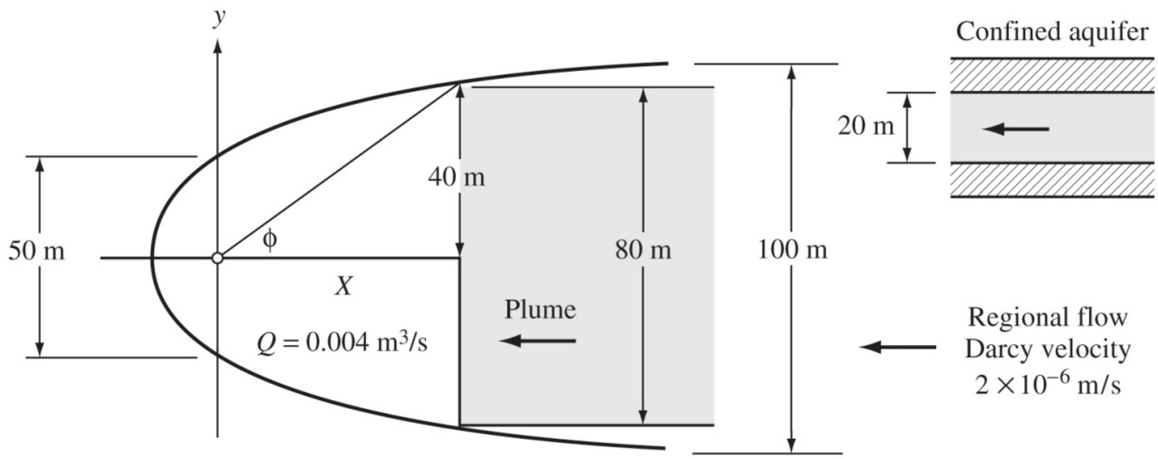
CVG 2132 Final Exam Review Problems

Groundwater Example

- Consider a confined aquifer having a thickness of 20 m, a hydraulic conductivity of 1.0 x 10^{-3} m/s and a regional hydraulic gradient equal to 0.002. The maximum pumping rate has been determined to be 0.004 m^3/s. The aquifer has been contaminated and, for simplicity, consider the plume to be rectangular, with a width of 80 m. Locate a single extraction well so that it can totally remove the plume.

<u>ANS</u>:



Given:

B = 20 m

K = 1.0 x 10^{-3} m/s (seems like clay)

dh/dL = 0.002

Q_{max} = 0.004 m^3/s

w_{plume} = 80 m

First determine Darcy velocity of the aquifer:

$q = K(dh/dL) = 1.0 \times 10^{-3} \times 0.002 = 2.0 \times 10^{-6}$ m/s

Second determine the width of the capture zone at well:

$w_{\text{capture zone at well}} = Q/(2qB) = 0.004\ m^3/s / [2 \times 20\ m \times 2.0 \times 10^{-6}\ m/s] = 50\ m$

Now determine the maximum width of the capture zone:

$w_{\text{max of capture zone}} = Q/(qB) = 0.004\ m^3/s / [20\ m \times 2.0 \times 10^{-6}\ m/s] = 100\ m$

Therefore an 80m wide plume will fit in the capture zone if the well is located an appropriate distance downgradient of the front of the plume.

Let Y = 40 m (see diagram above).

Calculate phi and use this value to calculate X.

$$Y = \frac{Q}{2qB}(1 - \frac{\phi}{\pi}) = 40 = 50(1 - \frac{\phi}{\pi})$$

$$\phi = 0.2\pi \text{ rad}$$

Now use phi to calculate X.

$$X = Y/(\tan \phi) = 55\ m$$

The extraction well should be placed 55 m ahead of the oncoming plume at the time of installation and the well should be directly in the middle of the plume.

Stream DO Example

- A stream with k_r = 0.31 d^{-1}(base 10), temperature of 28°C, and minimum river flow of 23.0 m^3/s receives 3.80 x 10^4 m^3/d of sewage from a city. The oxygen demand of the river before it intersects with the discharge pipe is negligible. *What is the maximum permissible BOD of the sewage if the DO content of the stream is never to fall below 4.0 mg/L and the DO content of the sewage is assumed to be zero and the temperature of the sewage is also 28°C.* ? The river is saturated with DO (7.83 mg/L) before the sewage outfall. Assume k_d = 0.17 d^{-1}(base 10). NOTE: both rate constants are base 10 and are given for a temperature of 28°C.

