Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Calculus III for Engineers
MAT 2322A - Fall 2011
Final Exam
Professor: Victor G. LeBlanc
Time limit: 3 hours. Closed books. No calculators.

Name: Solutions ID Number: __________

Instructions

- This exam has 15 pages and you have 3 hours to complete it.
- This is a closed book exam. Furthermore, all calculators, cell phones, pagers or any other electronic or communication devices are forbidden.
- Read each question carefully before answering.
- Questions 1 to 10 are multiple choice questions. These questions are worth 2 points each and no partial marks are possible. **Please write your answers in the corresponding boxes in the grid below entitled "Answers to multiple choice Qs".**
- Questions 11 to 16 are long answer questions and are worth 5 marks each, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstaple the test.
- Good luck!

Answers to multiple choice Qs

1	2	3	4	5	6	7	8	9	10
C	D/E	A	D	B	F	D	C	A	B

Grid below is used for grading
(do not write in this grid)

MCQ	11	12	13	14	15	16	Total
/20	/5	/5	/5	/5	/5	/5	/50

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1. Consider the function $f(x, y) = x^4 y + 2xy^2$. If we start at the point $(1, 2)$, along which direction should we move in order to obtain the maximum rate of change of this function?

A. $9\vec{i} + 16\vec{j}$

B. 25

C. $16\vec{i} + 9\vec{j}$

D. $\vec{i}$

E. $\vec{j}$

F. this function does not have a maximum rate of change

$$\vec{\nabla} f(1,2) = f_x(1,2)\vec{i} + f_y(1,2)\vec{j}$$
$$= (4x^3y + 2y^2)\Big|_{(1,2)}\vec{i} + (x^4 + 4xy)\Big|_{(1,2)}\vec{j}$$
$$= 16\vec{i} + 9\vec{j}$$

2. Which of the following corresponds to the equation for the tangent plane to the surface $z = 2x^2 - 4y^3$ at the point $(2, 1, 4)$?

A. $z = 8x - 12y + 4$

B. $z = 4x(x - 2) - 12y^2(y - 2) + 4$

C. $z = 4$

D. $z = 8x - 12y$ ← Also good

E. $z = 8(x - 2) - 12(y - 1) + 4$

F. this function is not differentiable at the given point, so there is no tangent plane.

$$z = f_x(2,1)(x-2) + f_y(2,1)(y-1) + 4$$
$$= 4x\Big|_{(2,1)}(x-2) - 12y^2\Big|_{(2,1)}(y-1) + 4$$
$$= 8(x-2) - 12(y-1) + 4$$

3. Consider the solid region which is bounded by the planes $z = 0$, $z = 10 - x - 2y$, $x = 0$, $x = 1$, $y = 0$ and $y = 2$. This solid has a mass density given by $\delta(x, y) = xy$. Then the total mass of this solid is

A. $\frac{20}{3}$

B. $\frac{19}{3}$

C. 1

D. 15

E. 7

F. 0

A

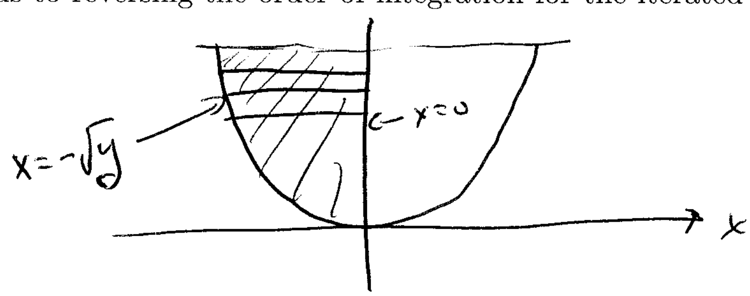
$$\int_{y=0}^{2}\int_{x=0}^{1}\int_{z=0}^{10-x-2y} xy\,dz\,dx\,dy = \int_0^2\int_0^1 xyz\Big|_0^{10-x-2y} dx\,dy$$

$$= \int_0^2\int_0^1 xy(10-x-2y)\,dx\,dy = \int_0^2\int_0^1 (10xy - x^2y - 2xy^2)\,dx\,dy$$

$$= \int_0^2 5x^2y - \frac{x^3y}{3} - x^2y^2\Big|_0^1 dy = \int_0^2 (5y - y/3 - y^2)\,dy$$

$$= 5y^2/2 - y^2/6 - y^3/3\Big|_0^2 = 10 - 2/3 - 8/3 = 20/3$$

4. Which of the following corresponds to reversing the order of integration for the iterated integral $\displaystyle\int_{x=-1}^{0}\int_{y=x^2}^{1} f(x,y)\,dy\,dx$?



