

ASSIGNMENT # 1 Solution

Q1

A grower estimates that if he picks his apple crop now, he will obtain 600 boxes of apples, which he can sell at \$5 a box. However, he thinks his crop will increase by 48 boxes of apples for each week he delays picking, but that the price will drop at a rate of 25¢ a box per week; in addition, he estimates approximately 10 boxes a week will spoil for each week he delays picking. When should he pick his crop to obtain the largest total cash return? How much will he receive for his crop at that time?

Solution

Let t = time from the present (in weeks)

Volume of apples at any time = $(600 + 48t - 10t) = 600 + 38t$

Price at any time = $\$5.00 - \$0.25t$

Total Cash Return (TCR) = $(3,000 + 38t) (\$5.00 - \$0.25t)$
 $= \$3,000 + \$40t - \$9.5t^2$

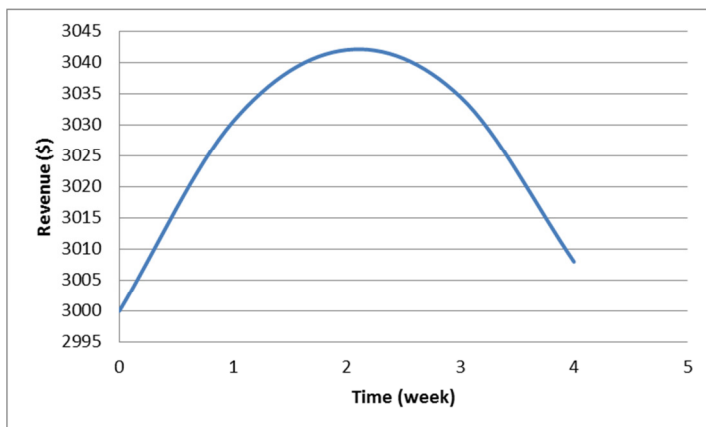
Set the first derivative equal to zero and solve for t .

$dTCR/dt = \$40 - \$19t = 0$

$t = \$40/\$9 = 2.1$ weeks

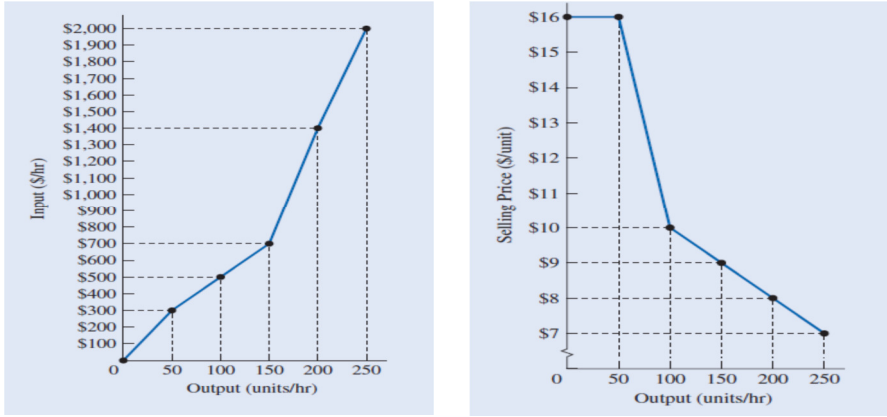
At $t = 2.1$ weeks:

Total Cash Return (TCR) = $\$3,000 + \$40(2.1) - \$9.5(2.1)^2 = \$3,042$



Q2

On her first engineering job, Joy Hayes was given the responsibility of determining the production rate for a new product. She has assembled data as indicated on two graphs:

