

**ADM 2304**  
**APPLIED STATISTICAL METHODS IN BUSINESS**

**Final Exam – December 13, 2008**

NAME (please print): \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

SECTION (please circle one):            A                                    B

**Time allowed:            3 hours**  
**Question booklet:    10 pages (all single-sided sheets, including this cover sheet)**  
**Appendices:            7 pages of output and 5 tables (on 6 double-sided sheets)**

**Instructions:**

1. Calculators, rulers, and one sheet of notes (8.5 x 14 in.) are allowed.
2. Please write clearly and legibly.
3. *Complete all hypothesis tests at the .05 level of significance unless otherwise indicated.*
4. You must hand everything back at the end of the exam (exam question booklet, appendices and stat-tables), but please keep your personal sheet of notes.

<b>Question</b>	<b>Value</b>	<b>Mark</b>
1	16	
2	11	
3	15	
4	25	
<b>Total</b>	<b>67</b>	

**Statement of Academic Integrity**

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: \_\_\_\_\_

**Question 1. [ 16 marks ]**

A Leger Marketing poll released on 4 December 2008 asked a random sample of Canadians whether they would choose an election or a coalition, if MPs were to vote non-confidence in the newly elected government led by Stephen Harper. The results were distributed as follows:

Region	Choice		
	Election	Coalition	Undecided
<b>B.C.</b>	73	39	23
<b>Alberta</b>	75	19	12
<b>Prairies</b>	35	17	8
<b>Ontario</b>	168	152	70
<b>Quebec</b>	60	139	41
<b>Maritimes</b>	19	29	13
<b>Totals</b>	430	395	167

- (a) Calculate a 95% confidence interval for the proportion of Canadians who supported an election.

[3]

- (b) Test whether there were more Canadians who supported an election than those who supported the coalition (ignore the undecided group).

[4]

- (c) Test whether there was a difference in the proportions of British Columbians and of Ontarians who supported the coalition.

[4]

- (d) Test whether the distribution of responses differed by region. Use the partial Minitab output below to minimize your calculations and start by filling in some of the blanks. What is the approximate p-value?

Expected counts are printed below observed counts  
Chi-Square contributions are printed below expected counts

	C1	C2	C3	C4	C5	C6	Total
1	73	75	35	168	60	19	430
	_____	45.95	26.01	169.05	104.03	26.44	
	_____	18.370	3.109	0.007	18.637	2.094	
2	39	19	17	152	139	29	395
	_____	42.21	23.89	155.29	95.56	24.29	
	_____	12.761	1.988	0.070	19.742	0.914	
3	23	12	8	70	41	13	167
	22.73	_____	10.10	65.66	40.40	_____	
	0.003	_____	0.437	0.288	0.009	_____	
Total	_____	_____	60	390	240	61	992

Chi-Sq = \_\_\_\_\_, DF = \_\_, P-Value = \_\_\_\_\_

[5]

**Question 2. [11 marks]**

To compare the performance of two recently hired financial advisors, a financial planning firm selected a random sample of portfolios from each advisor and examined the one-year performance for each portfolio. The results are given below, with the analyses in Appendix A. (Your answers can be chosen from the tests listed in the output.)

Analyst1	Analyst2	Diff1-2
-32.5	-30.4	-2.1
-26.8	-31.5	4.7
-32.2	-31.5	-0.7
-31.1	-35.5	4.4
-33.3	-29.3	-4
-33.3	-29.3	-4
-28.2	-32.4	4.2
-31.8	-31.6	-0.2
-29.6	-31.6	2
-31.5	-33.1	1.6
-26.4	-33.7	7.3
-28.4	-28.6	0.2
-25.6	-34.8	9.2
-26.3	-37.3	11
-26.5	-33.1	6.6

- (a) Are the two samples independent or paired? Explain briefly.

[1]

- (b) Given your answer in (a), identify the appropriate test that should be completed to determine if there is a difference in average (mean/median) performance between the two advisors. Explain briefly with specific reference to the appropriate graph(s).

[2]

- (c) Notwithstanding the above, calculate a 95% confidence interval to estimate the difference in mean performance between the two financial advisors.

[3]

- (d) Using the results in (c), perform a hypothesis test to determine if there is a significant difference in performance.

[3]

- (e) Explain briefly but completely why the corresponding non-parametric test would lead you to the same result as part (d).

[1]

- (f) Explain briefly what the W statistic represents in the Mann-Whitney test

[1]

**Question 3. [15 marks]**

A manufacturer is interested in studying the effects of temperature and supplier on the hardness of windshields. He has four suppliers of raw materials for making the plastic windshields and produces the windshields using three different temperatures.

Temperature	SupplierA	SupplierB	SupplierC	SupplierD
Low	67	60	90	54
	77	55	87	67
	58	48	98	56
Medium	87	55	101	67
	78	67	98	87
	92	60	89	70
High	89	70	104	78
	92	82	98	79
	78	67	110	81

Appendix B contains the output for the analysis of variance.

Rows: Temperature Columns: Supplier

	A	B	C	D	All
Low	67.33 3	54.33 3	91.67 3	59.00 3	68.08 12
Medium	85.67 3	60.67 3	96.00 3	74.67 3	79.25 12
High	86.33 3	73.00 3	104.00 3	79.33 3	85.67 12
All	79.78 9	62.67 9	97.22 9	71.00 9	77.67 36

Cell Contents: Hardness : Mean  
Count

(a) Draw the interactions plot (be sure to label everything properly).

