Exam 2016, questions - Math exam sample for midterm

Differential & Integral Calculus II (Concordia University)

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section**
Mathematics	205	CA
Examination	**Date**	**Duration**
Midterm	25 May, 2016	90 minutes
Instructor		**Course Examiners**
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Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks**	

[*Marks*]

[11] **1.** **(a)** Write the sigma notation formula for the right Riemann sum R_n of the function $f(x) = x^2 + 2x$ on the interval [-1,2] using n subintervals of equal length, and calculate the definite integral $\int_{-1}^{2} f(x)\,dx$ as the limit of R_n at $n \to \infty$.

(Reminder: $\sum_{k=1}^{n} k = n(n+1)/2, \qquad \sum_{k=1}^{n} k^2 = n\,(n+1)\,(2n+1)/6$)

(b) Calculate the derivative of $F(x) = \int_{\ln(x^2+1)}^{\ln 2} e^t\sqrt{t}\,dt$, and determine whether $F(x)$ is increasing or decreasing at the point $x = 1$.

[6] **2.** Find the antiderivative $F(x)$ of $f(x) = \dfrac{\sin(x) + \sqrt{1 + 2\tan(x)}}{\cos^2(x)}$ such that $F(0) = 0$.

[15] **3.** Calculate the following indefinite integrals

(**a**) $\displaystyle\int \frac{(x^{3/4} + x^{1/4})^2}{x}\,\mathrm{d}x$ (**b**) $\displaystyle\int \frac{x^3 - 4}{x^3 - 4x}\,\mathrm{d}x$ (**c**) $\displaystyle\int \frac{3^x}{3^{2x} + 9}\,\mathrm{d}x$

[12] **4.** Evaluate the following definite integrals (give the exact values, **do not approximate**):

(**a**) $\displaystyle\int_0^4 \frac{x}{\sqrt{2x+1}}\,\mathrm{d}x$ (**b**) $\displaystyle\int_0^1 x^2\cos(\pi x)\,\mathrm{d}x$

[6] **5.** Find the mean value of the function $f(x) = \cos^3(x)\sin^2(x)$ on the interval $[0, \pi/2]$.

[3] **Bonus question.** Assuming $f(x)$ is a continuous function of x, find $F'(x)$ for

$$F(x) = \int_1^x [x^2 + f(t)]dt$$

[NOTE: the answer is **not** $F'(x) = x^2 + f(x)$.]