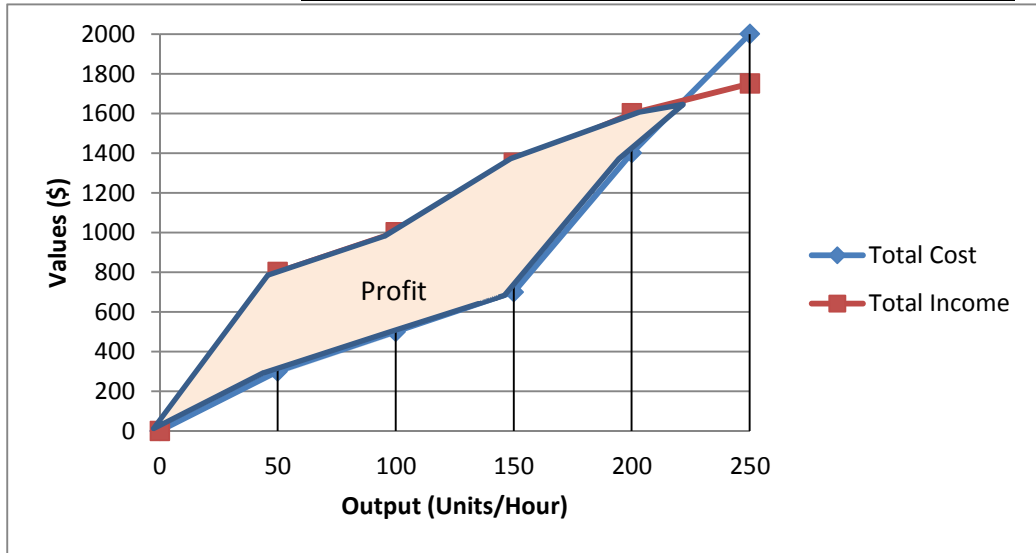


- (a) Select a suitable economic criterion and estimate the production rate based upon it.
- (b) Joy’s boss told her: “I want you to maximize output with minimum input.” Joy wonders if it is possible to meet her boss’s criterion. She asks your advice. What would you tell her?

Solution

The suitable criterion is to maximize the difference between output and input. Or simply, maximize net profit. The data from the graphs may be tabulated as follows:

Output Units/Hour	Total Cost	Total Income	Net Profit
0	0	0	0
50	300	800	500
100	500	1000	500
150	700	1350	650
200	1400	1600	200
250	2000	1750	-250



(b) Minimum input is, of course, zero, and maximum output is 250 units/hour (based on the graph). Since one cannot achieve maximum output with minimum input, the statement makes no sense.

Q3

Electricity is sold for \$0.12 per kilowatt-hour (kWh) for the first 15,000 units each month and \$0.09/kWh for all remaining units. If a firm uses 17,500kWh/month what is its average and marginal cost?

Solution

If firm uses 17,500 kWh/month, then cost is:

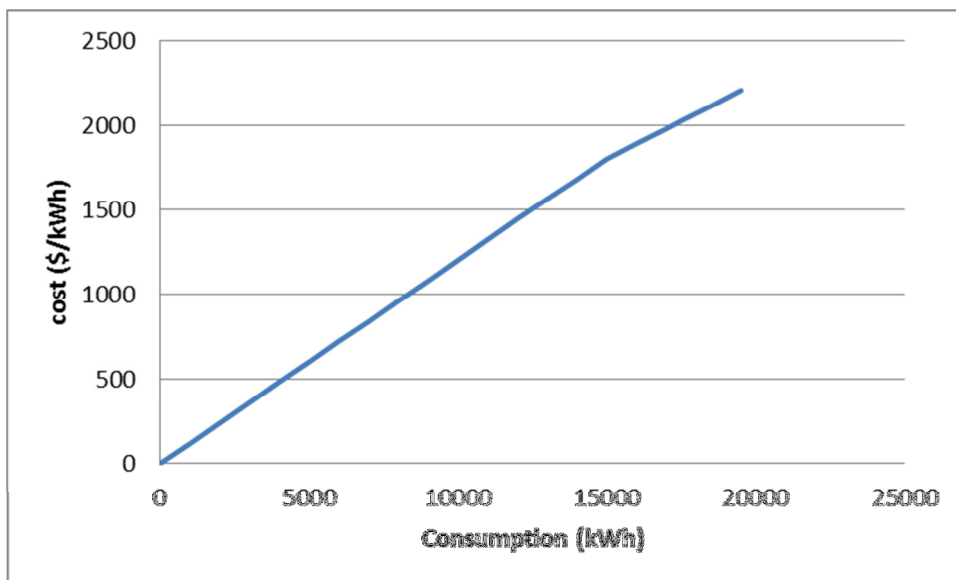
\$0.12/kWh for first 15,000 kWh

\$0.09/kWh for the remaining 2,500 kWh

Marginal Cost (cost for the next kWh) = **\$0.09** since we are in the 2nd bracket of the cost structure.