[3]



(b) What does the plot show about how the factors affect the response?

[2]

(c) Comment on whether the model assumptions are warranted.

[2]

(d) Test whether the effect of temperature on hardness depends on the supplier.

[4]

(e) After deciding that the suppliers have an effect on the hardness of the windshields, you decide to compare Supplier C with the other suppliers. Calculate the appropriate margin of error that would allow you to make these comparisons. What do you conclude?

[4]

**Question 4. [25 marks]**

Appendix A contains the Minitab output for a regression analysis to explain the variation in total Profits (\$millions) in 2007 among a sample of large multi-national companies using the following predictor variables:

Revenues (\$ millions)  
Assets (\$ millions)  
Equity (\$millions)  
Market Value (\$ millions)  
Employees

The complete data are listed after the second model.

- (a) Examine the residual plot for Model 1 and discuss the appropriateness of the model assumptions.

[2]

- (b) What might you do to improve the fit? How would you justify this if someone accused you of “playing around” with the data?

[2]

- (c) How do we get from Model 1 to Model 2?

[2]

(d) Discuss three reasons why Model 2 is considered better than Model 1.

[3]

(e) Looking at the data and the output, what other data manipulations would you recommend? Explain briefly.

[2]

(f) Using Model 2, test at the **.01** level of significance the usefulness of Model 2 for predicting profits.

[3]

- (g) Using Model 2, test at the **.01** level of significance the (marginal) usefulness of the Market Value variable for predicting Profits. State your conclusion as clearly as possible.

[3]

- (h) What is the meaning of the value of the coefficient for the Market Value variable in Model 2?

[2]

- (i) What is the 99% interval for the average profit of companies with revenues of \$56,114, equity values of \$14,920, market values of \$102,932 (all figures in millions), and 152,000 employees? Be sure to indicate the units of measurement.

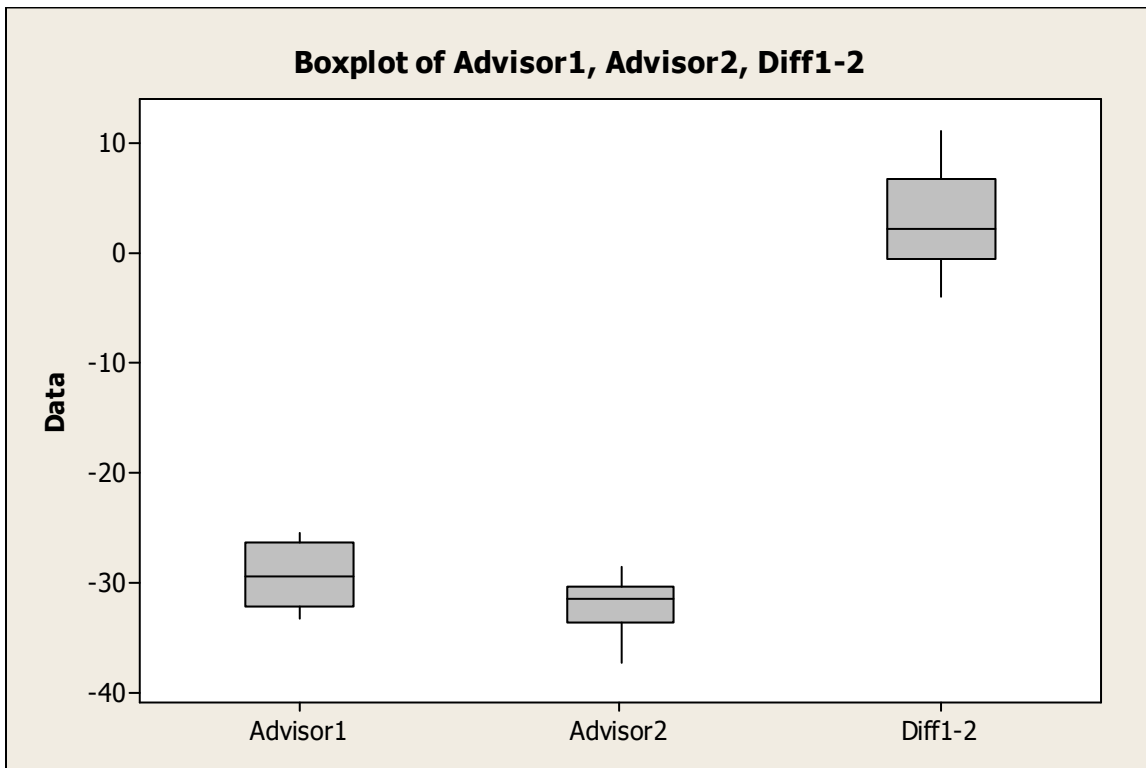
[3]

- (j) What is the 99% interval for the actual profit for a company with the characteristics above? How do you interpret the fact that your interval does not contain the actual profit for company 13 which has the same characteristics?

