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COMP1805 - B2
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1. Let x be the proposition “The data is secure”, y be the proposition “The computer has a virus”. Translate the following expressions into English.

(c) $y \oplus x$

≡ The data is secure or the computer has a virus, but not both

2. Translate the following logical propositions into English expressions. Let a be the proposition “The economy is improving”, b be “I make money”, c be “I have a job”, and d be “I finish my degree at Carleton”.

(c) $(\neg b \land \neg c) \rightarrow d$

≡ If I don't make money and I don’t have a job, then I finish my degree at Carleton.

3. Translate the following English expressions into logical statements. You must explicitly state what are the atomic propositions and then show their logical relation.

(d) If I can ski then I cannot skate, and, if I cannot ski and I cannot skate, then I do not like winter.

Let a = “I can ski”, b = “I can skate”, and c = “I like winter”
$\equiv (a \rightarrow \neg b \land (\neg a \land \neg b) \rightarrow \neg c))$

4. Translate the following statement into logic. Negate the logical statement. Translate the negated logical statement back into English. You must explicitly state what are the atomic propositions.

(a) If the company suffered a loss and the economy is not improving then employees lose their jobs.

Let a = “The company suffered a loss”, b = “The economy is not improving”, c = “Employees lose their jobs”

The logical equivalent to (a) would be $(a \land b) \rightarrow c$, and to easily negate it, we should remove the implication which would lead to
$\equiv \neg(a \land b) \lor c$ (Implication Relation)
$\equiv \neg a \lor \neg b \lor c$ (De Morgan’s Laws), and now we negate the logical statement to get

$\neg(\neg a \lor \neg b \lor c)$
$\equiv a \land b \land \neg c$ (De Morgan's Laws)

This would be translated to English as "The company suffered a loss and the economy is not improving and employees don't lose their jobs."
5. Determine which of the following statements are True and explain why or why not.

(d) If 2 + 5 > 23 then 9 > 20

Let a = "2 + 5 > 23" and b = "9 > 20", a is a false proposition, as 7 is not greater than 23, and b is also a false proposition, as 9 is not greater than 20. This statement this translated into logic as $a \to b$, and since both propositions are false, the statement is true.

6. Find a logical proposition that is equivalent to $((\neg p \to q) \leftrightarrow (q \land \neg r))$ but that only uses negation $\neg$ and the connectives $\lor$, $\land$. Prove your answer is correct.

$((\neg p \to q) \leftrightarrow (q \land \neg r))$
$\equiv ((\neg(\neg p) \lor q) \leftrightarrow (q \land \neg r))$ (Implication Relation)
$\equiv ((p \lor q) \leftrightarrow (q \land \neg r))$ (Double Negation)
$\equiv ((p \lor q) \to (q \land \neg r)) \land ((q \land \neg r) \to (p \lor q))$ (Biconditional)
$\equiv (\neg(p \lor q) \lor (q \land \neg r)) \land ((q \land \neg r) \to (p \lor q))$ (Implication Relation)
$\equiv (\neg(p \lor q) \lor (q \land \neg r)) \land (\neg(q \land \neg r) \lor (p \lor q))$ (Implication Relation)
$\equiv ((\neg p \land \neg q) \lor (q \land \neg r)) \land (\neg(q \land \neg r) \lor (p \lor q))$ (De Morgan's Laws)
$\equiv ((\neg p \land \neg q) \lor (q \land \neg r)) \land ((\neg q \lor r) \lor (p \lor q))$ (De Morgan's Laws)

$((\neg p \land \neg q) \lor (q \land \neg r)) \land ((\neg q \lor r) \lor (p \lor q)) \equiv ((\neg p \to q) \leftrightarrow (q \land \neg r))$, as proven with the rules of logic.

7. Determine, for each statement, whether it is a proposition. Justify your answer in either case.

(c) "Life is like a box of chocolates."

It is a proposition, as it is a declarative statement that says something about the world and is a truth bearer.