Total Cost = $\$0.12/\text{kWh} * 15,000 \text{ kWh} + \$0.09/\text{kWh} * 2,500 \text{ kWh} = \$2,025$

Average Cost = $\$2,025 / 17,500\text{kWh} = \$0.1157/\text{kWh}$



Q4

Two new rides are being compared by a local amusement park in terms of their annual operating costs. The two rides are assumed to be able to generate the same revenue (hence the focus on costs). The Tummy Tugger has fixed costs of \$13,000 a year and variable costs of \$1.750 per visitor. The Head Buzzer has fixed costs of \$3,000 per year and variable costs of \$3.5 per visitor. Answer the following questions so the amusement park can make the needed comparison.

(a) Determine mathematically the breakeven number of visitors per year for the two rides to have equal annual costs.

(b) Develop a graph that illustrates the following (*note*: put visitors per year on the horizontal axis and costs on the vertical axis):

- accurate total cost lines for the two alternatives (show line, slopes, and equations)
- the break-even point for the two rides in terms of number of visitors
- the ranges of visitors per year where each alternative is preferred

Solution

(a)

Let x = number of visitors per year

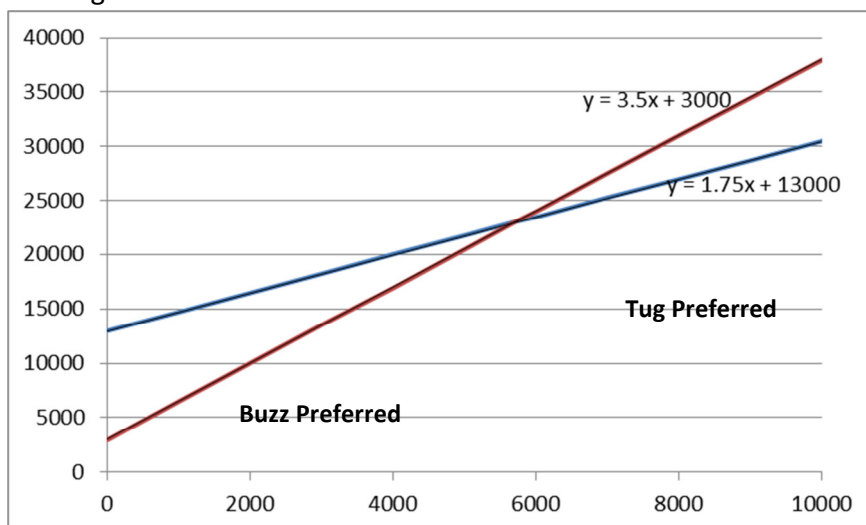
Break even when: Total Costs (Tugger) = Total Costs (Buzzer)

$$\$13,000 + \$1.75x = \$3,000 + \$3.5x$$

$x = 5714$ visitors is the breakeven quantity

X	Y1 (Tug)	Y2 (Buzz)
0	13000	3000
1,000	14750	6500
3,000	18250	13500
4000	20000	17000
5000	21750	20500
5714	22999.5	22999
6000	23500	24000
7000	25250	27500

(b) See the figure below



Q5

Assume that you save 5 cent a day for 50 years, that you deposit it in the bank at the end of each month, and that there are 30.5 days per month (you save 152.5 cents each month). How much do you have after 50 years, if:

- (a) The bank does not pay any interest?
 (b) The bank pays 1.5% per month interest?

Solution

(a) $F = (\$1.525/\text{month}) * (12 \text{ mon/yr}) * (50 \text{ yrs}) = \915

b)

$$F = A[((1+i)^N - 1)/i]$$

$$F = 1.525[((1+0.015)^{600} - 1)/0.015] = \$770453.85$$

OR

By using Excel with 600 rows(tabular method)

Q6

The Apex Company sold a water softener to Marty Smith. The price of the unit was \$495. Marty asked for a deferred-payment plan, and a contract was drawn up. Under the contract, the buyer could delay paying for the water softener if he bought the coarse salt for recharging the softener from Apex. At the end of two years, the buyer was to pay for the unit in a lump sum, with interest at a rate of 1.5% per quarter-year. According to the contract, if the customer stopped buying salt from Apex at any time before two years, the full payment due at the end of two years would automatically become due. Six months later, Marty decided to buy salt elsewhere and stopped buying from Apex, whereupon Apex asked for the full payment that was to have been due 18 months hence. Marty was unhappy about this, so Apex offered as an alternative to accept the \$495 with interest at 10% per semi-annual period for the six months that Marty had been buying salt from Apex. Which of these alternatives should Marty accept? Explain.