[3]

**Appendix A**

Advisor1	Advisor2	Diff1-2
-32.5	-30.4	-2.1
-26.8	-31.5	4.7
-32.2	-31.5	-0.7
-31.1	-35.5	4.4
-33.3	-29.3	-4
-33.3	-29.3	-4
-28.2	-32.4	4.2
-31.8	-31.6	-0.2
-29.6	-31.6	2
-31.5	-33.1	1.6
-26.4	-33.7	7.3
-28.4	-28.6	0.2
-25.6	-34.8	9.2
-26.3	-37.3	11
-26.5	-33.1	6.6



**Two-Sample T-Test and CI: Advisor1, Advisor2**

Two-sample T for Advisor1 vs Advisor2

	N	Mean	StDev	SE Mean
Advisor1	15	-29.57	2.82	0.73
Advisor2	15	-32.25	2.42	0.63

Difference = mu (Advisor1) - mu (Advisor2)

Estimate for difference: 2.68000

95% CI for difference: \_\_\_\_\_

T-Test of difference = 0 (vs not =): T-Value = \_\_\_\_\_ P-Value = \_\_\_\_\_ DF = \_\_\_\_

Both use Pooled StDev = \_\_\_\_\_

**Paired T-Test and CI: Advisor1, Advisor2**

Paired T for Advisor1 - Advisor2

	N	Mean	StDev	SE Mean
Advisor1	15	-29.5667	2.8185	0.7277
Advisor2	15	-32.2467	2.4239	0.6259
Difference	15	2.68000	4.60887	1.19001

95% CI for mean difference: \_\_\_\_\_

T-Test of mean difference = 0 (vs not = 0): T-Value = \_\_\_\_\_ P-Value = \_\_\_\_\_

**Mann-Whitney Test and CI: Advisor1, Advisor2**

	N	Median
Advisor1	15	-29.600
Advisor2	15	-31.600

Point estimate for ETA1-ETA2 is 2.800

95.4 Percent CI for ETA1-ETA2 is \_\_\_\_\_

W = 286.0

Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.0279

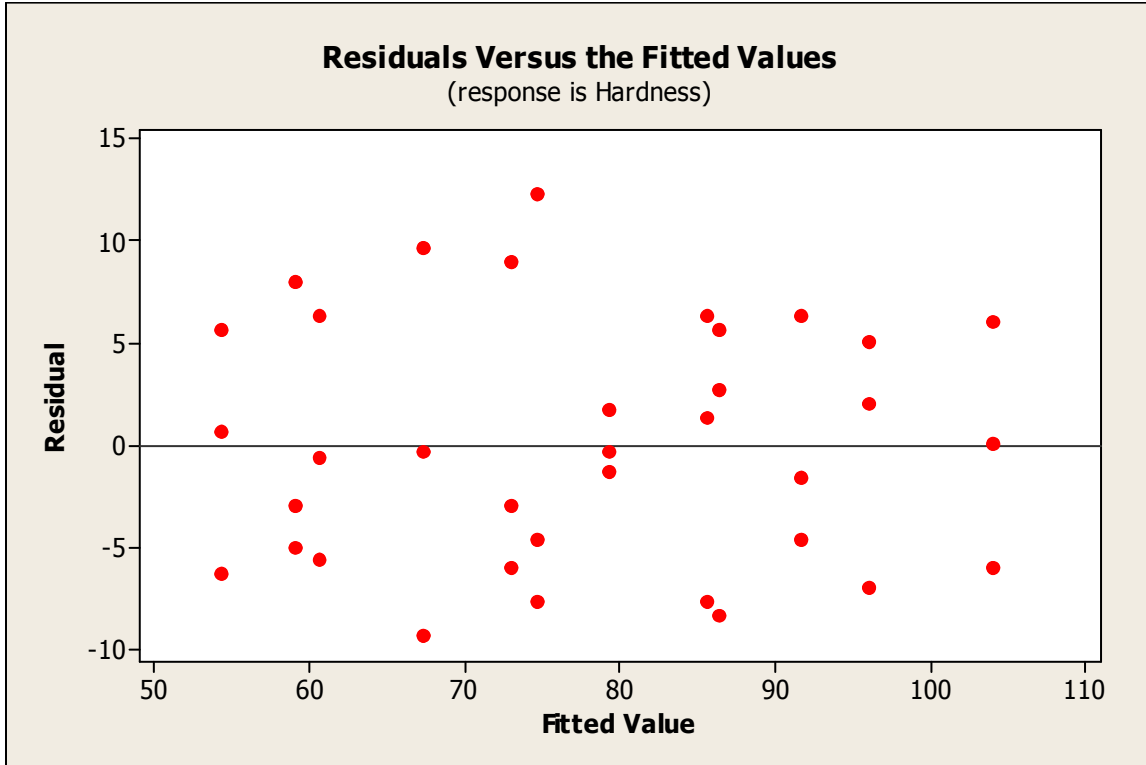
The test is significant at 0.0278 (adjusted for ties)

