10-Hypothesis tests for *p*

- vocabulary for hypothesis tests
  - hypothesis, hypothesis test
  - null and alternative hypotheses
  - test statistic
  - significance level
  - P-value
- Type I and Type II errors
  - power
- procedure for hypothesis test for *p*
- link to confidence intervals

Hypothesis Tests

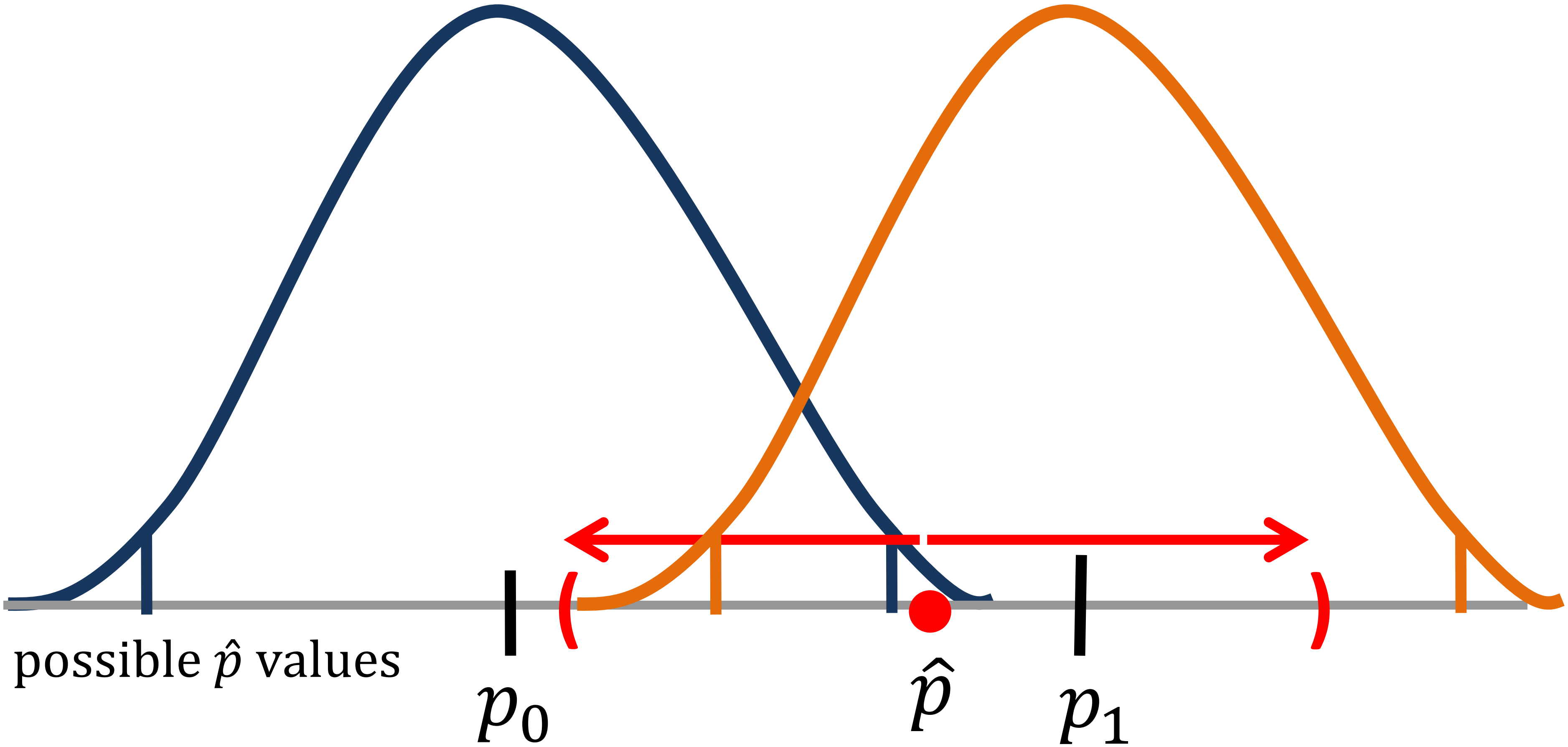
a standard procedure for testing a claim about a property of a population (i.e. a 'hypothesis')

Rare Event Rule:

If the probability—under a given assumption—of a particular observed event is extremely small, then the assumption is probably not correct.

