MAT2377 (Fall 2012) - Assignment 1

Total number of points: 33

Important:

- The assignments have to be handed in the lobby of the Department of Mathematics and Statistics (in the box marked MAT 2377) on the due date, no later than 14:30. Late assignments cannot be accepted. Since the assignments will not be distributed after marking, students are advised to make a copy of each assignment before submitting it.
- Electronic submission of assignments is not accepted!
- According to the university policy regarding the confidentiality of students' work, graded assignments cannot be brought to class for students to pick them up.
- Students should keep a copy of each assignment, check the solution posted on the Virtual Campus, and verify their grades (using also the Virtual Campus). If they have any doubts regarding the way they have been graded on any particular assignment, they are welcome to see their assignment in the instructor's office, during the office hours.
- Write clearly.

Q1. Pieces of aluminium are classified according to the finishing of the surface and according to the finishing of edge. The results from 85 samples are summarized as follows:

	Edge	
Surface	excellent	good
excellent	60	5
good	16	4

Let A denote the event that a selected piece has "excellent" surface, and let B denote the event that a selected piece has "excellent" edge. If samples are elected randomly, determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A^c)$

(d) $P(A \cap B)$ (e) $P(A \cup B)$ (f) $P(A^c \cup B)$

(g) If the selected piece has excellent edge finishing, what is the probability that it has excellent surface finishing?

(h) If the selected piece has good surface finishing, what is the probability that it has excellent edge finishing?

(i) Are A and B independent?

Solution to Q1:

(a) $P(A) = 65/85$

(b) $P(B) = 76/85$

(c) $P(A^c) = 1 - 65/85 = 20/85$

(d) $P(A \cap B) = 60/85$

(e) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 65/85 + 76/85 - 60/85 = 81/85$

(f) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B) = 20/85 + 76/85 - 16/85 = 80/85$

(g) $P(A|B) = P(A \cap B)/P(B) = 60/76$

(h) $P(B|A^c) = P(B \cap A^c)/P(A^c) = 16/20$

(i) $P(A|B) \neq P(A)$ - events are not independent.

Marking scheme for Q1:

1 point for each correct answer. Total - 9 points.

Q2. If $P(A) = 0.1$, $P(B) = 0.3$, $P(C) = 0.3$, and events A, B, C are mutually exclusive, determine the following probabilities:

(a) $P(A \cup B \cup C)$ (b) $P(A \cap B \cap C)$ (c) $P(A \cap B)$

(d) $P((A \cup B) \cap C)$ (e) $P(A^c \cap B^c \cap C^c)$ (f) $P[(A \cup B \cup C)^c]$

Solution to Q2:

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.7$, since the events are mutually exclusive.

(b) $P(A \cap B \cap C) = 0$

(c) $P(A \cap B) = 0$

(d) $P((A \cup B) \cap C) = 0$

(e) $P(A^c \cap B^c \cap C^c) = P[(A \cup B \cup C)^c] = 1 - P(A \cup B \cup C) = 0.3$ (draw Venn diagram)

(f) $P[(A \cup B \cup C)^c] = 1 - P(A \cup B \cup C) = 0.3$

Marking scheme for Q2:

Correct answer for each part - 1 point. Total - 6 points.

Q3. Probability that an electrical switch, which is kept in dryness, fails during the guarantee period, is 1%. If the switch is humid, the failure probability is 8%. Assume that 90% of switches are kept in dry conditions, whereas remaining 10% are kept in humid conditions.

(a) What is the probability that the switch fails during the guarantee period?

(b) If the switch failed during the guarantee period, what is the probability that it was kept in humid conditions?

Solution to Q3:

Let F-"failure", H-"humid", D-"dry". Given: $P(F|D) = 0.01$, $P(F|H) = 0.08$, $P(D) = 0.9$, $P(H) = 0.1$

(a)

$$P(F) = P(F|D)P(D) + P(F|H)P(H) = 0.01 * 0.9 + 0.08 * 0.1 = 0.009 + 0.008 = 0.017$$

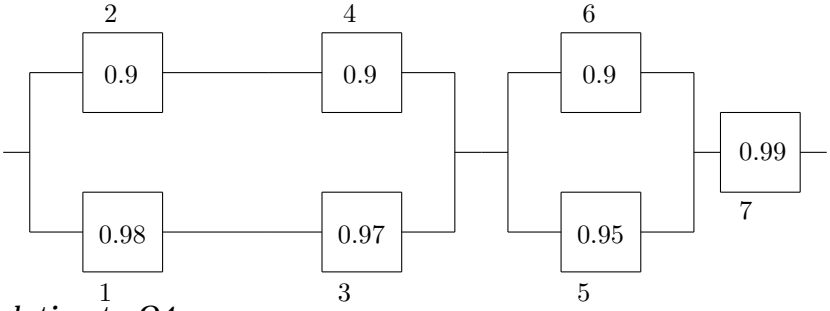
(b)

$$P(H|F) = \frac{P(H \cap F)}{P(F)} = \frac{P(F|H)P(H)}{P(F)} = 0.4706$$

Marking scheme for Q3:

1 point for each part. Total - 2 points.

Q4. The following system operates only if there is a path of functional device from left to the right. The probability that each device functions is as shown. What is the probability that the circuit operates? Assume independence.



Solution to Q4:

Let Box A: components 1,2,3,4; Box B: components 5,6; Box C: component 7.

$$P(\text{system works}) = P(\text{A works})P(\text{B works})P(\text{C works}).$$

Now, B is just parallel system, so that

$$P(\text{B works}) = 0.9 + 0.95 - 0.9 * 0.95 = 1.85 - 0.855 = 0.995.$$

Furthermore, $P(2 \text{ and } 4 \text{ work}) = 0.9 * 0.9 = 0.81$, $P(1 \text{ and } 3 \text{ work}) = 0.9506$. Now, A is the parallel system of $2, 4$ and $1, 3$, thus

$$P(\text{A works}) = P(2 \text{ and } 4 \text{ work}) + P(1 \text{ and } 3 \text{ work}) - P(2 \text{ and } 4 \text{ work})P(1 \text{ and } 3 \text{ work}) = 0.9906.$$

Final answer: 0.9578.

Marking scheme for Q4:

Total - 3 points.

Q5. An inspector working for a manufacturing company has a 95% chance of correctly identifying defective items and 2% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 1% of nonconforming items.

(a) What is the probability that an item selected for inspection is classified as defective?

(b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?

Solution to Q5:

Let A - the event that an item is classified as defective, D - the event that an item is defective; so that D^c is the event that an item is 'good'. What is known is: $P(D) = 0.01$; $P(A|D) = 0.95$, $P(A|D^c) = 0.02$.

(a)

$$P(A) = P(A \cap D) + P(A \cap D^c) = P(A|D)P(D) + P(A|D^c)P(D^c) \approx 0.0293.$$

(b) To compute $P(D^c|A^c)$. From Bayes' formula:

$$P(D^c|A^c) = \frac{P(A^c|D^c)P(D^c)}{P(A^c)} = \frac{(1 - P(A|D^c))P(D^c)}{1 - P(A)} \approx 0.999$$

Marking scheme for Q5:

1 point for each correct value of $P(A|D), P(D), P(A|D^c), P(A^c|D^c)$. For part (a): 1 point for formula $P(A|D)P(D)+P(A|D^c)P(D^c)$. For part (b): 1point for the correct Bayes' formula. Total - 6 points.

Q6. In a group of 16 candidates for for laboratory research positions, 7 are chemists and 9 are physicians. In how many ways one can choose 2 chemists and 3 physicians?

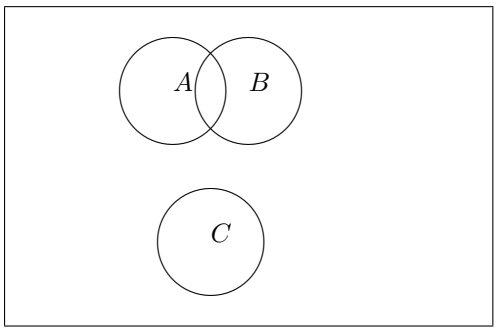
Solution to Q6:

$$\begin{pmatrix} 7 \\ 2 \end{pmatrix} * \begin{pmatrix} 9 \\ 3 \end{pmatrix} = 21 * 84 = 1764$$

Marking scheme for Q6:

1 point for use of binomial coefficients. 1 point for the correct answer. Total - 2 points.

Q7. The three events are shown on the Venn diagram:



Reproduce the figure and shade the region corresponding to the following events:

(a) A^c (b) $(A \cap B) \cup (A \cap B^c)$ (c) $(A \cap B) \cup C$

(d) $(B \cup C)^c$ (e) $(A \cap B)^c \cup C$

Marking scheme for Q7:

1 point for each correct answer. Total - 5 points.