**Wilcoxon Signed Rank Test: Diff1-2**

Test of median = 0.000000 versus median not = 0.000000

	N	for Test	Wilcoxon Statistic	P	Estimated Median
Diff1-2	15	15	94.5	0.053	2.600

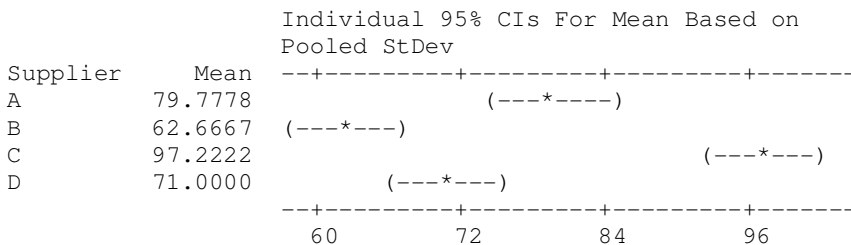
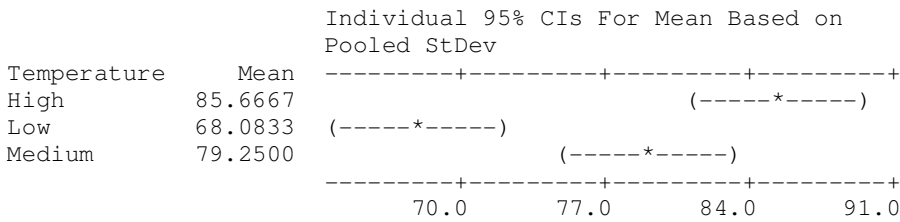
Appendix B



**Two-way ANOVA: Hardness versus Temperature, Supplier**

Source	DF	SS	MS	F	P
Temperature	2	1900.17	950.08	_____	_____
Supplier	3	5906.89	1968.96	_____	_____
Interaction	6	253.61	42.27	_____	_____
Error	24	1211.33	50.47		
Total	35	9272.00			

S = 7.104    R-Sq = 86.94%    R-Sq(adj) = 80.95%



## Appendix C.

### Model 1.

The regression equation is

$$\text{Profits} = 208 + 0.0340 \text{ Revenues} - 0.00048 \text{ Assets} + 0.0448 \text{ Equity} \\ + 0.0208 \text{ MktVal} - 0.00377 \text{ Nemployees}$$

Predictor	Coef	SE Coef	T	P
Constant	208.4	273.3	0.76	0.450
Revenues	0.034033	0.006349	5.36	0.000
Assets	-0.000482	0.001033	-0.47	0.643
Equity	0.04483	0.02042	2.20	0.033
MktVal	0.020803	0.002820		
Nemployees	-0.003766	0.001326	-2.84	0.007

S = 777.045    R-Sq = 85.1%    R-Sq(adj) = 83.4%

#### Analysis of Variance

Source	DF	SS	MS	F	P
Regression	5	151381384	30276277		
Residual Error	44	26567147	603799		
Total	49	177948532			

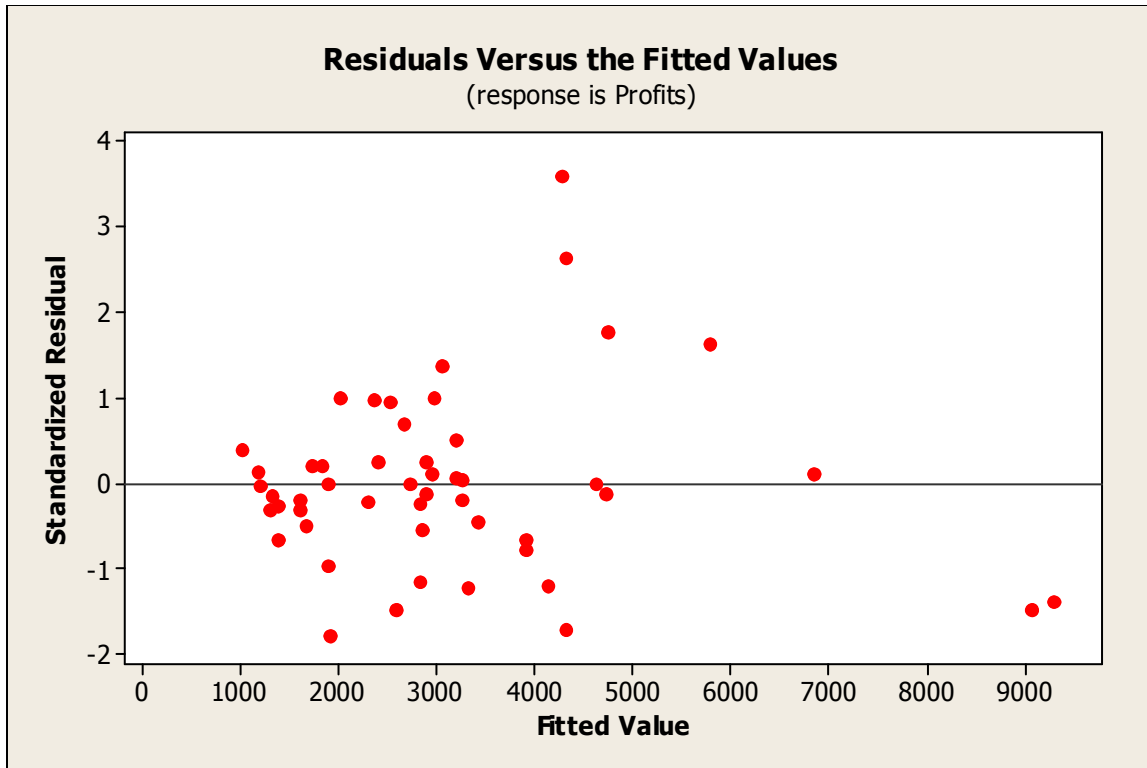
Source	DF	Seq SS
Revenues	1	85086029
Assets	1	3151
Equity	1	31928805
MktVal	1	29496369
Nemployees	1	4867031

#### Unusual Observations

Obs	Revenues	Profits	Fit	SE Fit	Residual	St Resid
1	90840	8203	9070	517	-867	-1.49 X
3	178174	6698	5785	533	913	1.62 X
4	122379	8460	9286	504	-826	-1.40 X
9	43957	3833	3068	538	765	1.37 X
13	56114	6310	4329	186	1981	2.63R
17	119299	3526	4335	617	-809	-1.71 X
32	25070	6945	4287	238	2658	3.59R

R denotes an observation with a large standardized residual.