$\hat{p}$ from a different distribution?

$$\hat{p} \sim N\left(p_0, \sqrt{\frac{pq}{n}}\right) \qquad \hat{p} \sim N\left(p_1, \sqrt{\frac{pq}{n}}\right)$$



Example: family history of breast cancer

Suppose that 240 of 10,000 women aged 50-54 years sampled (assume SRS) whose mothers had breast cancer, also had breast cancer themselves at some time in their lives. Given large studies, assume the prevalence rate of breast cancer for Canadian women of this age group is 2%.

How compatible is the sample rate of breast cancer with the population rate of 2%?

Example: family history of breast cancer

Hypothesis?

H_0: ⟵ 'null hypothesis'

H_a: ⟵ 'alternative hypothesis'

Requirements for hypothesis test for *p*:

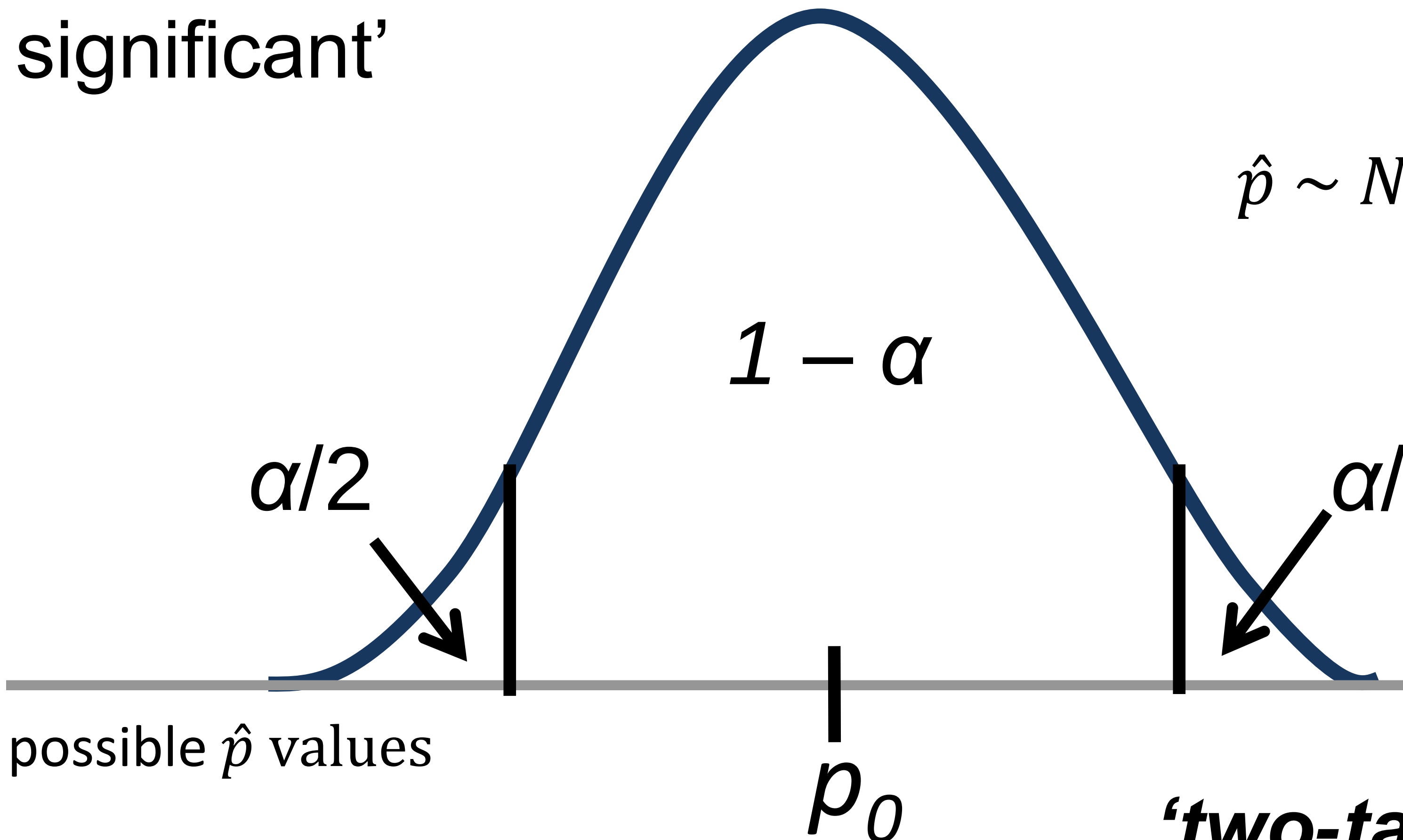
1. simple random sample
2. conditions for binomial distribution satisfied
3. normal approximation of binomial distribution is valid

   a) $np_0 \geq 5$ and b) $nq_0 \geq 5$

What is ‘rare’?

