# CONCORDIA UNIVERSITY
## Department of Mathematics & Statistics

| **Course** | **Number** | **Sections** |
|---|---|---|
| Mathematics | 205 | All |
| **Examination** | **Date** | **Pages** |
| Final | April 2016 | 2 |
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| **Special Instructions:** | Only approved calculators are allowed **Show all your work for full marks** | |

[*MARKS*]

[10] **1.** **(a)** Sketch a graph of the function

$$f(x) = \begin{cases} -\sqrt{4-x^2} & -2 \le x \le 0 \\ 2-2|x-2| & 0 < x \le 4 \end{cases}$$

on the interval $-2 \le x \le 4$ and calculate the definite integral $\int_{-2}^{4} f(x)\,\mathrm{d}x$ in terms of area (*do not antidifferentiate*).

**(b)** Use the Fundamental Theorem of Calculus to calculate the derivative of

$$F(x) = \int_{x}^{x^2} e^{\sin(\pi t)}\,\mathrm{d}t,$$

and determine whether $F$ is increasing or decreasing at $x = 1$.

[6] **2.** Find $F(x)$ such that $F'(x) = \dfrac{x^2+2x}{x^2+4}$ and $F(0) = 0$.

[11] **3.** Find the following indefinite integrals:

$$\textbf{(a)} \quad \int \frac{13-x}{x^2-x-6}\,dx \qquad \textbf{(b)} \quad \int x^{3/2}\ln^2(x)\,dx$$

[12] **4.** Evaluate the following definite integrals (give the **exact** answers):

$$\textbf{(a)} \int_0^1 \frac{2^x}{4^x+1}\,dx \qquad \textbf{(b)} \quad \int_1^2 \sqrt{4-x^2}\,dx\ .$$

[8] **5.** Evaluate the given improper integral or show that it diverges:

$$\textbf{(a)} \quad \int_e^{\infty} \frac{dx}{x\,\ln(x^2)} \qquad \textbf{(b)} \quad \int_0^1 \frac{dx}{(1-x)^{3/4}}$$

[18] **6. (a)** Sketch the curves $y = x\,(3 - x^2)$ and $y = -x$, and find the area enclosed by the two curves. (HINT: find first the points of intersection of the curves.)

**(b)** Sketch the region enclosed by $y = \cos(2x)$ and the $x$-axis on the interval $[0, \frac{\pi}{2}]$, and find the volume of revolution of this region about the axis $y = -1$.

**(c)** Find the average value of the function $f(x) = \sec^4(x)$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

[9] **7.** Find the limit of the sequence $\{a_n\}$ as $\to \infty$ or prove that it does not exist:

(**a**) $a_n = \dfrac{e^n - n^3}{3^n}$ (**b**) $a_n = \dfrac{(-1)^n n}{\sqrt{1+4n^2}}$ (**c**) $a_n = \ln(n + 2n^2) - \ln(2n + n^2)$.

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, then whether absolutely or conditionally convergent:

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2/3}}{1+2n}$ (**b**) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \sin\left(\frac{1}{n}\right)$ (**c**) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{n+1}}{n!}$

[6] **9.** Find **(a)** the radius and **(b)** the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^{3n}}{n^2\, 8^n}$

[8] **10. (a)** Derive the Maclaurin series of $f(x) = x^2\, e^{3x}$.
(HINT: start with the series for $e^z$ and then let $z = 3x$).

**(b)** Find the values of $x$ for which the following series converges

$$\sum_{n=0}^{\infty} \frac{(x^2+1)^n}{2^{n+1}}$$

and, for these values of $x$, find the sum of the series as a function of $x$.

**Bonus question** [5]. Find the values of $p$ (if any) for which the series

$$\sum_{n=5}^{\infty} \frac{1}{n \ln n (\ln(\ln n))^p} \quad \text{is convergent}$$