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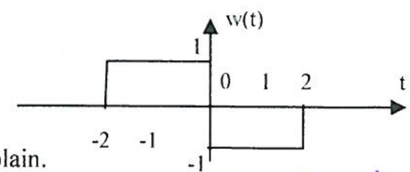
SYSC3501 Communications Theory, *1st in-class quiz, section B*

February 1, 2018, 1:00pm-1:30pm

Problem 1 (2.5points)



Find the average value and Fourier transform of the signal w(t).

Is this an energy or a power signal? Compute the energy or power and explain.

$\langle w(t)\rangle = 1/4 \int_{-2}^{2} w(t)\,dt = 0$. This is a timelimited signal therefore it is energy signal. $E_w = \int_{-2}^{2} w^2(t)\,dt = \int_{-2}^{2} 1^2 dt = 4$ Joules.

$\mathcal{F}(w(t)) = \mathcal{F}\left[\Pi\left(\frac{t+1}{2}\right) - \Pi\left(\frac{t-1}{2}\right)\right] = 2Sa(\pi f 2)\, e^{j2\pi f} - 2Sa(\pi f 2)\, e^{-j2\pi f}$

Note: $Sa(x) = \frac{\sin x}{x}$.

Here we apply the time-shift property of Fourie Transform.

Problem 2 (2 points)

A signal of 10Watts is passed trough an attenuator that has a power gain of -20db. What is the power at the attenuator output in Watts and dbm?

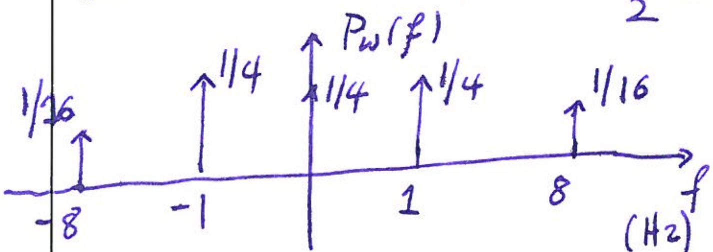
$(10 \text{ Watts})_{dbm} = 10 \log \frac{10\,W}{10^{-3}W} = 40\,dbm$. $40\,dbm \rightarrow \boxed{-20db} \rightarrow 20\,dbm$

Since the attenuation is -20db the output signal is 40-20= $= 20\,dbm = 10 \log \frac{P_{out}}{10^{-3}} \Rightarrow \log \frac{P_{out}}{10^{-3}W} = 2 \Rightarrow \frac{P_{out}}{10^{-3}W} = 100 \Rightarrow P_{out} = 0.1$ Watts

Problem 4 (2.5 points)

Consider the signal $w(t) = \sin^2(8\pi t) + \cos(2\pi t)$ Volts, find and plot its power spectral density. What is the DC power and what is the total power of the signal? What is the period of the signal?

We know $\sin^2(8\pi t) = \frac{1-\cos(2\cdot 8\pi t)}{2}$. Therefore: $w(t) = \frac{1}{2} - \frac{1}{2}\cos(16\pi t) + \cos(2\pi t)$



$P_w(f) = \frac{1}{4}\delta(f) + \frac{1}{4}\delta(f\pm 1) + \frac{1}{16}\delta(f\pm 8)$

$P_{DC} = \frac{1}{4}$ Watts

$P_{total} = \frac{1}{4} + 2\frac{1}{4} + 2\frac{1}{16} = \frac{1}{4} + \frac{1}{2} + \frac{1}{8} = 0.875$ W.

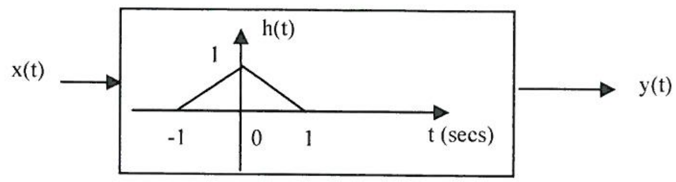
Problem 3 (1points)

Find the value of $\int_{-\infty}^{+\infty} e^{\pm j2\pi ft}\,dt$ and provide a brief explanation.

$\int_{-\infty}^{+\infty} e^{\pm j2\pi ft}\,dt = \delta(f)$. We know that $1 \xrightarrow{\mathcal{F}} \delta(f) \Rightarrow$

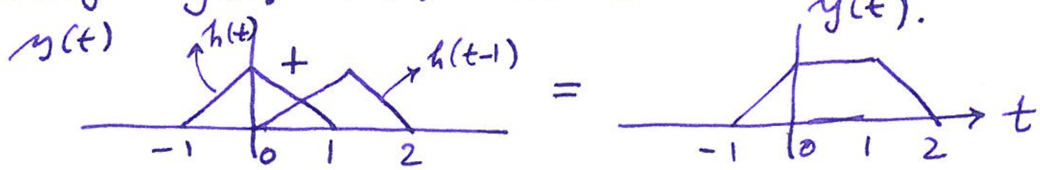
$\Rightarrow \int_{-\infty}^{+\infty} e^{-j2\pi ft}\,dt = \delta(f)$. By variable substitution $t \rightarrow -t$ it follows $\int_{-\infty}^{+\infty} e^{j2\pi ft} = \delta(f)$

Problem 5 (2 points)
A linear system has the impulse response function h(t) shown in the figure below



Is the system causal or non-causal? Plot the output y(t) if $x(t) = \delta(t) + \delta(t-1)$ and explain.

System is non-causal since $h(t) \neq 0$ for $t<0$. Because of linearity $y(t) = h(t) + h(t-1)$



USEFUL FORMULAE

$$cos(2x) = cos^2(x) - sin^2(x) = 1 - 2sin^2(x) = 2cos^2(x) - 1$$

TABLE 2–1 SOME FOURIER TRANSFORM THEOREMS*

Operation	Function	Fourier Transform
Linearity	$a_1 w_1(t) + a_2 w_2(t)$	$a_1 W_1(f) + a_2 W_2(f)$
Time delay	$w(t - T_d)$	$W(f)\, e^{-j\omega T_d}$
Scale change	$w(at)$	$\frac{1}{\lvert a\rvert} W\left(\frac{f}{a}\right)$
Conjugation	$w^*(t)$	$W^*(-f)$
Duality	$W(t)$	$w(-f)$
Real signal frequency translation	$w(t)\cos(\omega_c t + \theta)$	$\frac{1}{2}[e^{j\theta}W(f-f_c) + e^{-j\theta}W(f+f_c)]$

TABLE 2–2 SOME FOURIER TRANSFORM PAIRS

Function	Time Waveform $w(t)$	Spectrum $W(f)$
Rectangular	$\Pi\left(\frac{t}{T}\right)$	$T[\mathrm{Sa}(\pi f T)]$
Triangular	$\Lambda\left(\frac{t}{T}\right)$	$T[\mathrm{Sa}(\pi f T)]^2$
Unit step	$u(t) \triangleq \begin{cases} +1, & t>0 \\ 0, & t<0 \end{cases}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
Signum	$\mathrm{sgn}(t) \triangleq \begin{cases} +1, & t>0 \\ -1, & t<0 \end{cases}$	$\frac{1}{j\pi f}$
Constant	1	$\delta(f)$
Impulse at $t = t_0$	$\delta(t - t_0)$	$e^{-j2\pi f t_0}$