$-\sqrt{y} \le x \le 0$

$0 \le y \le 1$

A. $\displaystyle\int_{x=-1}^{0}\int_{y=x^2}^{1} f(y,x)\,dx\,dy$

B. $\displaystyle\int_{y=x^2}^{1}\int_{x=-1}^{0} f(x,y)\,dx\,dy$

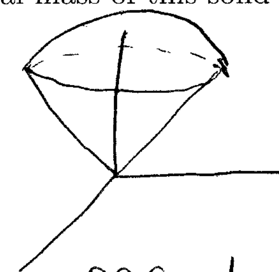
C. $\displaystyle\int_{0}^{1}\int_{x=0}^{\sqrt{y}} f(x,y)\,dx\,dy$

D. $\displaystyle\int_{y=0}^{1}\int_{x=-\sqrt{y}}^{0} f(x,y)\,dx\,dy$

E. $\displaystyle\int_{y=0}^{1}\int_{x=-\sqrt{y}}^{0} f(y,x)\,dx\,dy$

F. $\displaystyle\int_{x=0}^{-1}\int_{y=1}^{x^2} f(x,y)\,dy\,dx$

5. Consider the solid region (ice cream cone) bounded above by the hemisphere $z = \sqrt{1-x^2-y^2}$ and bounded below by the cone $z = \sqrt{x^2+y^2}$. This region has a mass density given by $\delta(x,y,z) = x^2+y^2+z^2$. Then the total mass of this solid is



$0 \le \rho \le 1$
$0 \le \phi \le \pi/4$
$0 \le \theta \le 2\pi$

A. $\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/4}\int_{\rho=0}^{1} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

B. $\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/4}\int_{\rho=0}^{1} \rho^4 \sin\varphi \, d\rho \, d\varphi \, d\theta$

C. $\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/4}\int_{\rho=0}^{1} \rho^4 \, d\rho \, d\varphi \, d\theta$

D. $\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/4}\int_{\rho=0}^{1} \rho^2 \, d\rho \, d\varphi \, d\theta$

E. $\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/2}\int_{\rho=0}^{1} \rho^4 \sin\varphi \, d\rho \, d\varphi \, d\theta$

F. $\displaystyle\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/4}\int_{\rho=0}^{1} \rho^3 \sin\varphi \, d\rho \, d\varphi \, d\theta$

$$\iiint \underset{\rho^2}{\delta} \, \underbrace{dV}_{\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}$$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{1} \rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$$

6. Consider the parametrized curve $\vec{r}(t) = 2t\,\vec{i} + \sin 3t\,\vec{j} - \cos 3t\,\vec{k}$, $0 \le t \le \pi/3$. The total arclength of this curve is

$$\vec{r}\,'(t) = 2\vec{i} + 3\cos 3t\,\vec{j} + 3\sin 3t\,\vec{k}$$

$$\|\vec{r}\,'(t)\| = \sqrt{4 + 9(\cos^2 3t + \sin^2 3t)}$$
$$= \sqrt{4+9} = \sqrt{13}$$

$$L = \int_0^{\pi/3} \|\vec{r}\,'(t)\| \, dt = \int_0^{\pi/3} \sqrt{13} \, dt = \sqrt{13}\,\frac{\pi}{3}$$