8. Determine if the following are tautologies, contradictions or contingencies. You may use truth tables.

(c) $\neg((x \to y) \to (\neg y \to \neg x))$
$\equiv \neg((\neg(x \to y) \lor (\neg y \to \neg x))$ (Implication Relation)
$\equiv \neg((\neg(\neg x \lor y) \lor (\neg y \to \neg x))$ (Implication Relation)
$\equiv \neg((\neg(\neg x \lor y) \lor (\neg(\neg y) \lor \neg x))$ (Implication Relation)
$\equiv \neg((\neg(\neg x \lor y) \lor (y \lor \neg x))$ (Double Negation)

≡ ¬((x ∧ ¬y) ∨ (y ∨ ¬x)) (De Morgan's Laws)
≡ (¬x ∨ y) ∧ (¬y ∧ x) (De Morgan's Laws)
≡ (¬x ∧ (¬y ∧ x)) ∨ ((y ∧¬y) ∧ x) (Distributive Laws/Associative Laws)
≡ (¬x ∧ (¬y ∧ x)) ∨ (F ∧ x) (Contradiction)
≡ ((¬x ∧ x) ∧ ¬y) ∨ (F ∧ x) (Associative Laws)
≡ (F ∧ ¬y) ∨ (F ∧ x) (Contradiction)
≡ F ∨ F (Domination Laws)
≡ F (Identity Laws) , therefore it is a Contradiction.

9. Determine if the following are tautologies, contradictions or contingencies. You cannot use truth tables to justify your answers. You must use logical equivalences (the rules).

(c) (¬y ∧ ¬z) → ¬((y ∨ x) ∧ (¬x ∨ z))
≡ ¬(¬y ∧ ¬z) ∨ ¬((y ∨ x) ∧ (¬x ∨ z)) (Implication Relation)
≡ (y ∨ z) ∨ ¬((y ∨ x) ∧ (¬x ∨ z)) (De Morgan's Laws)
≡ (y ∨ z) ∨ ((¬y ∧ ¬x) ∨ (x ∧ ¬z)) (De Morgan's Laws)
≡ y ∨ (¬y ∧ ¬x) ∨ z ∨ (x ∧ ¬z) (Commutativity Laws)
≡ (y ∨ ¬y) ∧ (y ∨ ¬x) ∨ z ∨ (x ∧ ¬z) (Distributive Laws)
≡ (y ∨ ¬y) ∧ (y ∨ ¬x) ∨ (z ∨ x) ∧ (z ∨ ¬z) (Distributive Laws)
≡ T ∧ (y ∨ ¬x) ∨ (z ∨ x) ∧ (z ∨ ¬z) (Law of the Excluded Middle)
≡ T ∧ (y ∨ ¬x) ∨ (z ∨ x) ∧ T (Law of the Excluded Middle)
≡ T ∧ (y ∨ ¬x) ∨ (z ∨ x) (Identity Laws)
≡ (y ∨ ¬x) ∨ (z ∨ x) (Identity Laws)
≡ (y ∨ z) ∨ (¬x ∨ x) (Commutativity Laws)
≡ (y ∨ z) ∨ T (Law of Excluded Middle)
≡ y ∨ z (Identity Laws)

Given y and z being given value for both true and false, then (¬y ∧ ¬z) → ¬((y ∨ x) ∧ (¬x ∨ z)) is a contingency.

10. Translate the following into English, where K(x) is "x Knows how to drive", O(x) is "x owns a motorcycle" and A(x) is "x has a ticket to the auto show". The universe of discourse is all humans

(d) ¬∀x(¬O(x) → ¬(K(x) ∧ A(x)))

¬∀x(¬O(x) → ¬(K(x) ∧ A(x)))
≡ ¬∀x(¬¬O(x) ∨ ¬(K(x) ∧ A(x))) (Implication Relation)
≡ ¬∀x(O(x) ∨ ¬(K(x) ∧ A(x))) (Double Negation)
≡ ¬∀x(O(x) ∨ (¬K(x) ∨ ¬A(x))) (De Morgan's Laws)
≡ ∃x(¬O(x) ∧ K(x) ∧ A(x)) (De Morgan's Law)

$\exists x(\neg O(x) \land (K(x) \land A(x)))$ translated into English is “There exists a human who doesn’t own a motorcycle and knows how to drive and has a ticket to the auto show.”

11. Let H(x) be “x can ski”, S(x) be “x can skate”, W(x) be “x loves winter” and P(x) be “x plays soccer”. The universe of discourse is all humans. State the following logically.

(d) There is exactly one person who can skate and can ski.

$\exists x(S(x) \land H(x))$

12. Express the negation of the following statements:

(b) There is at least one person in Carleton who does not know logic.

Let C(x) = “x in Carleton”, K(x) = “x knows logic”

“There is at least one person in Carleton who does not know logic" is translated into logic as $\exists x(C(x) \land \neg K(x))$, to which it is negated as $\neg\exists x(C(x) \land \neg K(x))$
$\equiv \forall x(\neg C(x) \lor K(x))$