ANS:

NOTE: in the solution below k_d is denoted as k_1'and k_r is denoted as k_2'since they are base 10 coefficients.

Given:

Q_w = 3.80 × 10^4 m^3/d, T = 28°C, DO = zero

Q_r = 23.0 m^3/s, T = 28°C, DO = Saturated

L_r = zero

At 28°C, Saturated = 7.83 mg/L

The primed coefficients refer to the fact that they are base 10:

$$k_1' = 0.17\ d^{-1};\ k_2' = 0.31\ d^{-1}$$

Use the conservative assumption that the sewage DO is 0.

D_w = 7.83 – 0.0 = 7.83 mg/L

The critical deficit is

D_c = 7.83 – 4.0 = 3.83 mg/L

Now calculate known values:

$$Q_w = \left(3.80 \times 10^4 \frac{\text{m}^3}{\text{d}}\right)\left(\frac{1\,\text{d}}{86\,400\,\text{s}}\right) = 0.44 m^3/s \quad (1)$$

$$D_0 = \frac{Q_w D_w + Q_r D_r}{Q_w + Q_r} = \frac{\left(0.44\frac{m^3}{s}\right)\left(7.83\frac{mg}{L}\right) + \left(23.0\frac{m^3}{s}\right)(0)}{23.0\frac{m^3}{s} + 0.44\frac{m^3}{s}} = 0.15 mg/L \quad (2)$$

$$L_0 = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r} = \frac{\left(0.44\frac{m^3}{s}\right)(L_w) + \left(23.0\frac{m^3}{s}\right)(0)}{23.0\frac{m^3}{s} + 0.44\frac{m^3}{s}} \quad (3)$$

At this point we need to determine L_0; once L_0 is determined, L_w is easily found.

D_c occurs at t_c. D_c & D_0 & k_1' & k_2' are known. We need equations that involve L_0.

$$\frac{dD}{dt} = k_1' L_c - k_2' D_c = 0$$ …dD/dt = 0 at D_c (4)

$$k_1' L_c - k_2' D_c = 0 \quad (5)$$

$$L_c = L_0 10^{-k_1' t_c}$$ …from class, now in base 10 (6)

$$k_1' L_0 10^{-k_1' t_c} = k_2' D_c$$ (Using 6 in 5) (7)

$$L_0 = \frac{k_2'}{k_1'} D_c 10^{k_1' t_c} \quad (8)$$

$$t_c = \left(\frac{1}{k_2' - k_1'}\right) \log\left[\frac{k_2'}{k_1'}\left(1 - D_0 \frac{k_2' - k_1'}{k_1' L_0}\right)\right] \quad (9a)$$

$$= \left(\frac{1}{0.31\,d^{-1} - 0.17\,d^{-1}}\right) \log\left\{\frac{0.31\,d^{-1}}{0.17\,d^{-1}}\left[1 - (0.15\,mg/L)\frac{0.31\,d^{-1} - 0.17\,d^{-1}}{(0.17\,d^{-1})L_0}\right]\right\}$$

$$= 7.14\left\{\log\left[1.82\left(1-\frac{0.12}{L_0}\right)\right]\right\} \quad (9b)$$

Using (9b) in (8):

$$L_0 = \left(\frac{0.31\,\mathrm{d}^{-1}}{0.17\,\mathrm{d}^{-1}}\right)(3.83\,\mathrm{mg/L})10^{0.17[7.14\log(1.82-0.22/L_0)]} = 6.98\left(1.82-\frac{0.22}{L_0}\right)^{1.21} \quad (10)$$

Solving the above equation by trial and error (which also could be done by using the solver routine in a spreadsheet):

L_0 = 14.5 mg/L

$t_c = 6.98\log\left[1.82-\frac{0.22}{14.5}\right]$ = 1.79 d

Now (3) can be used to find L_w:

$$L_0 = \frac{Q_r L_r + Q_w L_w}{Q}$$

$$L_w = \frac{L_0(Q_r + Q_w) - Q_r L_r}{Q_w} = \frac{(14.5\,\mathrm{mg/L})(23.44\,\mathrm{m^3/s}) - 0}{0.44\,\mathrm{m^3/s}}$$ = 772 mg/L