CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Duration**
Midterm Test	21 October, 2017	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks**	

1. (6+5 marks):

(a) Write the sigma notation formula for the *left* Riemann sum L_n of the function $f(x) = (1+x)^2$ on the interval [-1,2] using n subintervals of equal length. Use that formula to calculate the integral $\int_{-1}^{2} f(x)\,dx$ as the limit of L_n at $n \to \infty$.

(Reminder: $\sum_{k=1}^{n} k = \frac{n\,(n+1)}{2}$, and $\sum_{k=1}^{n} k^2 = \frac{n\,(n+1)\,(2n+1)}{6}$.)

(b) Graph the function $f(x) = |x-3| - 1$, and use it to evaluate the integral $\int_{-2}^{4} f(x)\,\mathrm{d}x$ in terms of area.

2. (5 marks): Use the Fundamental Theorem of Calculus to find the derivative $F'(x)$ of

$$F(x) = x^2 + \int_{-2x}^{2x} [3 + \sin(t^2)]\,dt.$$

3. (10 marks): Calculate the following indefinite integrals

$$\textbf{(a)} \quad \int \frac{x}{x^2 - 3x + 2}\,dx \qquad \textbf{(b)} \quad \int \cos^4(x)\,dx$$

4. (6 marks): Find the antiderivative $F(t)$ of $f(t) = \ln^2(x)$ such that $F(1) = 0$.

5. (12 marks): Evaluate the following definite integrals (*do not approximate*):

$$\textbf{(a)} \quad \int_{0}^{\ln 4} \frac{e^x}{e^{2x} + 16}\,dx \qquad \textbf{(b)} \quad \int_{0}^{3} x^2\sqrt{1+x}\;dx$$

6. (6 marks): Sketch the graphs of the functions $y = |x| - 2$ and $y = 4 - x^2$, and find the area of the region enclosed by these graphs.

Bonus. (3 marks): Calculate the definite integral $\int_{-\pi}^{\pi} \frac{\sin x}{1+x^2}\,dx$.