A. $\dfrac{2\pi}{3}$

B. $\dfrac{\pi}{3}$

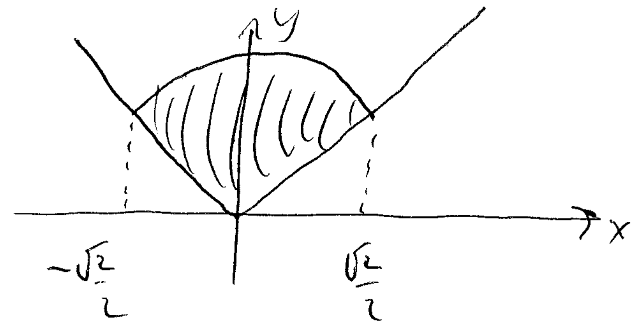
C. $\dfrac{\pi\sqrt{3}}{3}$

D. $\dfrac{-2\pi}{3}\vec{i}$

E. π

F. $\dfrac{\pi\sqrt{13}}{3}$

7. Which of the following is the result of having converted $\displaystyle\int_{x=-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{y=|x|}^{\sqrt{1-x^2}} (x^2 - y)\, dy\, dx$ into polar coordinates?



A. $\displaystyle\int_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, dr\, d\theta$

B. $\displaystyle\int_{\theta=0}^{\frac{\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

C. $\displaystyle\int_{\theta=0}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

D. $\displaystyle\int_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

E. $\displaystyle\int_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{r=0}^{1} (r^2 \sin^2 \theta - r \cos \theta)\, r\, dr\, d\theta$

F. $\displaystyle\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{1} (r^2 \cos^2 \theta - r \sin \theta)\, r\, dr\, d\theta$

$0 \le r \le 1$

$\pi/4 \le \theta \le 3\pi/4$

$(x^2 - y) = r^2 \cos^2\theta - r\sin\theta$

$dy\, dx = r\, dr\, d\theta$

8. Let D be a region in the plane on which Green's theorem holds, and let C denote the boundary of D oriented positively with respect to D. Which of the following line integrals will be equal to the area of D? Recall that the area of D is given by $\displaystyle\int\int_D dx\, dy$.

A. $\displaystyle\oint_C x\, dx + y\, dy$

B. $\displaystyle\oint_C y\, dx + x\, dy$

C. $\displaystyle\oint_C x\, dy$

D. $\displaystyle\oint_C y\, dx$

E. $\displaystyle\oint_C x^2\, dx + y^2\, dy$

F. $\displaystyle\oint_C y^2\, dx - x^2\, dy$

$$\oint P\,dx + Q\,dy = \iint \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{1} dx\,dy$$

$Q = x,\ P = 0 \Rightarrow \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 1$

9. Which of the following corresponds to the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ of the vector field $\vec{F}(x, y, z) = (y + z)\,\vec{\imath} + 2y\,\vec{\jmath} - z\,\vec{k}$ along the straight line segment C which starts at the point $(0, 0, 0)$ and ends at the point $(1, 1, 1)$?

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{2}$

D. $\frac{9}{2}$

E. $\frac{-3}{2}$

F. $\frac{-5}{2}$

$\vec{r}(t) = t\vec{\imath} + t\vec{\jmath} + t\vec{k} \qquad 0 \le t \le 1$

$\vec{r}'(t) = \vec{\imath} + \vec{\jmath} + \vec{k}$

$\vec{F}(\vec{r}(t)) = 2t\vec{\imath} + 2t\vec{\jmath} - t\vec{k}$

$\vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) = 2t + 2t - t = 3t$

$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 3t\,dt = \frac{3t^2}{2}\Big|_0^1 = 3/2$

10 Which of the following corresponds to the value of the surface (flux) integral $\iint_S \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F}(x, y, z) = (y + z)\,\vec{\imath} + 2y\,\vec{\jmath} - z\,\vec{k}$ over the square S defined by $0 \le x \le 2$, $0 \le y \le 2$, $z = 3$, and oriented in the direction of the vector $\vec{k}$?

A. -11

B. -12

C. -13

D. -14

E. -15

F. -16

$0 \le x \le 2, \quad 0 \le y \le 2$

$\vec{r}(x, y) = x\vec{\imath} + y\vec{\jmath} + 3\vec{k}$

$\vec{r}_x \times \vec{r}_y = \vec{\imath} \times \vec{\jmath} = \vec{k}$

$\vec{F}(\vec{r}(x, y)) = (y + 3)\vec{\imath} + 2y\vec{\jmath} - 3\vec{k}$

$\iint_S \vec{F} \cdot d\vec{S} = \int_0^2 \int_0^2 \vec{F}(\vec{r}(x, y)) \cdot \vec{k}\,dx\,dy = -\int_0^2 \int_0^2 3\,dx\,dy$

