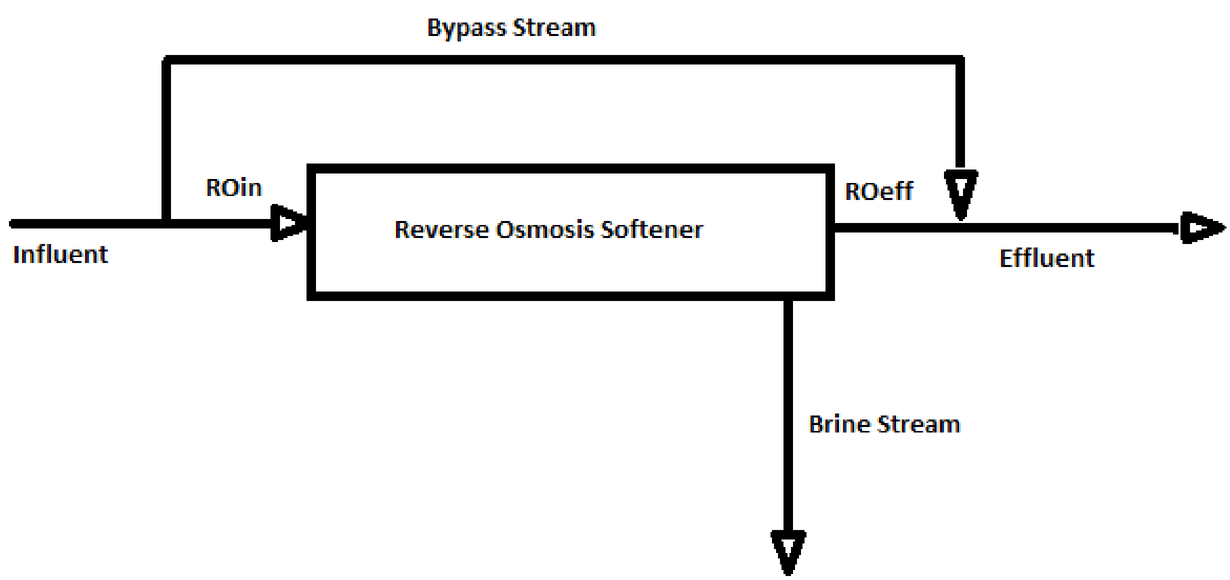
CVG2132 – FUNDAMENTALS OF ENVIRONMENTAL ENGINEERING

Homework 2 SOLUTIONS: Professor: Rob Delatolla

Due Date: Friday, October 6, 2017 (3:00pm) – Dropbox « CVG 2132 », Mezzanine A (0.5) CBY

Question 1. You have been hired to design a reverse osmosis water softening system that will be used in an office building in Kitchener (Average water hardness = 360 mg $CaCO_3$/L). You will design for an average influent concentration of 360 mg $CaCO_3$/L and a daily flow of 12000 L/d. The reverse osmosis softener removes all of the hardness from the influent and outputs a pure water stream and a concentrated brine stream. The brine stream has an effluent concentration of 2750 mg $CaCO_3$/L. However, low concentrations of hardness are not harmful so not all of the hardness needs to be removed. Therefore, as a cost saving measure, a bypass stream will be used to create a final effluent concentration of 85 mg $CaCO_3$/L. Your task is to determine the effluent flow rate, the bypass flow rate and the brine stream flow rate. Assume dilute streams.



Solution: The objective of this problem is to familiarise you with the manipulation of mass balances. As mass balances can be very open ended, there are multiple ways to solve this problem.

Note: Bypass stream is denoted by B, Brine stream is denoted by Br, Influent for Reverse Osmosis is Rin and effluent of Reverse Osmosis is Ro

Step I) State all assumptions

Assumptions:

1. As there is no indication of change with respect to time, steady state conditions apply
2. Unless stated otherwise assume that all streams are dilute and have the density of water
3. No reactions take place and $CaCO_3$ is conserved

Step II) List all given information

All of the following information was gathered from the problem statement.