X denotes an observation whose X value gives it large influence.



**Model 2.**

The regression equation is

$$\text{Profits} = 136 + 0.0345 \text{ Revenues} + 0.0476 \text{ Equity} + 0.0177 \text{ MktVal} - 0.00347 \text{ Nemployees}$$

48 cases used, 2 cases contain missing values

Predictor	Coef	SE Coef	T	P
Constant	136.2	188.7	0.72	0.474
Revenues	0.034493	0.004704	7.33	0.000
Equity	0.04755	0.01479	3.22	0.002
MktVal	0.017686	0.002110		
Nemployees	-0.0034739	0.0009736	-3.57	0.001

S = 571.463    R-Sq = 90.8%    R-Sq(adj) = 89.9%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	4	138087333	34521833		
Residual Error	43	14042526	326570		
Total	47	152129859			

Source	DF	Seq SS
Revenues	1	87141333
Equity	1	26772256
MktVal	1	20015929
Nemployees	1	4157815

Unusual Observations

Obs	Revenues	Profits	Fit	SE Fit	Residual	St Resid
1	90840	8203.0	8549.5	366.2	-346.5	-0.79 X
3	178174	6698.0	5961.7	390.6	736.3	1.77 X
4	122379	8460.0	8964.1	355.4	-504.1	-1.13 X
8	78508	6093.0	4589.4	128.9	1503.6	2.70R
9	43957	3833.3	3189.8	379.2	643.5	1.51 X
17	119299	3526.0	4276.6	453.6	-750.6	-2.16RX
43	37073	610.0	2078.2	175.5	-1468.2	-2.70R

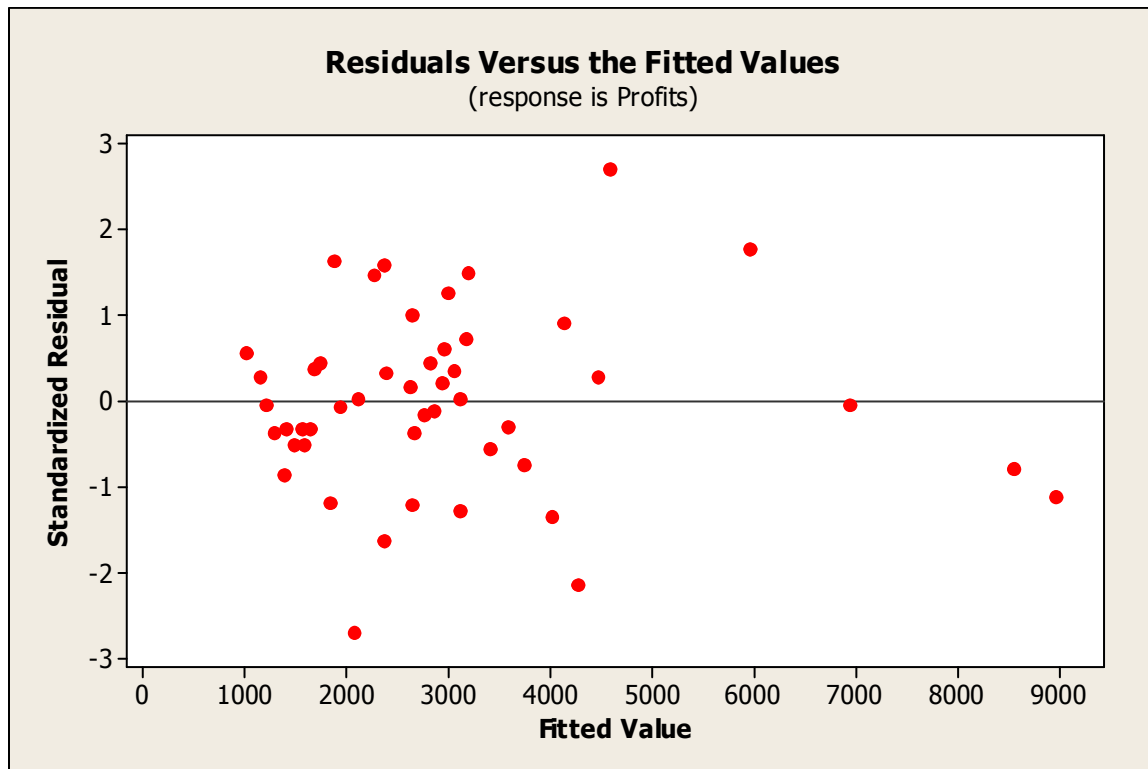
R denotes an observation with a large standardized residual.  
 X denotes an observation whose X value gives it large influence.

