1. **(13 marks)** Consider the following seven observations for the two random variables X and Y.

$(x-\bar{x})^2$	$(x-\bar{x})$	X	Y	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$
653.82	25.57	120	7	-1.85	-47.30
31.02	5.57	100	3	-5.85	-32.58
1974.02	-44.43	50	2	-6.85	304.35
208.22	-14.43	80	9	0.15	-2.16
2076.62	45.57	140	22	13.15	599.25
1476.86	-38.43	56	10	1.15	44.20
423.12	20.50	115	9	0.15	3.09

a. **(2 marks)** Calculate the sample mean of X.

1 $\cancel{\bar{X} = \frac{\Sigma X}{N}}$ $\quad \bar{X} = \frac{\Sigma X}{n} = \frac{120+100+50+80+140+56+115}{7}$

$$= \frac{661}{7} = 94.43$$

1

b. **(2 marks)** Calculate the sample mean of Y.

1 $\bar{y} = \frac{\Sigma Y}{n} = \frac{7+3+2+9+22+10+9}{7}$

$$= \frac{62}{7} = 8.86$$

1

c. **(3 marks)** Calculate the sample variance of X.

1 $s^2 = \frac{\Sigma(x-\bar{x})^2}{n}$ or $s^2 = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}$

$$s^2 = \frac{6843.68}{6} = 1140.61$$ 2

d. **(3 marks)** Calculate the covariance between X and Y.

1 $Cov(X,Y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n-1} = \frac{780.45}{6}$

2 $= 130.08$

e. **(3 marks)** Show if X and Y are independent.

3 Since $Cov(X,Y) \neq 0 \Rightarrow X$ and Y are not independent.

2. **(8 marks)** The coaching staff for a national soccer team has to select 25 players to participate in the upcoming World Cup in 2018. They have a list of 30 talented players from which to choose.

   a. **(4 marks)** Calculate how many different combinations of 25 players are possible.

   4 $^{30}C_{25} = 142506$

   b. **(4 marks)** Of the 30 available players, 20 are white. Calculate the probability that the final selected team would have 17 white players.

   2 $\frac{^{20}C_{17} \cdot {^{10}C_8}}{^{30}C_{25}} = \frac{(1140)(45)}{142506} = 0.40$ 2

3. **(10 marks)** A manager has 16 employees who could be assigned to a project-monitoring task. Three of the employees are women and thirteen are men. Four of the men are brothers. The manager is to make the assignment at random so that each of the 16 employees is equally likely to be chosen. Let A be the event "chosen employee is a man" and B be the event "chosen employee is one of the brothers."

   a. **(3 marks)** Calculate the probability of A.

$$P(A) = P(\text{man}) = \frac{13}{16} = 0.82$$

3

   b. **(3 marks)** Calculate the probability of B.

$$P(B) = P(\text{brother}) = \frac{4}{16} = 0.25$$

3

   c. **(2 marks)** Calculate the probability of A intersect B.

$$P(A \cap B) = P(B) = 0.25$$

2

   d. **(2 marks)** Calculate the probability of A union B.

1

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(B)$$
$$= P(A) = 0.82$$

1

4. **(8 marks)** Assume that the number of network errors experienced in a day on a local area network (LAN) is distributed as a Poisson random variable. The mean number of network errors experienced in a day is 1.8.

   a. **(4 marks)** Calculate the probability that, in any given day, less than two network errors will occur.

$$\lambda = 1.8$$

$$P(X<2) = P(X \leq 1) = F(1 \mid \lambda = 1.8)$$

$$\Rightarrow P(X<2) = 0.4628 \text{ (from table)}$$

   b. **(4 marks)** Assume that the mean time between two network errors in a given day is 2.3 hours. Calculate the probability that three hours would pass before the next network error occurs.

$$\frac{1}{\lambda} = 2.3 \Rightarrow \lambda = \frac{1}{2.3} = 0.43$$

$$P(T>3) = 1 - P(T<3)$$

$$= 1 - (1 - e^{-\lambda t})$$

$$= e^{-\lambda t}$$

$$= e^{-0.43(3)}$$

$$= e^{-1.29}$$

$$= 0.28$$

5. **(9 marks)** The number of computers sold per day at a computer store is defined by the probability distribution below.

