1. [1 point] What is the domain of the function $f(x) = \sqrt{4 - x^2}$?

(A) $x \geq 2$ (B) $-2 \leq x \leq 2$ (C) $x \leq -2$

(D) $x < 2$ (E) all x (F) $x \leq -2$ or $x \geq 2$

We need $4 - x^2 \geq 0$

$\Rightarrow x^2 \leq 4$

$\Rightarrow |x| \leq 2$

So $-2 \leq x \leq 2$

2. [1 point] Find a formula for the inverse of $f(x) = \ln(x+6)$.

(A) e^{x+6} (B) $e^x + 6$ (C) $e^x - 6$

(D) $x + 6$ (E) e^{x-6} (F) $6 - e^x$

Swap $x \Leftrightarrow y$ in $y = \ln(x+6)$:

$x = \ln(y+6)$

$\Rightarrow e^x = y + 6$

$\Rightarrow y = e^x - 6$

3. [1 point] What is the equation of the tangent line to the curve $y = f(x) = 2x + 3x^2$ at the point $(1, 5)$?

(A) $y = 32x - 27$ (B) $y = 6x - 1$ (C) $y = 8x - 3$

(D) $y = 3x + 2$ (E) $y = 2x + 3$ (F) $y = 2x + 3x^2$

$m = f'(1)$ $f'(x) = 2 + 6x$

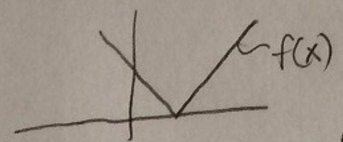
$f'(1) = 8$

Since only one answer has a slope of 8, it must be that one.

4. [1 point] If a function $f(x)$ is continuous at $x = a$, then it must be differentiable there.

(A) TRUE (B) FALSE

e.g.



abs. value fcn is cts everywhere, but not differentiable on its corner.

5. [1 point] What is $\lim\limits_{x\to\infty} \dfrac{5x^3 + 7x^2 - 8x + 9}{11x^2 + 7x - 6}$?

(A) 3/2 (B) 0 (C) ∞ (D) $-\infty$ (E) 5/11 (F) 11/5

$$\lim_{x\to\infty} \frac{5x^3+7x^2-8x+9}{11x^2+7x-6} = \lim_{x\to\infty} \frac{5x+7-8/x+9/x^2}{11+7/x-6/x^2}$$

$= +\infty$

6. [2 points] Find the limit $\lim_{x \to 2} \frac{\sqrt{x^2+12}-4}{x-2}$. You must do this properly, showing all steps.

$$\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2} = \lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2} \cdot \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}$$

$$= \lim_{x\to 2} \frac{x^2+12-16}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(\sqrt{x^2+12}+4)}$$

$$= \lim_{x\to 2} \frac{x+2}{\sqrt{x^2+12}+4}$$

$$= \frac{4}{4+4} = \frac{1}{2}$$

7. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{2x^2}{x+3}$. Then verify your answer with the Quotient Rule.

$$f'(x) = \lim_{h\to 0} \frac{\left(\frac{2(x+h)^2}{x+h+3} - \frac{2x^2}{x+3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\left[\frac{2(x+h)^2(x+3) - 2x^2(x+h+3)}{(x+h+3)(x+3)}\right]}{h}$$

$$= \lim_{h\to 0} \frac{(2x^2+4xh+2h^2)(x+3) - 2x^3 - 2x^2h - 6x^2}{h(x+h+3)(x+3)}$$

$$= \lim_{h\to 0} \frac{2x^3+4x^2h+2xh^2+6x^2+12xh+6h^2-2x^3-2x^2h-6x^2}{h(x+h+3)(x+3)}$$

$$= \lim_{h\to 0} \frac{2x^2h+2xh^2+12xh+6h^2}{h(x+h+3)(x+3)}$$

$$= \frac{2x^2+0+12x+0}{(x+3)^2} = \frac{2x^2+12x}{(x+3)^2}$$

Check:

$$f'(x) = \frac{4x(x+3) - 2x^2 \cdot 1}{(x+3)^2}$$

$$= \frac{4x^2+12x-2x^2}{(x+3)^2} = \frac{2x^2+12x}{(x+3)^2}$$

✓

8. [5 points] Find the first derivatives of the following functions.

(a) $f(x) = e^{-3x}\sin(4x^2)$

$$f'(x) = e^{-3x}(-3)\sin(4x^2) + e^{-3x}\cos(4x^2)\cdot 8x$$

(b) $g(t) = \sqrt{3t^2 + 7t - 2}$

$$g'(t) = \frac{1}{2\sqrt{3t^2+7t-2}}\cdot(6t+7)$$

(c) $\varphi(\theta) = \tan^2(3e^\theta)$

$$\varphi'(\theta) = 2\tan(3e^\theta)\cdot\sec^2(3e^\theta)\cdot 3e^\theta$$

(d) $p(t) = 5^{\sqrt{t}}$

$$p'(t) = 5^{\sqrt{t}}\ln 5\cdot\frac{1}{2\sqrt{t}}$$

(e) $y = e^{x\cos(2x)}$

$$y' = e^{x\cos(2x)}\cdot\left(\cos(2x) + x(-\sin(2x))\cdot 2\right)$$