Predicted Values for New Observations

New Obs	Fit	SE Fit	99% CI	99% PI
1	4073.7	136.1	_____	_____

Values of Predictors for New Observations

New Obs	Revenues	Equity	MktVal	Nemployees
1	56114	14920	102932	152000



Number	Profits	Revenues	Assets	Equity	MktVal	Nemployees
1	8203	90840	304012	34438	260147	276000
2	6920	153627	279097	30734	73923	363892
3	6698	178174	228888	17506	54244	608000
4	8460	122379	96064	43660	158784	80000
5	3591	34697	310897	21196	66105	93700
6	3708	30381	365521	21742	58151	69033
7	3104	37609	386555	20893	69420	67250
8	6093	78508	81499	19816	98322	269465
9	3833.27	43957	103626	37635.4	0	72655
10	3332.34	30520	163971	24001.1	87964	40000
11	3055.8	27777	391673	13793	67593	3500
12	4638	53261	58635	22647	105879	128000
13	6310	56114	55947	14920	102932	152000
14	2586	27132	302287	13956	46842	47277
15	1906	31731	292819	8329	29152	56600
16	3210	23585	260159	19837	57840	77000
17	3526	119299	45525	18502	113731	825000
18	2805	61147	60418	11362	28369	121000
19	3077	21734	264562	21337	68437	80360
20	3272	59978	43559	19461	58410	42700
21	3105	24949	80918	15610	40657	51400
22	1226.61	29348	214296	5776.6	0	4824
23	2405	41304	42942	11270	77019	98396
24	2454.9	30194	53964	12789.1	77650	141000
25	3256	36376	35473	17472	54852	39362
26	1203	24374	201907	14007	0	44979
27	3119	42895	31749	16155	65060	121900
28	1991	17760	120003	9574	45270	74000
29	1465	17701	262159	11404	23450	16943
30	2720	32836	32489	16319	41329	43451
31	2664	45187	29600	12766	31577	29313
32	6945	25070	28880	19295	125741	64000
33	3415	35764	27544	12046	113635	106000
34	2793.57	23260	42142	8037.6	54187	114359
35	3261	20561	36301	15165	61132	81000
36	1188	41296	38700	5862	22574	296000
37	1896	14329	157274	12032	35837	43933
38	1086	20038	108199	7932	14738	47700
39	1474	24856	42132	9892	75224	118340
40	4614.1	23637	25812	12613.5	156965	53800
41	1395	14399	194597	7521	33447	3200
42	1966	22473	37776	17285	72754	108000
43	610	37073	259482	19718	0	79000
44	1332	13305	131743	6085	12498	25000
45	901.1	18540	96001	11195.4	12420	40300
46	1305.7	13219	115901	10376	39869	56600
47	1300	28069	28361	5176	22465	173000
48	3303	22629	21453	12359	100631	90500
49	1180	29794	27278	13272	32601	150000
50	2142	29292	20101	6936	63351	136000

## Standard Normal Distribution

$P(0 \leq Z \leq z)$										
<b>z</b>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