Solution

Lump Sum Payment = \$495 (F/P, 1.5%, 8)
 = \$495 (1.126)
 = \$557.37

Alternate Payment = \$495 (F/P, 10%, 1)
 = \$495 (1.100)
 = \$544.50

Choose the alternate payment plan.

Q7

The local garbage company charges \$5 a month for garbage collection. It had been the company's practice to send bills to its 120,000 customers at the end of each two month period. Thus, at the end of February it would send a bill to each customer for \$10 for garbage collection during January and February. Recently the firm changed its billing date: it now sends out the two-month bills after one month's service has been performed. Bills for January and February, for example, are sent at the end of January. The local newspaper points out that the firm is receiving half its money before the garbage collection. This unearned money, the newspaper says, could be invested temporarily for one month at 1.5% per month interest by the garbage company to earn extra income. Compute how much extra income the garbage company could earn each year if it invested the money as described by the newspaper.

Solution

The garbage company sends out bills only six times a year. Each time they collect one month's bills one month early.

$$120,000 \text{ customers} \times \$5.00 \times 1.5\% \text{ per month} \times 6 \text{ times/yr} = \$54,000$$

Q8

Jack deposited \$450,000 in a bank for six months. At the end of that time, he withdrew the money and received \$470,000. If the bank paid interest based on continuous compounding,

(a) What was the effective annual interest rate?

(b) What was the nominal annual interest rate?

Solution

$$(a) \text{ Interest Rate per 6 months} = (470,000 - 450,000) / 450,000 = 0.044 = 4.4\%$$

$$\text{Effective Interest Rate per yr.} = (1 + 0.044)^2 - 1 = 0.09$$

$$= 9\%$$

(b) For continuous compounding:

$F = Pe^{r/k}$ (k is the number of payments per year, jack deposited the money once at 6 months, therefore k for this case is 1 per six months or 2 per year)

$$\$470,000 = \$450,000 e^{r/1}$$

$$r_{(\text{nominal interest per 6 months or per payment})} = \ln(\$470,000 / \$450,000) = 0.043$$

$$= 4.3\% \text{ per 6 months}$$

$$\text{Nominal Interest Rate (per year)} = 4.3\% (2) = 8.6\% \text{ per year}$$

Q9

The I've Been Moved Corporation (IBM) receives a constant flow of funds from its worldwide operations. This money (in the form of cheques) is deposited continuously in many banks with the goal of earning as much interest as possible for IBM. One billion dollars is deposited each month, and the money earns an average of 0.75% interest a month, compounded continuously. Assume all the money remains in the accounts until the end of the month.

(a) How much interest does IBM earn each month?

(b) How much interest would IBM earn each month if it held the cheques and made deposits to its bank accounts just four times a month?

Solution

(a) Interest rate = 0.75% per month

Principal = \$1,000 million

$$I = i_p \times P/2 = (e^{r/k} - 1) \times 1 \times 10^9 \times 1/2 = (e^{0.0075} - 1) \times 1 \times 10^9 \times 1/2 = \mathbf{\$3,764,097}$$
 every month

Note: In the above calculation, the actual principal has been halved, as the interest is paid on the average sum.

Money is being paid continuously, which is 0 at the beginning of the month, and 1 billion at the end of the month.

(b) Cheques are distributed 4 times a month.

$$i_p = e^{r/k} - 1$$

$$i_p = e^{0.09/48} - 1$$

Interest rate per payment, $i_p = 0.187\%$

Principal 1000 million/4 = \$250 million every week

Thus, $A = \$250$ million, $n = 4$

$$F = A [F/A, 0.187\%, 4] = 250 \text{ million} \times 4.0112$$

Thus, $F = \$1,002.8$ million

Interest accumulated = \$2,800,000 every month

Q10

A forklift truck costs \$36,000. A company agrees to purchase such a truck with the understanding that it will make a single payment for the balance due in three years. The vendor agrees to the deal and offers two different interest schedules. The first schedule uses an annual effective interest rate of 12%. The second schedule uses 12.75% compounded continuously.

(a) Which schedule should the company accept?

(b) What would be the size of the single payment?