Knowns: Q_{in}, C_{in}, C_{Br}, C_{Eff}

Q_{in} = 12000 L/d

C_{in} = 360 mg $CaCO_3$/L

C_{Br} = 2750 mg $CaCO_3$/L

C_{Eff} = 85 mg $CaCO_3$/L

Unknowns: Q_{Rin}, C_{Rin}, Q_{Br}, Q_B, C_B, Q_{Eff}, Q_{Ro}, C_{Ro}

Step III) Determine the flow rate of the Bypass Stream Q_B

Overall Mass Balance around the system

Acc = In –Out

Acc = 0 due to steady state

$0 = In - Out = \rho Q_{in} - \rho Q_{Br} - \rho Q_{Eff}$

As ρ are equal we can divide it out so we get:

$0 = Q_{in} - Q_{Br} - Q_{Eff}$

$Q_{in} = Q_{Br} + Q_{Eff}$ (i)

Overall Mass balance around the Reverse Osmosis Softener

Acc = In –Out

Acc = 0 due to steady state

$0 = In - Out = \rho Q_{Rin} - \rho Q_{Br} - \rho Q_{Ro}$

As ρ are equal we can divide it out so we get:

$0 = Q_{Rin} - Q_{Br} - Q_{Ro}$

$Q_{Rin} = Q_{Br} + Q_{Ro}$ (ii)

Hardness Mass Balance around the Reverse Osmosis Softener

Acc = In –Out + Generation – Consumption

Acc = 0 due to steady state

Generation = Consumption = 0

$0 = Q_{Rin}C_{Rin} - Q_{Br}C_{Br} - Q_{Ro}C_{Ro}$

As all of the hardness is removed $C_{Ro} = 0$

Therefore $Q_{Ro}C_{Ro} = 0$ and

$Q_{Rin}C_{Rin} = Q_{Br}C_{Br}$

Additionally $C_{Rin} = C_B = C_{in} = 360$ mg $CaCO_3$/L as the Rin and Bypass streams were separated by a mechanical split therefore the concentration of hardness in the Influent stream, Rin stream and Bypass stream is the same

$Q_{Br} = Q_{Rin}C_{Rin}/C_{Br} = Q_{Rin} * (360 \text{ mg } CaCO_3/L \ / \ 2750 \text{ mg } CaCO_3/L)$

$Q_{Br} = 0.131\ Q_{Rin}$ (iii)

Substitute eqn (iii) into eqn (ii)

$Q_{Rin} = Q_{Br} + Q_{Ro}$

$Q_{Ro} = Q_{Rin} - Q_{Br}$

$Q_{Ro} = Q_{Rin} – 0.131\ Q_{Rin}$

$Q_{Ro} = 0.869\ Q_{Rin}$ (iv)

<u>Overall Mass Balance around the Mixing point</u>

Acc = In – Out

Acc = 0 due to steady state

$0 = In – Out = \rho Q_B + \rho Q_{Ro} – \rho Q_{Eff}$

As ρ are equal we can divide it out so we get:

$0 = Q_B + Q_{Ro} – Q_{Eff}$

$Q_{Eff} = Q_B + Q_{Ro}$ (v)

<u>Hardness Mass Balance around the Mixing point</u>

Acc = In –Out + Generation – Consumption

Acc = 0 due to steady state

Generation = Consumption = 0

$0 = Q_BC_B – Q_{Eff}C_{Eff} + Q_{Ro}C_{Ro}$

As all of the hardness is removed $C_{Ro} = 0$

Therefore $Q_{Ro}C_{Ro} = 0$ and

$Q_BC_B = Q_{Eff}C_{Eff}$

We know $C_B = 360$ mg $CaCO_3$/L and $C_{Eff} = 85$ mg $CaCO_3$/L

$Q_{Eff} = Q_B C_B / C_{Eff}$

$Q_{Eff} = Q_B$*(360 mg $CaCO_3$/L / 85 mg $CaCO_3$/L)