## Student's t distribution

v	$t_{\alpha}$										
	$\alpha = P(t \geq t_{\alpha})$										
	0.250	0.200	0.150	0.100	0.075	0.050	0.030	0.025	0.010	0.005	0.001
1	1.00	1.38	1.96	3.08	4.17	6.31	10.58	12.71	31.82	63.66	318.31
2	0.82	1.06	1.39	1.89	2.28	2.92	3.90	4.30	6.96	9.92	22.33
3	0.76	0.98	1.25	1.64	1.92	2.35	2.95	3.18	4.54	5.84	10.21
4	0.74	0.94	1.19	1.53	1.78	2.13	2.60	2.78	3.75	4.60	7.17
5	0.73	0.92	1.16	1.48	1.70	2.02	2.42	2.57	3.36	4.03	5.89
6	0.72	0.91	1.13	1.44	1.65	1.94	2.31	2.45	3.14	3.71	5.21
7	0.71	0.90	1.12	1.41	1.62	1.89	2.24	2.36	3.00	3.50	4.79
8	0.71	0.89	1.11	1.40	1.59	1.86	2.19	2.31	2.90	3.36	4.50
9	0.70	0.88	1.10	1.38	1.57	1.83	2.15	2.26	2.82	3.25	4.30
10	0.70	0.88	1.09	1.37	1.56	1.81	2.12	2.23	2.76	3.17	4.14
11	0.70	0.88	1.09	1.36	1.55	1.80	2.10	2.20	2.72	3.11	4.02
12	0.70	0.87	1.08	1.36	1.54	1.78	2.08	2.18	2.68	3.05	3.93
13	0.69	0.87	1.08	1.35	1.53	1.77	2.06	2.16	2.65	3.01	3.85
14	0.69	0.87	1.08	1.35	1.52	1.76	2.05	2.14	2.62	2.98	3.79
15	0.69	0.87	1.07	1.34	1.52	1.75	2.03	2.13	2.60	2.95	3.73
16	0.69	0.86	1.07	1.34	1.51	1.75	2.02	2.12	2.58	2.92	3.69
17	0.69	0.86	1.07	1.33	1.51	1.74	2.02	2.11	2.57	2.90	3.65
18	0.69	0.86	1.07	1.33	1.50	1.73	2.01	2.10	2.55	2.88	3.61
19	0.69	0.86	1.07	1.33	1.50	1.73	2.00	2.09	2.54	2.86	3.58
20	0.69	0.86	1.06	1.33	1.50	1.72	1.99	2.09	2.53	2.85	3.55
21	0.69	0.86	1.06	1.32	1.49	1.72	1.99	2.08	2.52	2.83	3.53
22	0.69	0.86	1.06	1.32	1.49	1.72	1.98	2.07	2.51	2.82	3.50
23	0.69	0.86	1.06	1.32	1.49	1.71	1.98	2.07	2.50	2.81	3.48
24	0.68	0.86	1.06	1.32	1.49	1.71	1.97	2.06	2.49	2.80	3.47
25	0.68	0.86	1.06	1.32	1.49	1.71	1.97	2.06	2.49	2.79	3.45
26	0.68	0.86	1.06	1.31	1.48	1.71	1.97	2.06	2.48	2.78	3.43
27	0.68	0.86	1.06	1.31	1.48	1.70	1.96	2.05	2.47	2.77	3.42
28	0.68	0.85	1.06	1.31	1.48	1.70	1.96	2.05	2.47	2.76	3.41
29	0.68	0.85	1.06	1.31	1.48	1.70	1.96	2.05	2.46	2.76	3.40
30	0.68	0.85	1.05	1.31	1.48	1.70	1.95	2.04	2.46	2.75	3.39
31	0.68	0.85	1.05	1.31	1.48	1.70	1.95	2.04	2.45	2.74	3.37
32	0.68	0.85	1.05	1.31	1.47	1.69	1.95	2.04	2.45	2.74	3.37
33	0.68	0.85	1.05	1.31	1.47	1.69	1.95	2.03	2.44	2.73	3.36
34	0.68	0.85	1.05	1.31	1.47	1.69	1.95	2.03	2.44	2.73	3.35
35	0.68	0.85	1.05	1.31	1.47	1.69	1.94	2.03	2.44	2.72	3.34
36	0.68	0.85	1.05	1.31	1.47	1.69	1.94	2.03	2.43	2.72	3.33
37	0.68	0.85	1.05	1.30	1.47	1.69	1.94	2.03	2.43	2.72	3.33
38	0.68	0.85	1.05	1.30	1.47	1.69	1.94	2.02	2.43	2.71	3.32
39	0.68	0.85	1.05	1.30	1.47	1.68	1.94	2.02	2.43	2.71	3.31
40	0.68	0.85	1.05	1.30	1.47	1.68	1.94	2.02	2.42	2.70	3.31
50	0.68	0.85	1.05	1.30	1.46	1.68	1.92	2.01	2.40	2.68	3.26
100	0.68	0.85	1.04	1.29	1.45	1.66	1.90	1.98	2.36	2.63	3.17
200	0.68	0.84	1.04	1.29	1.45	1.65	1.89	1.97	2.35	2.60	3.13

## Chi-Square Distribution

v	$\chi^2_{\alpha}$									
	$\alpha = P(\chi^2 \geq \chi^2_{\alpha})$									
	0.100	0.050	0.025	0.010	0.005	0.001	0.0001	0.00001	0.000001	0.0000005
1	2.71	3.84	5.02	6.63	7.88	10.83	15.14	19.51	23.93	25.26
2	4.61	5.99	7.38	9.21	10.60	13.82	18.42	23.03	27.63	29.02
3	6.25	7.81	9.35	11.34	12.84	16.27	21.11	25.90	30.66	32.09
4	7.78	9.49	11.14	13.28	14.86	18.47	23.51	28.47	33.38	34.84
5	9.24	11.07	12.83	15.09	16.75	20.52	25.74	30.86	35.89	37.39
6	10.64	12.59	14.45	16.81	18.55	22.46	27.86	33.11	38.26	39.79
7	12.02	14.07	16.01	18.48	20.28	24.32	29.88	35.26	40.52	42.09
8	13.36	15.51	17.53	20.09	21.95	26.12	31.83	37.33	42.70	44.30
9	14.68	16.92	19.02	21.67	23.59	27.88	33.72	39.34	44.81	46.43
10	15.99	18.31	20.48	23.21	25.19	29.59	35.56	41.30	46.86	48.51
11	17.28	19.68	21.92	24.72	26.76	31.26	37.37	43.21	48.87	50.54
12	18.55	21.03	23.34	26.22	28.30	32.91	39.13	45.08	50.83	52.53
13	19.81	22.36	24.74	27.69	29.82	34.53	40.87	46.91	52.75	54.47
14	21.06	23.68	26.12	29.14	31.32	36.12	42.58	48.72	54.64	56.38
15	22.31	25.00	27.49	30.58	32.80	37.70	44.26	50.49	56.49	58.26
16	23.54	26.30	28.85	32.00	34.27	39.25	45.92	52.24	58.32	60.12
17	24.77	27.59	30.19	33.41	35.72	40.79	47.57	53.97	60.13	61.95
18	25.99	28.87	31.53	34.81	37.16	42.31	49.19	55.68	61.91	63.75
19	27.20	30.14	32.85	36.19	38.58	43.82	50.80	57.37	63.68	65.53
20	28.41	31.41	34.17	37.57	40.00	45.31	52.39	59.04	65.42	67.30
21	29.62	32.67	35.48	38.93	41.40	46.80	53.96	60.70	67.15	69.04
22	30.81	33.92	36.78	40.29	42.80	48.27	55.52	62.34	68.86	70.77
23	32.01	35.17	38.08	41.64	44.18	49.73	57.07	63.97	70.55	72.48
24	33.20	36.42	39.36	42.98	45.56	51.18	58.61	65.58	72.23	74.18
25	34.38	37.65	40.65	44.31	46.93	52.62	60.14	67.18	73.89	75.86
26	35.56	38.89	41.92	45.64	48.29	54.05	61.66	68.77	75.55	77.54
27	36.74	40.11	43.19	46.96	49.64	55.48	63.16	70.35	77.19	79.19
28	37.92	41.34	44.46	48.28	50.99	56.89	64.66	71.92	78.82	80.84
29	39.09	42.56	45.72	49.59	52.34	58.30	66.15	73.47	80.44	82.48
30	40.26	43.77	46.98	50.89	53.67	59.70	67.63	75.02	82.04	84.10
35	46.06	49.80	53.20	57.34	60.27	66.62	74.93	82.64	89.95	92.08
40	51.81	55.76	59.34	63.69	66.77	73.40	82.06	90.08	97.65	99.86
50	63.17	67.50	71.42	76.15	79.49	86.66	95.97	104.54	112.61	114.96
100	118.50	124.34	129.56	135.81	140.17	149.45	161.32	172.10	182.13	185.03
150	172.58	179.58	185.80	193.21	198.36	209.26	223.11	235.60	247.15	250.48
200	226.02	233.99	241.06	249.45	255.26	267.54	283.06	297.00	309.84	313.53
224	251.52	259.91	267.35	276.16	282.27	295.14	311.39	325.96	339.36	343.22
250	279.05	287.88	295.69	304.94	311.35	324.83	341.83	357.04	371.02	375.04
500	540.93	553.13	563.85	576.49	585.21	603.45	626.24	646.48	664.96	670.25