Significance level (α): a ‘cut-off’ probability (set in advance) for evaluating whether observed results should be considered ‘statistically significant’

$$\hat{p} \sim N\left(p, \sqrt{\frac{pq}{n}}\right)$$



‘two-tailed test’

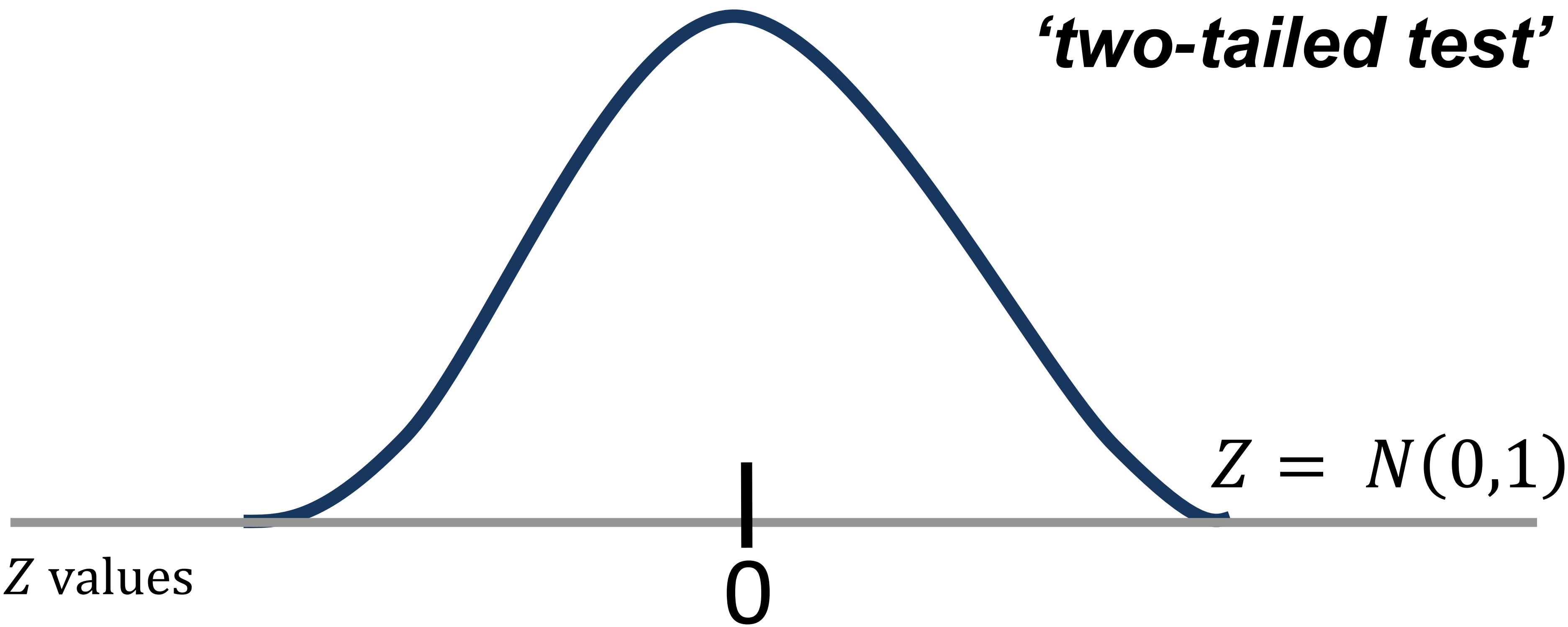
Example: family history of breast cancer

What is the probability of a sample proportion $\hat{p}$ *<u>as extreme or more extreme</u> than 0.024 (i.e. 2.4%), if* p_0 *= 0.02 (i.e. 2%)?*

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$z =$ ⟵ 'test statistic': a standardized version of an estimate

Example: family history of breast cancer



P-value: *probability of observing results* ***as extreme or more extreme*** *as the results actually observed, assuming* H_0 *is true.*

Type I and II errors

		Truth	
		H_0 is true	H_0 is false
Decision from hypothesis test	Reject H_0	Type I error (probability of)	correct decision
	Fail to reject H_0	correct decision	Type II error (probability of β)

Power (1 –): *test's effectiveness at rejecting a false null hypothesis*