$Q_{Eff} = 4.235\ Q_B$ (vi)

We can substitute eqn (vi) into eqn (v) to get

$4.235\ Q_B = Q_B + Q_{Ro}$

$Q_{Ro} = 3.235 Q_B$ (vii)

Now we have Q_{Ro} in terms of two other variables: Q_B and Q_{Rin} therefore

$3.235 Q_B = 0.869 Q_{Rin}$

Now we set one variable in terms of the other

$Q_{Rin} = Q_B$*(3.235/0.869) (viii)

<u>Overall Mass Balance around the Split</u>

Acc = In –Out

Acc = 0 due to steady state

$0 = \text{In} - \text{Out} = \rho Q_{in} - \rho Q_B - \rho Q_{Rin}$

As ρ are equal we can divide it out so we get:

$0 = Q_{in} - Q_B - Q_{Rin}$

$Q_{in} = Q_B + Q_{Rin}$

Substitute in eqn (viii)

$Q_{in} = Q_B + Q_B$*(3.235/0.869)

From the given information we know that Q_{in} = 12000 L/d so we can isolate for Q_B

$Q_B = Q_{in}$/ (4.723) = 12000 L/d / 4.723 = 2541 L/d

Now that we have Q_B, we have a number of variables in terms of Q_B that require simple substitutions

<u>Step IV) Determine Q_{Rin}, Q_{Ro} and Q_{Eff}</u>

From eqn (viii)

$Q_{Rin} = Q_B$ * (3.235/0.869)

Q_{Rin} = 2541 L/d * (3.235/0.869)

Q_{Rin} = 9459 L/d

From eqn (vii)

$Q_{Ro} = 3.235Q_B$

$Q_{Ro} = 3.235*2541$ L/d

$Q_{Ro} = 8220$ L/d

From eqn (vi)

$Q_{Eff} = 4.235Q_B$

$Q_{Eff} = 4.235*2541$ L/d

$Q_{Eff} = 10761$ L/d

Step V) Determine the Brine stream flow rate Q_{Br}

From eqn (i) we have:

$Q_{in} = Q_{Br} + Q_{Eff}$

Which can be rearranged to

$Q_{Br} = Q_{in} – Q_{Eff}$

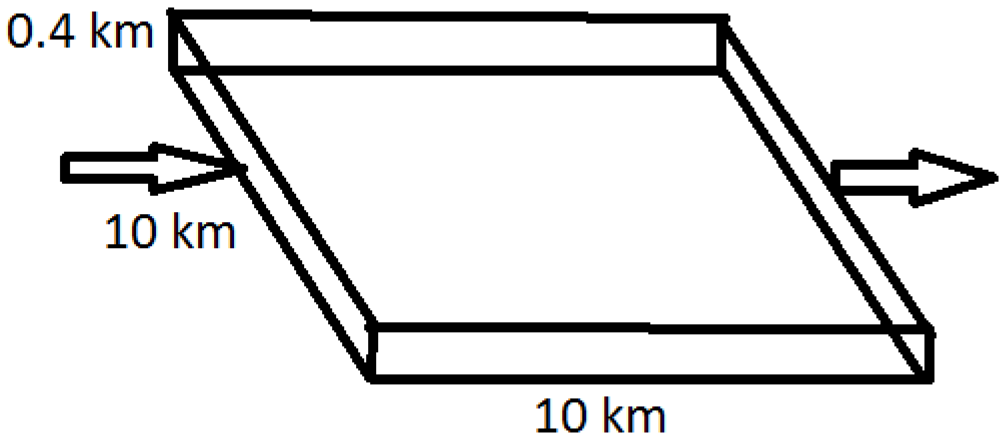
$Q_{Br} = 12000$ L/d – 10761 L/d

$Q_{Br} = 1239$ L/d

Question 2. A company wishes to attain approval to start up an oil refinery operation. Before operation can begin, the company needs to confirm that their projected airborne particulate output would be less than 0.5 mg/m^3. The area of interest is a 10 km by 10 km plot of land with an average ceiling height of 0.4 km. If the daily airborne particulate production was 176 kg/km^2, and the average wind velocity was 0.5 m/s, determine if the projected output meet the requirement. Assume that the influent air contains no particulate, the particulate is evenly distributed throughout the projected area and steady state conditions apply. Make sure to include a system diagram.