$= -3(2)^2 = -12$

11. Consider the vector field $\vec{F}(x,y,z) = (z\,e^x \sin y)\,\vec{i} + (z\,e^x \cos y)\,\vec{j} + (e^x \sin y)\,\vec{k}$. Show that the vector field is conservative, and then find a scalar function $f(x,y,z)$ such that $\vec{F}(x,y,z) = \vec{\nabla} f(x,y,z)$. Finally, compute

$$\int_C \vec{F} \cdot d\vec{r},$$

where C is some continuous curve that starts at the point $(0,0,0)$ and ends at the point $\left(1, \frac{\pi}{2}, 2\right)$.

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z e^x \sin y & z e^x \cos y & e^x \sin y \end{vmatrix} = \begin{array}{l} \vec{i}\,(e^x \cos y - e^x \cos y) \\ -\vec{j}\,(e^x \sin y - e^x \sin y) \\ +\vec{k}\,(z e^x \cos y - z e^x \cos y) \end{array}$$

$= \vec{0}$. So $\vec{F}$ is Conservative. $\Rightarrow \exists$ scalar f s.t.

$\vec{F} = \vec{\nabla} f$ i.e. $\frac{\partial f}{\partial x} = z e^x \sin y$, $\frac{\partial f}{\partial y} = z e^x \cos y$, $\frac{\partial f}{\partial z} = e^x \sin y$

$\Rightarrow f(x,y,z) = z e^x \sin y + h(y,z) \Rightarrow \frac{\partial f}{\partial y} = z e^x \cos y + \frac{\partial h}{\partial y} = z e^x \cos y$

$\Rightarrow \frac{\partial h}{\partial y} = 0 \Rightarrow h(y,z) = G(z)$

$\Rightarrow f(x,y,z) = z e^x \sin y + G(z) \Rightarrow \frac{\partial f}{\partial z} = e^x \sin y + G'(z) = e^x \sin y$

$\Rightarrow G'(z) = 0 \Rightarrow G(z) = K$ (constant)

So all potentials are of the form

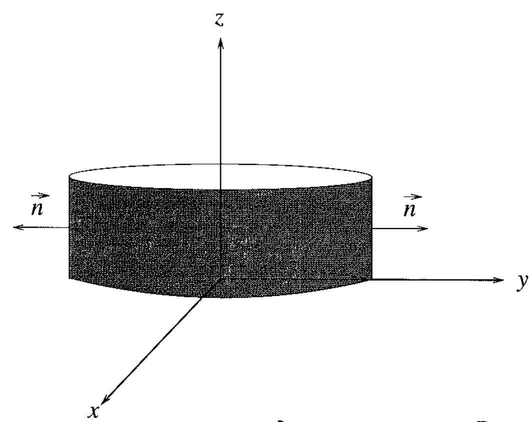
$f(x,y,z) = z e^x \sin y + K$

By F.T.C. for line integrals

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{\nabla} f \cdot d\vec{r} = f(1, \pi/2, 2) - f(0,0,0) = 2e^1 \sin \tfrac{\pi}{2} - 0$$

$= 2e$

12. Consider the vector field $\vec{F}(x,y,z) = (x-y)\,\vec{\imath} + (y+x)\,\vec{\jmath} + \vec{k}$. Let S denote the oriented cylindrical strip $x^2+y^2=9$, $0 \le z \le 2$, with normal vector $\vec{n}$ pointing away from the z-axis, as illustrated. Compute the surface (flux) integral $\displaystyle\iint_S \vec{F}\cdot d\vec{S}$.



