Trigonometry Module Review **Name: SOLUTIONS**

1) Complete each of the following angle conversions: (1 mark each)

a) $106.75° = 106.75° \times \frac{\pi\, rad}{180°}$
$= 1.86314\, rad$

b) $2.075\, rad = 2.075\, rad \times \frac{180°}{\pi\, rad}$
$= 118.89°$

c) $126.735° = 126.735° \times \frac{\pi\, rad}{180°}$
$= 2.211943028\, rad$
$= 2.21\, rad$

d) $2.25\pi\, rad = 2.25\pi\, rad \times \frac{180°}{\pi\, rad}$
$= 2.25 \times 180°$
$= 405°$

e) $-36.55° = -36.55° \times \frac{\pi\, rad}{180°}$
$= -0.6379\, rad$

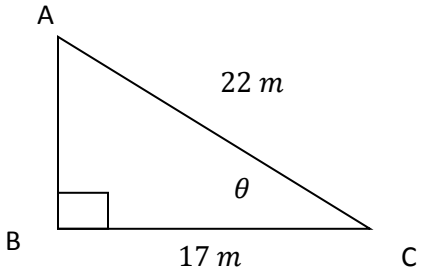
f) $\frac{4}{9}\pi\, rad = \frac{4}{9}\pi\, rad \times \frac{180°}{\pi\, rad}$
$= \frac{4}{9} \times 180°$
$= 80°$

g) $152° = 152° \times \frac{\pi\, rad}{180°}$
$= \frac{152}{180}\, \pi\, rad$
$= \frac{38}{45}\, \pi\, rad\ (= 0.844\pi\, rad)$

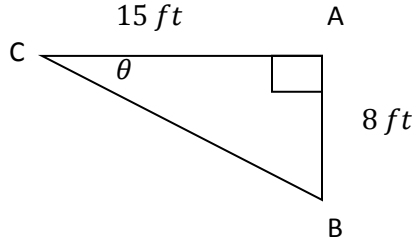
h) $67.53° = 67.53° \times \frac{\pi\, rad}{180°}$
$= 67.53° \times \frac{\pi\, rad}{180°}$
$= 0.37516666 \times \pi\, rad$
$= 0.375\pi\, rad$

2) For each triangle below state the ratio for each of the primary trigonometric functions for angle θ.

a)



b)



$c = \sqrt{22^2 - 17^2} = \sqrt{195} \cong 13.96$

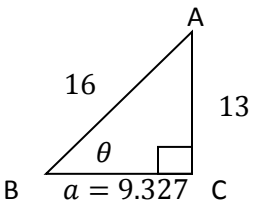
$\sin(\theta) = \frac{\sqrt{195}}{22}$
$\cos(\theta) = \frac{17}{22}$
$\tan(\theta) = \frac{\sqrt{195}}{17}$

$a = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

$\sin(\theta) = \frac{8}{17}$
$\cos(\theta) = \frac{15}{17}$
$\tan(\theta) = \frac{8}{15}$

3) Given that $\csc(\theta) = \frac{16}{13}$. Draw the triangle labelling all 3 sides.

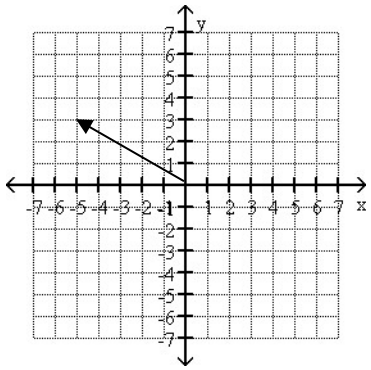
$\csc(\theta) = \frac{16}{13} = \frac{Hypotenuse}{Opposite}$



$a = \sqrt{16^2 - 13^2} = \sqrt{87} \cong 9.327$
$\theta = \sin^{-1}\left(\frac{13}{16}\right) = 54.34°$
$\angle A = 90° - 54.34° = 35.66°$

4) For the following terminal arms, state both the positive and negative values of the angle in standard position.