Solution: The objective of this problem is to familiarise you with the generation term in a non-reaction setting.

Step I) Draw the System Diagram



Step II) State all Assumptions

1. As stated in the problem statement, steady state conditions apply
2. Particulate produced by the factory is conservative and does not react
3. All streams can be assumed to be dilute and have the density of air (1.225 kg/m^3)
4. The contribution of the factory to the **overall mass** is negligible
5. There is no particulate in the influent

Step III) List all given information

Knowns: v, L, W, H, Gen, C_{in}

v = 0.5 m/s

= 0.5 m/s * 60 s/min * 60 min/h *24 h/d * 1 km/1000 m = 43.2 km/d

L = 10 km

W = 10 km

H = 0.4 km

Gen = 176 kg/km^2d

C_{in} = 0

Unknowns: Q_{in}, Q_{eff}, C_{eff}

Step IV) Determine Q_{in} and Q_{eff}

Q_{in} = v*H*W = 43.2 km/d * 0.4 km * 10 km

Q_{in} = 172.8 km^3/d

Overall Mass Balance around the System

Acc = In – Out

Acc = 0 b/c steady state

$0 = In - Out = \rho_{in}Q_{in} - \rho_{out}Q_{out}$

Due to the dilute streams assumption we can divide out the ρ term to get

$0 = Q_{in} - Q_{out}$

$Q_{in} = Q_{out}$

Step V) Determine C_{eff}

Particulate Mass Balance around the System

Acc = In – Out + Gen – Cons

Acc = 0 b/c steady state

Cons = 0 as no reaction takes place

$0 = Q_{in}C_{in} - Q_{out}C_{out} + Gen$

So we need to determine the Gen term

We know that 176 kg of particulate is produced per km^2 per day therefore:

Gen = 176 kg/km^2d * Surface area

Gen = 176 kg/km^2d * L*W

Gen = 176 kg/km^2d * 10 km *10km

Gen = 17600 kg/d

And we have $C_{in} = 0$ as there is no particulate in the influent

Thus we have:

$0 = 172.8\ km^3/d * 0 - 172.8\ km^3/d * C_{eff} + 17600\ kg/d$

$C_{eff} = 17600\ kg/d\ /\ 172.8\ km^3/d$

$C_{eff} = 101.9\ kg/km^3$

Step VI) Determine if the effluent concentration meets the requirement

$C_{eff} = 101.9\ kg/km^3 * 1000\ g/kg * 1000\ mg/g * 1\ km^3/1000000000\ m^3$

$C_{eff} = 0.102\ mg/m^3$

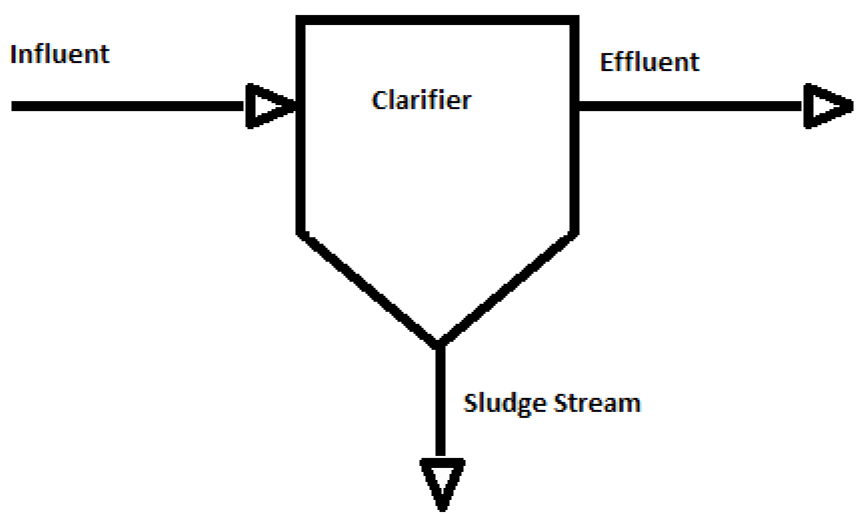
Therefore the effluent concentration meets the restriction.