Parametrization of surface : $\vec{r}(z,\theta) = 3\cos\theta\,\vec{\imath} + 3\sin\theta\,\vec{\jmath} + z\,\vec{k}$
$0 \le \theta \le 2\pi,\quad 0 \le z \le 2$

$$\vec{r}_\theta \times \vec{r}_z = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{\imath}\,(3\cos\theta) + \vec{\jmath}\,(3\sin\theta)$$

$$\vec{F}(\vec{r}(z,\theta)) = (3\cos\theta - 3\sin\theta)\,\vec{\imath} + (3\sin\theta + 3\cos\theta)\,\vec{\jmath} + \vec{k}$$

$$\vec{F}(\vec{r}(z,\theta))\cdot(\vec{r}_\theta\times\vec{r}_z) = 9\cos^2\theta - 9\sin\theta\cos\theta + 9\sin^2\theta + 9\sin\theta\cos\theta$$
$$= 9(\cos^2\theta + \sin^2\theta) = 9$$

$$\iint_S \vec{F}\cdot d\vec{S} = \int_0^2\int_0^{2\pi} \vec{F}(\vec{r}(z,\theta))\cdot(\vec{r}_\theta\times\vec{r}_z)\,d\theta\,dz = \int_0^2\int_0^{2\pi} 9\,d\theta\,dz =$$

$$9\cdot 2\cdot 2\pi = 36\pi$$

13. Find the global extrema of the function $z = f(x, y) = 3x^2 - 2y^2$ on the disk of radius one, centered at the origin, $0 \le x^2 + y^2 \le 1$.

Critical points: $f_x = 6x$, $f_y = -4y$ $\Rightarrow$ $(0,0)$ is the only critical point, and it is in the domain.

On boundary: $x^2 + y^2 = g(x,y) = 1$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$$\boxed{\begin{aligned} 6x &= \lambda 2x \\ -4y &= \lambda 2y \end{aligned}}$$

$x(3-\lambda) = 0 \Rightarrow x = 0$ or $\lambda = 3$

$x = 0 \Rightarrow y = \pm 1$

$\lambda = 3 \Rightarrow -4y = 6y \Rightarrow y = 0$

$\Rightarrow x = \pm 1$

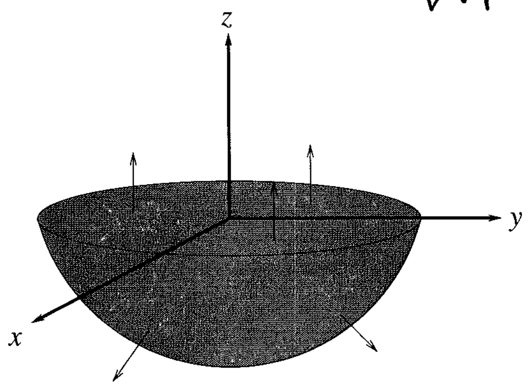
Must check the following points:

$(0,0)$, $(1,0)$, $(-1,0)$, $(0,1)$, $(0,-1)$.

(x,y)	$f(x,y)$	
$(0,0)$	0	
$(1,0)$	3	global max
$(-1,0)$	3	
$(0,1)$	-2	global min
$(0,-1)$	-2	

14. For the vector field $\vec{F}(x,y,z) = \overbrace{(x^3-y^2)}^{P}\,\vec{\imath}+\overbrace{(y^3+x)}^{Q}\,\vec{\jmath}+\overbrace{(z^3+x)}^{R}\,\vec{k}$, compute the divergence of $\vec{F}$, i.e. compute $\vec{\nabla}\cdot\vec{F}$. Then, **using Gauss' divergence theorem**, compute the surface (flux) integral $\displaystyle\int\!\!\int_S \vec{F}\cdot d\vec{S}$, where S is the oriented surface illustrated below, consisting of two parts: the hemisphere $z=-\sqrt{1-x^2-y^2}$, and the disk $0\le x^2+y^2\le 1$ in the plane $z=0$.

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 3x^2+3y^2+3z^2 = 3(x^2+y^2+z^2)$$



Gauss' divergence theorem: $\displaystyle\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \vec{\nabla}\cdot\vec{F}\, dV$

Using spherical coordinates, E is described as

$0\le\rho\le 1$, $0\le\theta\le 2\pi$, $\pi/2\le\phi\le\pi$

$dV=\rho^2\sin\phi\, d\phi\, d\rho\, d\theta$, $\vec{\nabla}\cdot\vec{F} = 3(x^2+y^2+z^2) = 3\rho^2$

$$\Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^1\int_{\pi/2}^{\pi} 3\rho^4\sin\phi\, d\phi\, d\rho\, d\theta = \int_0^{2\pi}\int_0^1 (-3\rho^4\cos\phi)\Big|_{\pi/2}^{\pi} d\rho\, d\theta$$