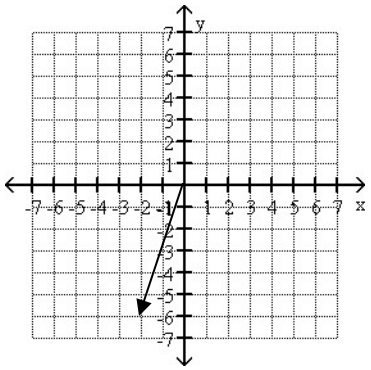
a)



$\theta = 180° - \tan^{-1}\left(\frac{3}{5}\right) = 149.04°$

Or $\theta = 149.04° - 360° = -210.96°$

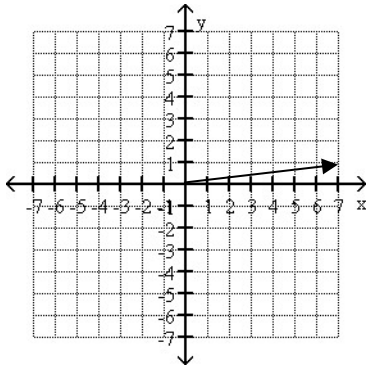
b)



$\theta = 180° + \tan^{-1}\left(\frac{6}{2}\right) = 251.57°$

Or $\theta = 251.57° - 360° = -108.43°$

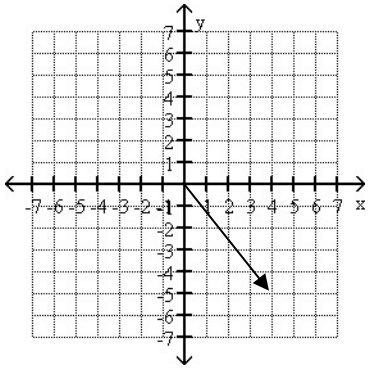
c)



$\theta = \tan^{-1}\left(\frac{1}{7}\right) = 8.13°$

Or $\theta = 8.13° - 360° = -351.87°$

d)



$\theta = 360° - \tan^{-1}\left(\frac{5}{4}\right) = 308.66°$

Or $\theta = 308.66° - 360° = -51.34°$

5) Using a calculator, determine the values of each trig function below:

a) $\sin(5°) = 0.0872$

b) $\cos(-45°) = 0.707$

c) $\tan(112°) = -2.475$

d) $\sin(215°) = -0.574$

e) $\cos(308°) = 0.616$

f) $\tan(-250°) = -2.747$

g) $\sin(5) = -0.959$

h) $\cos(-1.25) = 0.315$

i) $\tan\left(\frac{\pi}{2}\right) = \infty$ (undefined)

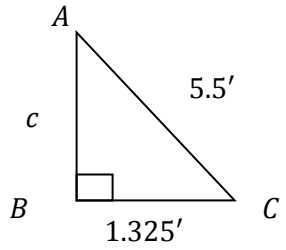
j) $\sin\left(\frac{2\pi}{3}\right) = 0.866$

k) $\cos\left(\frac{3}{5}\pi\right) = -0.309$

l) $\tan(-2\pi) = 0$

6) Solve the following triangles for all missing sides and angles:

a)



$\sin(C) = \frac{c}{5.5}$

$\tan(C) = \frac{c}{1.325}$

$\cos(C) = \frac{1.325}{5.5}$

$\rightarrow C = \cos^{-1}\left(\frac{1.325}{5.5}\right) = 76.06°$

$c = 1.325\tan(76.06°) = 5.338′$

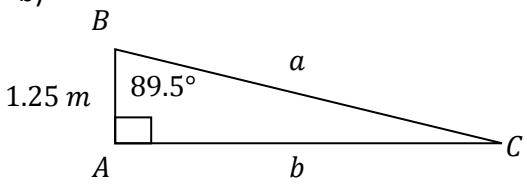
Giving

$A = 180° - 90° - 76.06° = 13.94°$

$C = 76.06°$

$c = 5.34′$

b)



$\sin(89.5°) = \frac{b}{a}$

$\cos(89.5°) = \frac{1.25}{a} \rightarrow a = \frac{1.25}{\cos(89.5°)} = 143.241\ m$