Question 3. Recently, there has been a blue-green algae bloom in the Rideau Canal. The algae bloom needs to be treated as they produce cyanotoxins. A solution was proposed to remove the algae without releasing the cyanotoxins as they were deemed too harmful to allow. This method adds a chemical coagulant (alum) that causes the bacteria to become clumps of TSS that settle in a clarifier rather than lysing the cells. The design parameters, after chemical addition and the clumps have formed, are: an influent flow rate of 32 m^3/s, an influent TSS concentration of 177 g/m^3, a sludge density of 1050 kg/m^3, a sludge solids fraction of 4%, and an effluent parameter of 1 μg TSS/L.

a) Design the system diagram of the clarifier.
b) Determine the effluent flowrate, and the sludge flow rate.

Solution: The objective of this problem is to familiarise you with using solids/mass fractions and different densities.

Question 3a. Design the system diagram.



Question 3b. Determine the effluent flowrate and the sludge flow rate.

Step I) State all Assumptions

1. As there is no indication of change with respect to time steady state conditions apply
2. Unless otherwise stated, assume streams are dilute and have the density of water (1000 kg/m^3)
3. Assume that no reaction takes place in the clarifier

Step II) List all given information

Known: Q_{in}, C_{in}, ρ_{sludge}, $x_{TSS,sludge}$, C_{eff}

Q_{in} = 32 m^3/s

C_{in} = 177 g TSS/m^3

ρ_{sludge} = 1050 kg/m^3

$x_{TSS,sludge}$ = 4% = 0.04 kg TSS/kg total

C_{eff} = 1 μg TSS/L *1 g/1000000 μg * 1000 L/m^3 = 10^{-3} g TSS/m^3

Unknowns: Q_{sludge}, Q_{eff}

<u>Step III) Determine Q_{eff}</u>

<u>Overall Mass Balance around the system</u>

Acc = In – Out

Acc = 0 b/c steady state

0 = In – Out

$0 = Q_{in}\rho_{in} - Q_{eff}\rho_{eff} - Q_{sludge}\rho_{sludge}$

Due to dilute streams $\rho_{in} = \rho_{eff}$ = 1000 kg/m^3

$0 = Q_{in}(1000\ kg/m^3) – Q_{eff}(1000\ kg/m^3) – Q_{sludge}(1050\ kg/m^3)$

Dividing out the density

$0 = Q_{in} – Q_{eff} – 1.05*Q_{sludge}$ (i)

We do not know Q_{eff} and Q_{sludge} therefore we will use the equation to define Q_{eff}

$Q_{eff} = Q_{in} – 1.05*Q_{sludge}$ (ii)

<u>Step IV) Determine Q_{sludge}</u>

<u>TSS Mass Balance around the system</u>

Acc = In – Out + Gen – Con

Acc = 0 b/c steady state

There is no reaction taking place so Gen = 0 and Con = 0

0 = In – Out

$0 = Q_{in}C_{in} – Q_{eff}C_{eff} – Q_{sludge}C_{sludge}$ (iii)

$C_{sludge} = \rho_{sludge}*x_{TSS,sludge}$

C_{sludge} = 1050 kg/m^3*0.04 kg TSS/kg total * 1000 g/m^3

C_{sludge} = 42000 g/m^3

We can substitute eqn ii into eqn iii

$0 = Q_{in}C_{in} – (Q_{in} – 1.05Q_{sludge})C_{eff} – Q_{sludge}C_{sludge}$

$0 = Q_{in}C_{in} – Q_{in}C_{eff} + 1.05Q_{sludge}C_{eff} – Q_{sludge}C_{sludge}$