X	0	1	2	3	4	5	6
P(x)	0.020	0.090	0.200	0.005	0.050	0.300	0.335
$xP(x)$	0	0·09	0·4	0·015	0·2	1·5	2·01

a. **(3 marks)** Calculate the probability that the store sells more than three computers in a given day.

$$P(X>3) = P(X=4) + P(X=5) + P(X=6)$$
$$= 0.05 + 0.3 + 0.335$$
$$= 0.685$$

3

If include 3 −1

b. **(3 marks)** Calculate the probability that the store sells between two and four computers inclusive in a given day.

$$P(2 \leq X \leq 4) = P(X=2) + P(X=3) + P(X=4)$$
$$= 0.2 + 0.005 + 0.05$$
$$= 0.255$$

c. **(3 marks)** Calculate the expected number of computers to be sold in a given day.

$$E(X) = \Sigma x\, P(x) = 4.215$$

6. **(15 marks)** Consider the following discrete joint probability distribution:

X \ Y	100	200	300	400	500
1	0.01	0.20	0.07	0.02	0.03
2	0.06	0.03	0.06	0.15	0.04
3	0.15	0.04	0.04	0.01	0.09

a. **(3 marks)** Calculate the marginal probability densities of X and Y.

x	$P(x)$
1	0.33
2	0.34
3	0.33

y	$P(y)$
100	0.22
200	0.27
300	0.17
400	0.18
500	0.16

b. **(3 marks)** Calculate the E(X).

$$E(x) = \Sigma x\, p(x)$$
$$= (1 \times 0.33) + 2(0.34) + 3(0.33)$$
$$= 0.33 + 0.68 + 0.99 = 2.$$

c. **(3 marks)** Calculate P (X=2, Y=200).

$P(X=2, Y=200) = 0.03$

d. **(3 marks)** Calculate P (X=3| Y=400).

$P(X=3 | Y=400) = \frac{P(3, 400)}{P(Y=400)} = \frac{0.01}{0.18}$

$= 0.06$

e. **(3 marks)** Calculate E(X|Y=400).

$E(X|Y=400) = \Sigma(x|y)\, P(x|y=400)$

$= (1 \times 0.11) + 2(0.83) + 3(0.06)$

$= 0.11 + 1.66 + 0.08 = 1.95$

$P(X=2 | Y=400) = \frac{0.15}{0.18} = 0.83$

$P(X=1 | Y=400) = \frac{0.02}{0.18} = 0.11$

7. **(8 marks)** There are two types of traders: high ability and low ability. The probability that a trader is of high ability is 30 percent. Furthermore, the probability that a high-ability trader beats the market is 70 percent and the probability that a low-ability trader beats the market is 25 percent.

   a. **(4 marks)** If a newly hired trader has beaten the market, calculate the probability that the trader is a low-ability trader.

1 $P(H) = (0.3) \Rightarrow P(L) = 0.7 \quad P(B|H) = 0.7 \quad P(B|L) = 0.25$

1 $$P(L|B) = \frac{P(B \cap L)}{P(B)} = \frac{P(B|L) \cdot P(L)}{P(B|L) \cdot P(L) + P(B|H) \cdot P(H)}$$

$$= \frac{(0.25)(0.7)}{(0.25)(0.7) + (0.7)(0.3)}$$

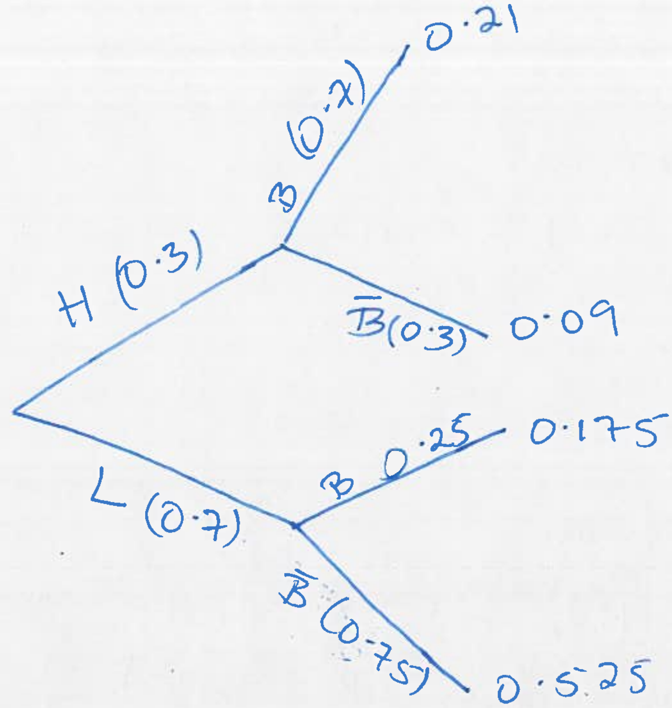
$$= \frac{0.175}{0.175 + 0.21} = \frac{0.175}{0.385} = 0.45$$

