CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2016	2
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Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[10] **1.** **a.** Sketch the graph of the function

$$f(x) = \begin{cases} 1-2x & \text{if } x \le 1 \\ \dfrac{|1-x|}{1-x} & \text{if } 1 < x \le 2 \\ x-3 & \text{if } x > 2 \end{cases}$$

and find the definite integral $\int_0^3 f(x)\,\mathrm{d}x$ in terms of area (do not antidifferentiate).

b Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{1-x}^{x^2} 2^{-t^2}\,\mathrm{d}t$.

[16] **2.** Find the following indefinite integrals:

(**a**) $\displaystyle\int \frac{(x^{2/3}+x^{-2/3})^2}{x}\,dx$ (**b**) $\displaystyle\int \arcsin(x)\,dx$ (**c**) $\displaystyle\int \frac{x^3+4}{x^3-4x}dx$

[12] **3.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^{\pi/2} x^2\cos(2x)\,dx$ (**b**) $\displaystyle\int_0^4 x\sqrt{16-x^2}\,dx$

[6] **4.** Find $F(t)$ such that $F'(t) = \sin^3(t)\cos^5(t)$ and $F\left(\frac{\pi}{2}\right) = 0$.

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_0^{1/e} \frac{\mathrm{d}x}{x\ln^2(x)}$ (**b**) $\displaystyle\int_0^{\infty} \frac{e^x}{e^x+4}\,dx$

[16] **6.** **a.** Sketch the curves $y = x + \dfrac{3}{x}$ and $y = 4$, and find the area enclosed.

b. Find the volume of a solid obtained by rotating the region bounded by the curve $y = \sin(x)$ and the x-axis on the interval $[0, \pi]$ about the axis $y = -1$

c. Find the exact average value of $f(x) = \tan^2 x$ on the interval $[0, \pi/4]$.

[6] **7.** Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

(a) $a_n = \dfrac{(3^n + 1)^2}{6^n}$ **(b)** $a_n = \ln(1 + 2n^2) - 2\ln(1 + n)$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally :

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{1 + n^3}}{n^2}$ **(b)** $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{5 + e^n}$ **(c)** $\displaystyle\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2}$

[6] **9.** Find **(a)** the radius of convergence, and **(b)** the interval convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)\,2^n}$.

[8] **10.** **(a)** Use the integrability of the power series to express the function

$F(x) = \displaystyle\int_0^x \left(\sum_{n=1}^{\infty} n\, t^{2n-1} \right) \mathrm{d}t$ as an elementary function

(i.e. sum the series for $F(x)$ within the radius of its convergence).

(b) Find the MacLaurin series for the function $x\, e^{-x^2}$.
(Hint: start with the series for e^z then replace z by $-x^2$)

[5] **Bonus Question.** A solid is generated by rotating about the x-axis the region under the curve $y = f(x)$, where f is a positive function and $x \geq 0$. The volume generated by the part of the curve from $x = 0$ to $x = b$ is πb^2 for all $b \geq 0$. Find $f(x)$.