$0 = Q_{in}(C_{in} – C_{eff}) – Q_{sludge}(C_{sludge} – 1.05C_{eff})$

$Q_{sludge} = Q_{in}(C_{in} – C_{eff})/(C_{sludge} – 1.05C_{eff})$

$Q_{sludge} = 32\ m^3/s*(177\ g/m^3 – 10^{-3}g/m^3)/(42000\ g/m^3 – 1.05*10^{-3}g/m^3)$

$Q_{sludge} = 0.135\ m^3/s$

<u>Step V) Determine Q_{eff} using Q_{sludge}</u>

Use eqn ii to solve for Q_{eff}

$Q_{eff} = Q_{in} – 1.05Q_{sludge}$

$Q_{eff} = 32\ m^3/s – 1.05*0.135\ m^3/s$

$Q_{eff} = 31.86\ m^3/s$

Question 4. A bench scale test of a new disinfectant chemical 'D' has produced the following data.

Time (minutes)	Concentration (mg D/L)
0	300
1	84.5
2	49
3	35
5	22
10	11
15	7.5
20	5.5
25	4.5
30	4

a) Determine if the reaction is zero, first or second order by using the graphical method (Make sure to plot all three possibilities and show all work)
b) Determine the reaction rate constant (including proper units)
c) Determine the half-life of the reaction using the initial conditions.

NOTE: Make sure to show all of your work (including tables) and the evaluations of each kinetic order.

Solution: The objective of this problem is to demonstrate how to determine the rate of the reaction

<u>Step I) Create a table that can be used to simulate the three required orders</u>

To ensure that you have the proper order of reaction you need to create a zero order plot, a first order plot and a second order plot.

Zero order is determined by a linear result when plotting C vs t .

First order is determined by a linear result when plotting LnC vs t

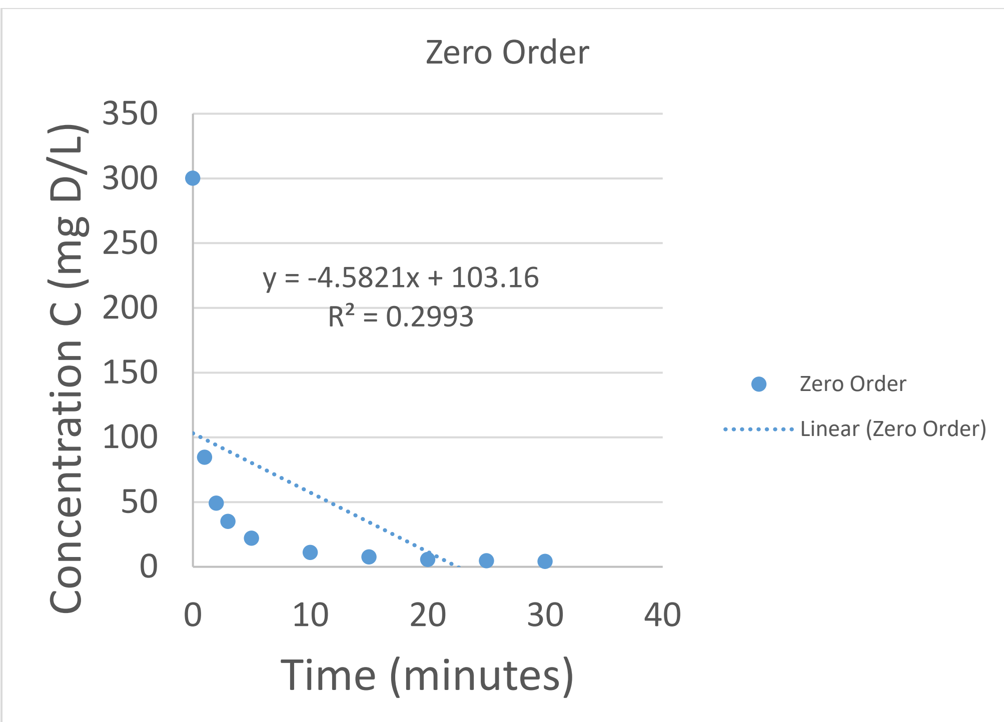
Second order is determined by a linear result when plotting 1/C vs t.