   b. **(4 marks)** If a newly hired trader has failed to beat the market, calculate the probability that the trader is a high-ability trader.

$$P(H|\bar{B}) = \frac{P(H \cap \bar{B})}{P(\bar{B})} = \frac{P(\bar{B}|H)P(H)}{P(\bar{B}|H)P(H) + P(\bar{B}|L)P(L)}$$

$$= \frac{(0.3)(0.3)}{0.09 + (0.75)(0.7)}$$

$$= \frac{0.09}{0.615} = 0.14$$


H (0·3)
B (0·7)
0·21
$\bar{B}$(0·3)
0·09
L (0·7)
B 0·25
0·175
$\bar{B}$ (0·75)
0·525

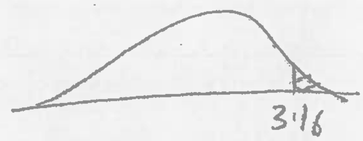
8. **(9 marks)** Stock returns, denoted by X, are normally distributed with a mean of 3 percent and a variance of 10 percent. $\Rightarrow \sigma = 3.16$

a. **(3 marks)** Calculate P (X > 13). Demonstrate your answer graphically.

$$P(X>13) = P\left(Z > \frac{13-3}{3.16}\right) = P(Z > 3.16)$$

$$= 1 - F(3.16) = 1 - 0.9992$$

$$= 0.0008$$



b. **(3 marks)** Calculate P ($1 \leq X \leq 5$). Demonstrate your answer graphically.

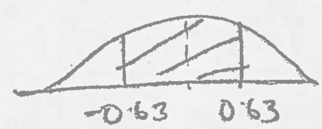
$$P(1 \leq X \leq 5) = P\left(\frac{1-3}{3.16} < Z < \frac{5-3}{3.16}\right)$$

$$= (-0.63 < Z < 0.63)$$

$$= F(0.63) - [1 - F(0.63)]$$

$$= 0.7357 - (1 - 0.7357)$$

$$= 0.7357 - 0.2643 = 0.4714$$



c. **(3 marks)** Calculate P ($-2 \leq X \leq 2$). Demonstrate your answer graphically.

$$P(-2 \leq X \leq 2) = P\left(\frac{-2-3}{3.16} < Z < \frac{2-3}{3.16}\right)$$

$$= (-1.58 < Z < -0.32)$$

$$= F(1.58) - F(0.32)$$

$$= 0.9429 - 0.6255$$

$$= 0.3174$$

9. **(10 marks)** A hamburger stand sells hamburgers for \$1.45 each. Daily sales have a normal distribution with a mean of 530 and a standard deviation of 69.

a. **(2 marks)** Calculate the mean of daily total revenue from the sales of hamburgers.

$Y = 1.45X. \quad E(Y) = 1.45\,E(X) = 768.5$

b. **(2 marks)** Calculate the standard deviation of daily total revenue.

$\sigma_Y = 1.45\sigma_X = 100.05$

c. **(2 marks)** What is the distribution of daily total revenue?

Total revenue will $\sim N(768.5, 100.05)$

d. **(2 marks)** What is the probability that daily revenue would exceed \$900?

$P(Y > 900) = P\left[Z > \dfrac{900 - 768.5}{100.05}\right] = P(Z > 1.31)$

$= 1 - F(1.31)$

$= 1 - 0.9049 = 0.0951$

e. **(2 marks)** What is the maximum daily revenue that would be achieved with 95 percent probability?

$Z = -1.645$

$\Rightarrow X = -1.645(100.05) + 768.5$

$= 603.92$