$\tan(89.5°) = \frac{b}{1.25}$

$\rightarrow b = 1.25\tan(89.5°) = 143.236\ m$

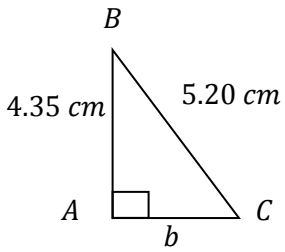
$C = 180° - 90° - 89.5° = 0.5°$

$C = 0.5°$

$a = 143.241\ m$

$b = 143.236\ m$

c)



$\sin(C) = \frac{4.35}{5.20} \rightarrow C = \sin^{-1}\left(\frac{4.35}{5.20}\right) = 56.78°$

$\cos(C) = \frac{b}{5.20}$

$\tan(C) = \frac{4.35}{b}$

$\cos(C) = \frac{b}{5.20} \rightarrow b = 5.2\cos(56.78°) = 2.85\ cm$

$B = 180° - 90° - 56.78° = 33.22°$

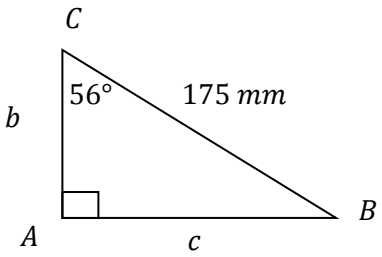
Giving

$b = 2.85\ cm$

$B = 33.22°$

$C = 56.78°$

d)



$\sin(56°) = \frac{c}{175}$

$\cos(56°) = \frac{b}{175}$

$\tan(56°) = \frac{c}{b}$

$\rightarrow c = 175\sin(56°) = 145.08\ mm$

$\rightarrow b = 175\cos(56°) = 97.86\ mm$

$B = 180° - 90° - 56° = 34°$

$b = 97.86\ mm$

$c = 145.08\ mm$

$B = 34°$

7) Find BOTH possible angles between 0 and 360 degrees given:

a) $\sin(\theta) = 0.150$

$\theta_1 = \sin^{-1}(0.150)$
$= 8.627°$ (By Calc)

$\theta_2 = 180° - 8.627°$
$= 171.373°$ (By Cast)

b) $\sin(\theta) = -0.532$

$\theta_1 = \sin^{-1}(-0.532)$
$= -32.141°$ (re-write as +)
$= 327.859°$ (By Calc)

$\theta_2 = 180° + 32.141°$
$= 212.141°$ (By Cast)

c) $\cos(\theta) = 0.821$

$\theta_1 = \cos^{-1}(0.821)$
$= 34.815°$ (By Calc)

$\theta_2 = 360° - 34.815°$
$= 325.185°$ (By Cast)

d) $\cos(\theta) = -0.973$

$\theta_1 = \cos^{-1}(-0.973)$
$= 166.656°$ (By Calc)

$\theta_2 = 180° + (180° - 166.656°)$
$= 193.344°$ (By Cast)

e) $\tan(\theta) = 0.275$

$\theta_1 = \tan^{-1}(0.275)$
$= 15.376°$

$\theta_2 = 180° + 15.376°$
$= 195.376°$ (By Cast)

f) $\tan(\theta) = -5.85$

$\theta_1 = \tan^{-1}(-5.85)$
$= -80.300°$ (rewrite)
$= 279.700°$ (By Calc)

$\theta_2 = 180° - 80.300°$
$= 99.700°$ (By Cast)

8) Find the correct angle given the following:

a) $\sin(\theta) = -0.25$ and $\cos(\theta) > 0$

$\theta_1 = \sin^{-1}(-0.25) = -14.478°$ (re − write)
$= 345.522°$ (By Calc)

Since $\cos(\theta) > 0$ in Quadrant IV
345.522° is the answer.

b) $\cos(\theta) = 0.285$ and $\tan(\theta) < 0$

$\theta_1 = \cos^{-1}(0.285) = 73.441°$

$\theta_2 = 360° - 73.441° = 286.559°$

Since $\tan(\theta) < 0$ in quadrant IV, 286.559°
is the answer

c) $\tan(\theta) = -1.732$ and $\sin(\theta) < 0$

$\theta_1 = \tan^{-1}(-1.732) = -59.999°$ (re-write)
$= 300.001°$ (By Calc)