Therefore C, Ln C and 1/C need to be determined for all times

T (min)	C (mg/L)	LnC	1/C
0	300	5.703782	0.003333
1	84.5	4.436752	0.011834
2	49	3.89182	0.020408
3	35	3.555348	0.028571
5	22	3.091042	0.045455
10	11	2.397895	0.090909
15	7.5	2.014903	0.133333
20	5.5	1.704748	0.181818
25	4.5	1.504077	0.222222
30	4	1.386294	0.25

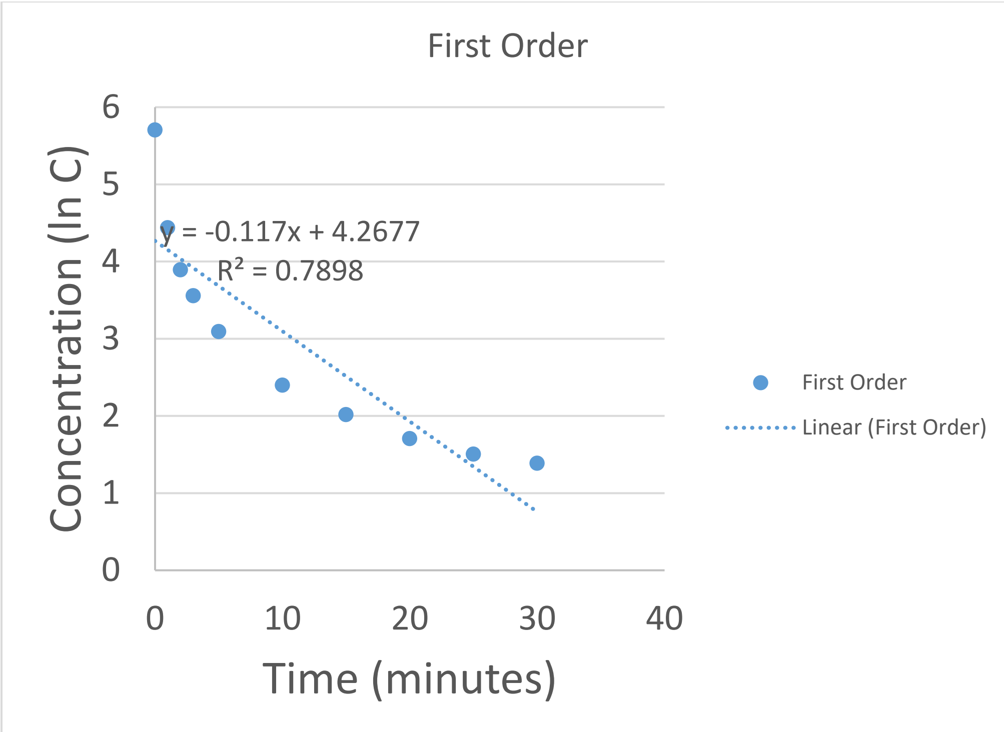
NOTE: When plotting the data in excel, use the ‘trendline function’ and select ‘linear’. Make sure to select the ‘display R^2’ and ‘display equation’ options.