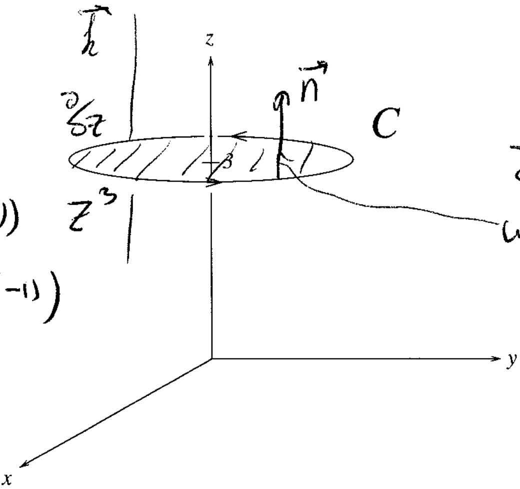
$$= \int_0^{2\pi}\int_0^1 3\rho^4\, d\rho\, d\theta = \int_0^{2\pi} \frac{3\rho^5}{5}\Big|_0^1 d\theta = \int_0^{2\pi}\frac{3}{5}\, d\theta = \frac{6\pi}{5}$$

15. For the vector field $\vec{F}(x,y,z) = (-y + e^{\sin x})\,\vec{\imath} + (x - \ln(\cos^2(e^y)))\,\vec{\jmath} + (z^3)\,\vec{k}$, compute the curl of $\vec{F}$, i.e. compute $\vec{\nabla} \times \vec{F}$. Then, **using Stokes theorem**, compute the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where C is the oriented curve $x^2 + y^2 = 1$, $z = 3$, illustrated below.

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y+e^{\sin x} & x - \ln(\cos^2(e^y)) & z^3 \end{vmatrix}$$

$$= \vec{\imath}(0) - \vec{\jmath}(0) + \vec{k}(1-(-1))$$

$$= 2\vec{k}$$



Stokes thm

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S}$$

where S is disk $x^2+y^2 \le 1$, $z=3$ oriented with $+\vec{k}$.

Parametrize S: $\vec{r}(a,\theta) = a\cos\theta\,\vec{\imath} + a\sin\theta\,\vec{\jmath} + 3\vec{k}$

$0 \le a \le 1$, $0 \le \theta \le 2\pi$

$$\vec{r}_a \times \vec{r}_\theta = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \cos\theta & \sin\theta & 0 \\ -a\sin\theta & a\cos\theta & 0 \end{vmatrix} = a\vec{k}$$

$$(\vec{\nabla}\times\vec{F})(\vec{r}(a,\theta)) = 2\vec{k} \Rightarrow (\vec{\nabla}\times\vec{F})\cdot(\vec{r}_a\times\vec{r}_\theta) = 2a$$

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S} = \int_0^{2\pi}\int_0^1 2a \, da\, d\theta = \int_0^{2\pi} a^2\Big|_0^1 d\theta$$

$$= \int_0^{2\pi} 1 \, d\theta = 2\pi$$

16. Find and classify the critical points of the function $f(x, y) = x^3 + y^3 + 3xy + 3$.

$f_x = 3x^2 + 3y$ $\qquad$ $f_y = 3y^2 + 3x$

$f_x = 0 \Rightarrow y = -x^2$ $\qquad$ $f_y = 0 \Rightarrow 3(-x^2)^2 + 3x = 0$

$x^4 + x = 0$

$x(x^3 + 1) = 0$

$x = 0$ or $x = -1$

$y = 0$ or $y = -1$

Two critical pts $(0,0)$ and $(-1,-1)$.

$f_{xx} = 6x \qquad f_{xy} = f_{yx} = 3 \qquad f_{yy} = 6y$

$\underline{(0,0)}$; $f_{xx} f_{yy} - f_{xy}^2 = -9 < 0 \Rightarrow (0,0)$ is saddle

$\underline{(-1,-1)}$; $f_{xx} f_{yy} - f_{xy}^2 = 36(-1)(-1) - 9 = 27 > 0$

$f_{xx} = 6(-1) = -6 < 0$

$\Rightarrow (-1,-1)$ is local $\underline{\text{max}}$

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