$\theta_2 = 180° - 59.999° = 120.001°$

Since $\sin(\theta) < 0$ in Quadrant IV
300.0° is the answer.

d) $\sin(\theta) = 0.68$ and $\tan(\theta) > 0$

$\theta_1 = \sin^{-1}(0.68) = 42.844°$

$\theta_2 = 180° - 42.844° = 137.156°$

Since $\tan(\theta) > 0$ in quadrant I, 42.844°
is the answer

e) $\cos(\theta) = -0.95$ and $\sin(\theta) > 0$

$\theta_1 = \cos^{-1}(-0.95) = 161.805°$ (by Calc)

$\theta_2 = 180° + (180° - 161.805°) = 198.195°$

Since $\sin(\theta) > 0$ in Quadrant II
161.805° is the answer.

f) $\tan(\theta) = 0.3$ and $\cos(\theta) < 0$

$\theta_1 = \tan^{-1}(0.3) = 16.699°$ (by Calc)

$\theta_2 = 180° + 16.699° = 196.699°$

Since $\cos(\theta) < 0$ in quadrant III, 196.699°
is the answer

g) $\sin(\theta) = -0.4$ and $\cos(\theta) < 0$

$\theta_1 = \sin^{-1}(-0.4) = -23.578°$ (re-write)
$= 336.422°$ (By Calc)

$\theta_2 = 180° + 23.578° = 203.578°$

Since $\cos(\theta) < 0$ in Quadrant III
203.578° is the answer.

h) $\cos(\theta) = 0.8$ and $\tan(\theta) < 0$

$\theta_1 = \cos^{-1}(0.8) = 36.870°$

$\theta_2 = 360° - 36.870° = 323.130°$

Since $\tan(\theta) < 0$ in quadrant IV, 323.130°
is the answer

9) Complete the following operations:

a) $\frac{\pi}{2}+\frac{5\pi}{4}=\frac{2\pi}{4}+\frac{5\pi}{4}=\frac{7\pi}{4}$

b) $\frac{3\pi}{2}+\frac{\pi}{8}=\frac{12\pi}{8}+\frac{\pi}{8}=\frac{13\pi}{8}$

c) $\frac{5}{8}\pi-\frac{3\pi}{4}=\frac{5\pi}{8}-\frac{6\pi}{8}=\frac{-\pi}{8}$

d) $-\frac{\pi}{4}+\frac{3}{2}\pi=\frac{-\pi}{4}+\frac{6\pi}{4}=\frac{5\pi}{4}$

e) $\frac{\pi}{8}\times\frac{2}{3}=\frac{\pi}{4}\times\frac{1}{3}=\frac{\pi}{12}$

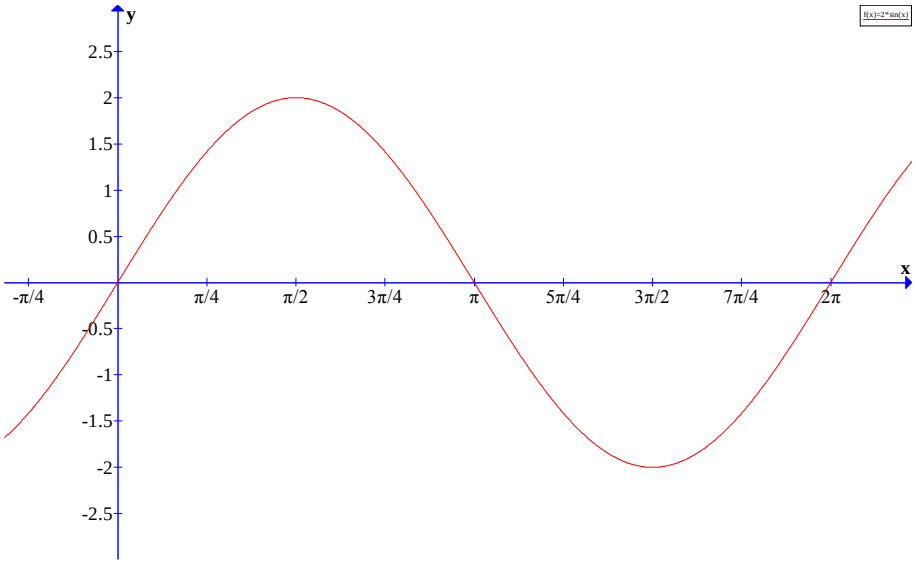
f) $-\frac{\pi}{4}\div\frac{4}{5}=\frac{-\pi}{4}\times\frac{5}{4}=\frac{-5\pi}{16}$