<u>Step II) Plot Zero Order (C vs t)</u>



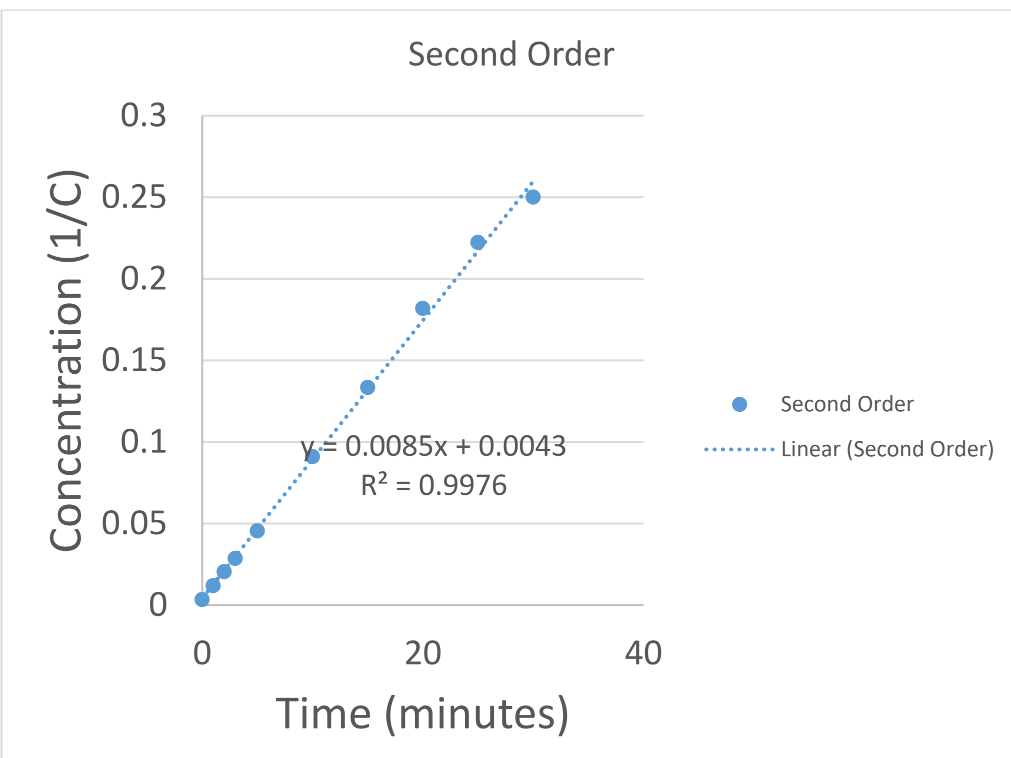
The curved, non-linear shape of the plot and the low R^2 value indicates that the data does not indicate zero order kinetics

<u>Step III) Plot First Order (ln C vs t)</u>



Similarly to the zero order plot, the first order plot also indicates a clear non-linear relationship. Additionally the plot does not have a high correlation (R^2) value that one would expect from a linear relationship.

<u>Step IV) Plot Second Order (1/C vs t)</u>



The plot clearly demonstrates a linear relationship with a high correlation (R^2) value therefore it is likely that the kinetics are second order.

The zero order and first order plots do not demonstrate a linear relationship while the second order plot demonstrates a strong linear relationship. Therefore it can be concluded that the consumption is of second order.

Step V) Determine the reaction rate constant

As the reaction was determined to be second order, the reaction rate constant 'k' can be determined from the slope of the graph. Therefore k = 0.0085. As it is second order, the units are L/mg*min, so we have k = 0.0085 L/mg*min.

To determine the required units, know that the final units of the reaction rate term must be concentration/time. For a second order reaction the rate equation is $r = kC^2$ which has units of

$mg/L*min = X\ (mg/L)^2$

$X = mg/L*min\ /\ mg^2/L^2 = L/mg*min$

Step VI) Determine the half-life of the reaction

From the lecture notes we know that the n order half-life equation is as follows:

$t_{1/2} = ((0.5C_0)^{1-n} – C_0^{1-n})/(-(1-n)k)$

For this question we know the reaction is second order so n=2 and C_0 = 300 mg/L

$t_{1/2} = ((0.5*300\ mg/L)^{1-2} – (300\ mg/L)^{1-2})/(-(1-2)*0.0085L/mg*min)$

$t_{1/2} = (0.00666667\ L/mg – 0.0033333\ L/mg)/(0.0085\ L/mg*min)$

$t_{1/2} = 0.39\ min *60\ s/min = 23.5\ seconds$

Therefore the half-life is 23.5 second for the bench scale test