Solution

$P = \$36,000$, $n = 3$ years, $F = ?$

(a) $i_a = 0.12$

$$F = P (1 + i)^n = \$36,000 (1.12)^3 = \$50,577$$

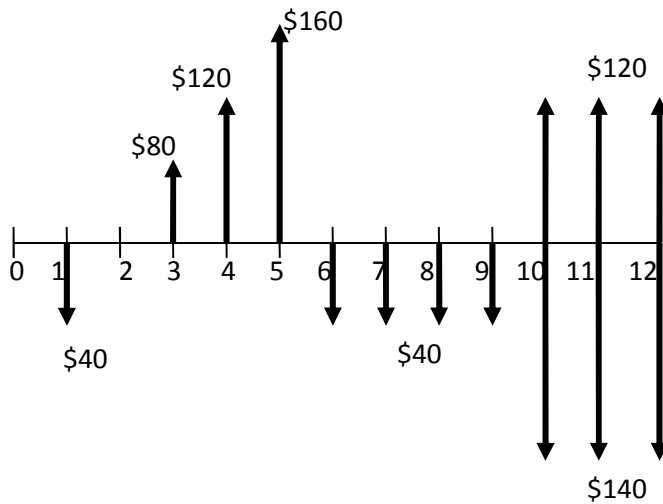
(b) $r = 0.1275$, $k = 1/3$

$$F = P e^{r/k} = \$36,000 e^{(0.1275)(3)} = \$36,000 (1.4659) = \$52,774$$

The correct choice for the company is to choose the 12% interest rate and discrete compounding.

Q11

Calculate the present worth of the cash flow shown in the accompanying diagram, using at most three kinds of interest factors at 2.75% interest compounded annually.

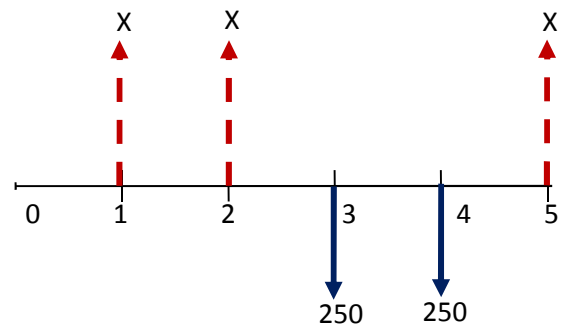
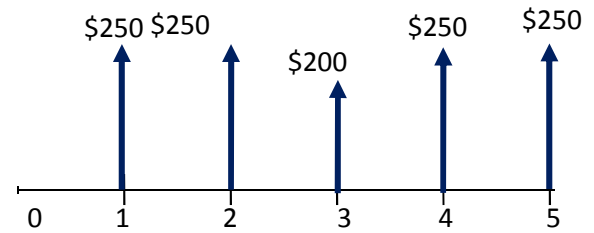
**Solution**

There are multiple ways to interpret the cash flows, below is one way to solve the cash flow diagram.

$$\begin{aligned}
 P &= -40(P/F, 0.0275, 1) + \{80 + 40(A/G, 0.0275, 3)\}(P/A, 0.0275, 3)(P/F, 0.0275, 2) \\
 &\quad - 40(P/A, 0.0275, 4)(P/F, 0.0275, 5) - 20(P/A, 0.0275, 3)(P/F, 0.0275, 9) \\
 &= \$107
 \end{aligned}$$

Q12

Find the value of X so that the two cash flows shown in the diagram are equivalent for an interest rate of 2%.

**Solution**

Establishing equivalence at $n = 5$

$$250(F/A, 2\%, 5) - 50(F/P, 2\%, 1) = X(F/A, 2\%, 5) - (250 + X)[(F/P, 2\%, 2) + (F/P, 2\%, 1)]$$

$$1250.0 = X(5.2040) - (250 + X)(1.02 + 1.0404)$$

$$1250.0 + 515.1 = 3.1436X$$

$$X = \$561.4$$

Q13

For the following transactions, draw the C.F.D and find the value of G that makes the deposit series equivalent to the withdrawal series at interest rate of 5.5%, compounded annually.

End of period	Deposit	Withdrawal
0	\$1000	
1	900	
2	700	
3	500	
4	300	
5	150	
6		G
7		2G
8		3G
9		4G
10		5G

Solution

$$G(P/G, 5.5\%, 6) = 900(F/A, 5.5\%, 4) + (1,000 - 200(P/G, 5.5\%, 4))(F/P, 5.5\%, 4) + 150*(P/F, 5.5\%, 1)$$

$$G = \{ 900*(4.342) + (1000 - 200*5.0233)*1.2388 + 150*0.9479 \} / 11.71$$

$$G = 345.36$$

