

This test paper has two parts and total of 30 marks.

Part I has 4 multiple choice questions. Part II has 3 long answer questions.

It cannot be taken from the examination room.

Only nonprogrammable calculators are allowed. Duration: 50 Minutes.

NAME :

STUDENT NO :

PART I: Multiple Choice Questions. Circle the correct answer. No partial marks.

[2] 1. Let A and B be two 3×3 matrices such that $\det A = 4$ and $\det B = 16$.
What is $\det(4A^2B^{-1}A^T)$?

(a) 4^3

(b) 4^4

(c) 4^5

(d) 4^6

(e) 4^8

[2] 2. Let A be a 6×10 matrix. If the rank of A is 3, what is the dimension of null space of A ?

(a) 2

(b) 3

(c) 4

(d) 5

(e) 7

[2] 3. Let $H = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix} \right\}$. What is the dimension of the subspace H ?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 0

[2] 4. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ -2 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 5 \\ 1 \\ 2 \end{bmatrix}$.

You are given that $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis for $H = \text{Span}\{v_1, v_2, v_3\}$ and $x \in H$.

If $[x]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$, what is c_2 ?

(a) -2

(b) 2

(c) 3

(d) 1

(e) -1

PART II: Long answer questions. Show all your work.

[6] 1. Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 3 & 2 \\ 0 & 1 & 7 & 8 \\ 3 & 0 & 0 & 9 \end{bmatrix}$. Calculate $\det A$

[8] 2. Let $A = \begin{bmatrix} 5 & 3 & 1 & 1 \\ 0 & 2 & 2 & 3 \\ 4 & 0 & 0 & 1 \\ 0 & 2 & 1 & 8 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 8 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$.

Use Cramer's Rule to solve for x_1 (without solving for x_2, x_3 and x_4) in the matrix equation $Ax = b$.

[8] 3. You are given that $A = \begin{bmatrix} 1 & -5 & 3 & 2 & 4 \\ -1 & 6 & -2 & 1 & -2 \\ 1 & -4 & 5 & 5 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 17 & 6 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} = B$.

(a) Find a basis for $\text{Col}A$.

(c) Find a basis for $\text{Nul}A$.

(b) What is $\dim(\text{Col}A)$?

(d) What is $\dim(\text{Null}A)$?