**Fisher's F distribution**

<b>F<sub>.05</sub></b>												
<b>P( F ≥ F<sub>.05</sub> ) = 0.050</b>												
<b>v<sub>1</sub></b>												
<b>v<sub>2</sub></b>	1	2	3	4	5	6	7	8	9	10	11	12
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.0	243.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.11	2.08
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	2.10	2.07
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	2.09	2.06
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	2.08	2.05
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07	2.04
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80

Studentized range values  $q(.05)$ 

df	k = number of groups								
	2	3	4	5	6	7	8	9	10
1	17.969	26.976	32.819	37.082	40.408	43.119	45.397	47.357	49.071
2	6.085	8.331	9.798	10.881	11.734	12.435	13.027	13.539	13.988
3	4.501	5.91	6.825	7.502	8.037	8.478	8.852	9.177	9.462
4	3.926	5.04	5.757	6.287	6.706	7.053	7.347	7.602	7.826
5	3.635	4.602	5.218	5.673	6.033	6.33	6.582	6.801	6.995
6	3.46	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493
7	3.344	4.165	4.681	5.06	5.359	5.606	5.815	5.997	6.158
8	3.261	4.041	4.529	4.886	5.167	5.399	5.596	5.767	5.918
9	3.199	3.948	4.415	4.755	5.024	5.244	5.432	5.595	5.738
10	3.151	3.877	4.327	4.654	4.912	5.124	5.304	5.46	5.598
11	3.113	3.82	4.256	4.574	4.823	5.028	5.202	5.353	5.486
12	3.081	3.773	4.199	4.508	4.75	4.95	5.119	5.265	5.395
13	3.055	3.734	4.151	4.453	4.69	4.884	5.049	5.192	5.318
14	3.033	3.701	4.111	4.407	4.639	4.829	4.99	5.13	5.253
15	3.014	3.673	4.076	4.367	4.595	4.782	4.94	5.077	5.198
16	2.998	3.649	4.046	4.333	4.557	4.741	4.896	5.031	5.15
17	2.984	3.628	4.02	4.303	4.524	4.705	4.858	4.991	5.108
18	2.971	3.609	3.997	4.276	4.494	4.673	4.824	4.955	5.071
19	2.96	3.593	3.977	4.253	4.468	4.645	4.794	4.924	5.037
20	2.95	3.578	3.958	4.232	4.445	4.62	4.768	4.895	5.008
21	2.941	3.565	3.942	4.213	4.424	4.597	4.743	4.87	4.981
22	2.933	3.553	3.927	4.196	4.405	4.577	4.722	4.847	4.957
23	2.926	3.542	3.914	4.18	4.388	4.558	4.702	4.826	4.935
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
25	2.913	3.523	3.89	4.153	4.358	4.526	4.667	4.789	4.897
26	2.907	3.514	3.88	4.141	4.345	4.511	4.652	4.773	4.88
27	2.902	3.506	3.87	4.13	4.333	4.498	4.638	4.758	4.864
28	2.897	3.499	3.861	4.12	4.322	4.486	4.625	4.745	4.85
29	2.892	3.493	3.853	4.111	4.311	4.475	4.613	4.732	4.837
30	2.888	3.486	3.845	4.102	4.301	4.464	4.601	4.72	4.824