from 167 mg BOD/L to the BOD concentration prior to the surge so that the lagoon operation can be restarted.

Note: When the lagoon is closed there is nothing flowing in nor out.

Solution: The objective of this question is to familiarise you with zero order cstr and batch/ unsteady state processes

Part A)

Step I) State assumptions

1. As there is no indication of change with respect to time we can assume steady state applies
2. Unless stated otherwise, assume dilute streams. Therefore the density of all streams is equivalent to the density of water i.e $\rho_W = \rho = 1000\ kg/m^3$

Step II) Derive the zero order CSTR equation

Based on the units of k= 0.007 mg/(L*min) we can determine that the CSTR operates at zero order therefore r=-k

So, performing a constituent mass balance around the system

As there is only a single stream entering and leaving the reactor and steady state conditions apply

$Q_{in} = Q_{eff} = Q$

Acc = In – Out – reaction

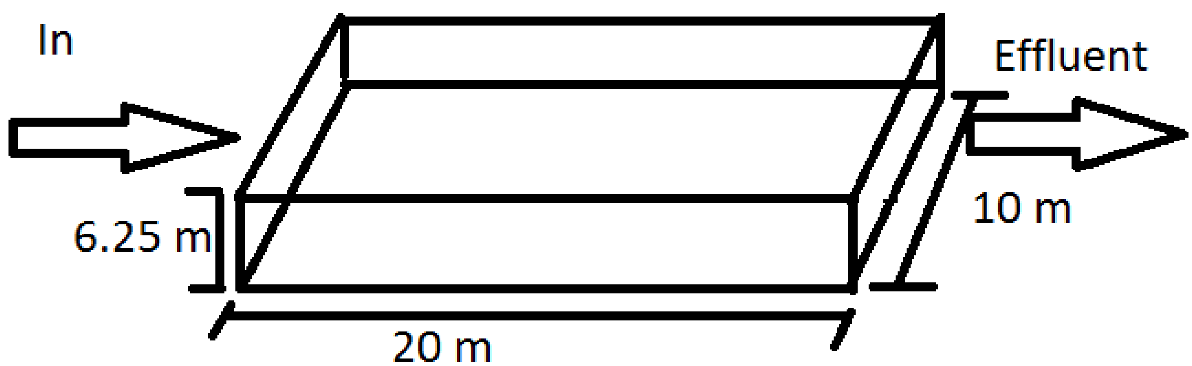
Acc = 0 due to steady state

$$0 = QC_0 - QC + rV_{CSTR}$$

$$0 = QC_0 - QC - kV_{CSTR}$$
Divide by Q
$$0 = C_0 - C - k\tau$$
$$\tau = \frac{(C_0 - C)}{k}$$

Part B)

Step III) Draw a system diagram



Step IV) Given information

Q = 280 m^3/d
C_{in} = 47 mg BOD/L
L = 20 m
W = 10 m
D = 6.25 m
V_{CSTR} = L*W*D = 20 m* 10 m* 6.25 m = 1250 m^3
k_0 = 0.007 mg/(L*min) = 0.007 g/(m^3*min)* 1440 min/d = 10.08 g/(m^3*d)
Unknowns: C_{eff}

Step V) Determine the normal effluent concentration

Using the equation previously derived we have:
τ= (C_0 – C)/k
Where C_0 = C_{in}, C = C_{eff}, τ = V_{CSTR}/Q and k = k_0
We can rearrange the equation to isolate C_{eff}
C_{eff} = C_{in} – $k_0 V_{CSTR}$/Q
C_{eff} = 47 g/m^3 – 10.08 g(m^3*d)*1250m^3/280 m^3/d = 2 g/m^3

Part C)

After the surge, the lagoon was closed so there was no influent or effluent flow. This means that the BOD in the lagoon can naturally degrade on its own. However with no influent or effluent to remove/replace the BOD, therefore the BOD only degrades. This means that the lagoon is no longer steady state and as there is no flow it is no longer a CSTR but rather a batch reactor.
To reopen the lagoon, the BOD concentration inside the system must return to the concentration that it was at steady state, in this case 2 g BOD/m^3.

Step VI) Determine the time for the BOD to return to normal

After the surge the concentration became C_0 = 167 g BOD/m^3.
Using the equation derived in the notes we have

$t = (C_0 – C)/k$
Where $C = 2\ g/m^3$ and $k = k_0 = 10.08\ g/(m^3*d)$
Therefore
$t = (167\ g/m^3 – 2\ g/m^3)/(10.08\ g/(m^3*d)) = 16.4\ d$

Therefore the lagoon can be reopened after 16.4 d. Note the concentration would decrease faster if there was influent and effluent flow however that would also expel effluent with concentrations much higher than the receiving water could handle.