g) $\frac{\pi}{3}\times\left(-\frac{4}{5\pi}\right)=\frac{1}{3}\times\left(-\frac{4}{5}\right)=\frac{-4}{15}$

h) $\frac{2\pi}{5}\div\frac{3\pi}{8}=\frac{2\pi}{5}\times\frac{8}{3\pi}=\frac{2}{5}\times\frac{8}{3}=\frac{16}{15}$

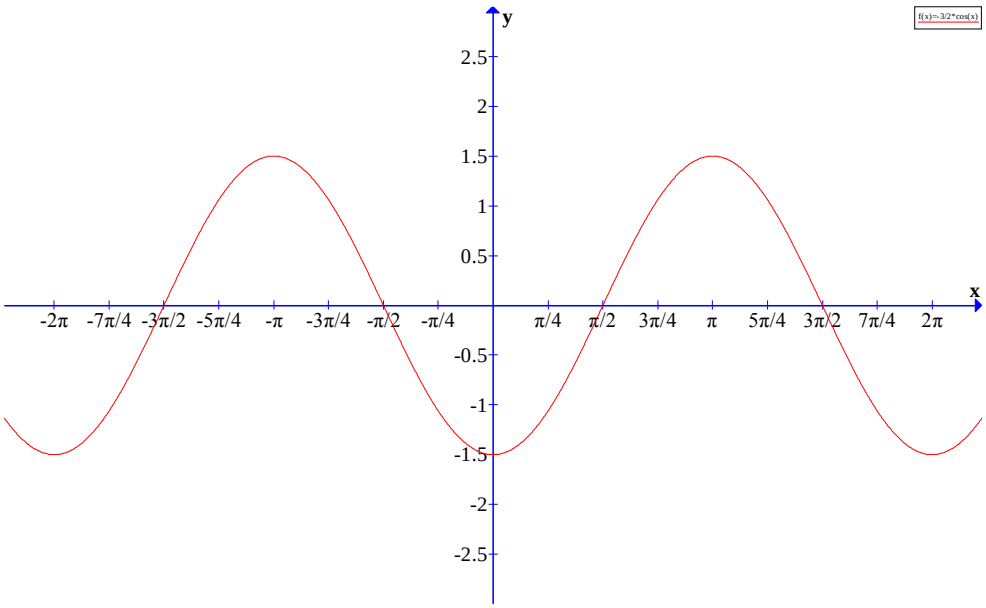
10) Complete the table of values below and graph the function $y=2\sin(\theta)$ for $0\leq\theta\leq2\pi$.

θ	$2\sin(\theta)$
0	0
$\frac{\pi}{4}$	1.414
$\frac{\pi}{2}$	2
$\frac{3\pi}{4}$	1.414
π	0
$\frac{5\pi}{4}$	-1.414
$\frac{3\pi}{2}$	-2
$\frac{7\pi}{4}$	-1.414
2π	0



11) Complete the table of values and graph for $y = -\frac{3}{2}\cos(\theta)$ for $-2\pi \le \theta \le 2\pi$.

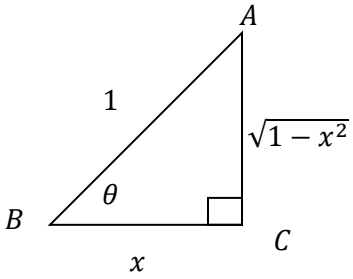
θ	-2π	$\frac{-7\pi}{4}$	$\frac{-3\pi}{2}$	$\frac{-5\pi}{4}$	$-\pi$	$\frac{-3\pi}{4}$	$\frac{-\pi}{2}$	$\frac{-\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$-\frac{3}{2}\cos(\theta)$	$-\frac{3}{2}$	-1.1	0	1.1	$\frac{3}{2}$	1.1	0	-1.1	$-\frac{3}{2}$	-1.1	0	1.1	$\frac{3}{2}$	1.1	0	-1.1	$-\frac{3}{2}$



12) Given that $\cos(\theta) = x$, determine $\tan(\theta)$.

$$\cos(\theta) = \frac{x}{1} = \frac{Adjacent}{Hypotenuse}$$

$$\tan(\theta) = \frac{Opposite}{Adjacent} = \frac{\sqrt{1-x^2}}{x}$$



Reach Ahead: (not part of this module, but used in some programs)

While most of us are used to working in decimal degrees, for very large objects (such as the earth) or very precise measurements (surveying), people sometimes use degrees, minutes and seconds instead of decimal degrees. In general $1\,° = 60\ minutes = 60'$ and $1' = 60\ seconds = 60"$. It is unfortunate the symbol for minutes is the same as feet and similarly the symbol for seconds is the same as the symbol for inches. The conversion method is identical. Consider the angle $23.152° = 23°\ 0.152\,° \times \frac{60'}{1°} = 23°\ 9.12' = 23°\ 9'\ 0.12' \times \frac{60"}{1'}$ giving $23.152° = 23°\ 9'\ 7.2"$ or rounded to $23.152° = 23°\ 9'\ 7"$.

1) Complete each of the following angle conversions: (1 mark each)

a) $126.735° = 126° + 0.735° \times \frac{60'}{1°}$

$= 126°\ 44.1'$

$= 126°\ 44' + 0.1' \times \frac{60"}{1'}$

$= 126°\ 44'\ 6"$

b) $276°\ 25'\ 38" = 276°\ 25' + 38" \times \frac{1'}{60"}$

$= 276° + 25.633' \times \frac{1°}{60'}$

$= 276.4272° \times \frac{\pi\ rad}{180°}$

$= 4.82456\ rad$

c) $2.85\ rad = 2.85\ rad \times \frac{180°}{\pi\ rad}$

$= 163.2929716°$

$= 163° + 0.2929716° \times \frac{60'}{1°}$

$= 163°\ 17' + 0.57829674' \times \frac{60"}{1'}$

$= 163°\ 17"\ 35"$

d) $67°\ 53'\ 13" = 67°\ 53' + 13" \times \frac{1'}{60"}$

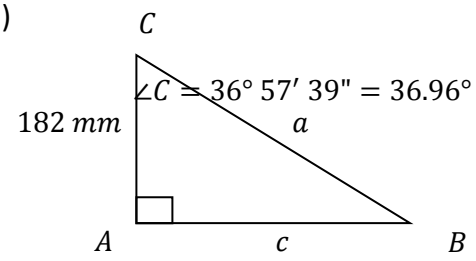
$= 67° + 53.2166' \times \frac{1°}{60'}$

$= 67.88694° \times \frac{\pi\ rad}{180°}$

$= 0.37715\pi\ rad$

2) Solve the following triangles for all missing sides and angles. (4 marks each)

a)



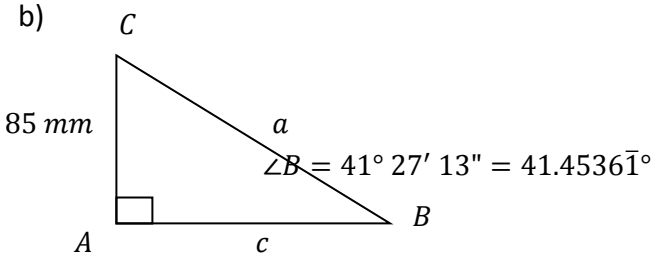
$\tan(36.96°) = \frac{c}{182}$

$c = 182 \tan(36.96°) = 136.948\ mm$

$\angle B = 90° - 36°\ 57'\ 39" = 53°\ 2'\ 21"$

$a = \sqrt{182^2 + 136.948^2} = 227.769\ mm$

b)



$\tan(41.45°) = \frac{85}{c}$

$c = \frac{85}{\tan(41.45°)} = 96.232\ mm$

$\angle C = 90° - 41°\ 27'\ 13" = 48°\ 32'\ 47"$

$a = \sqrt{85^2 + 96.232^2} = 128.396\ mm$