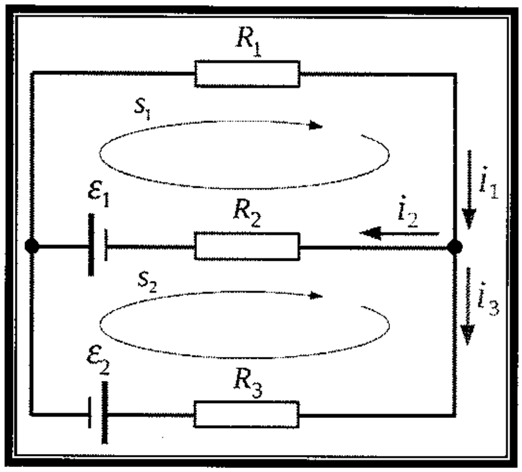


Nodal and mesh analysis.
(wikipedia.org).

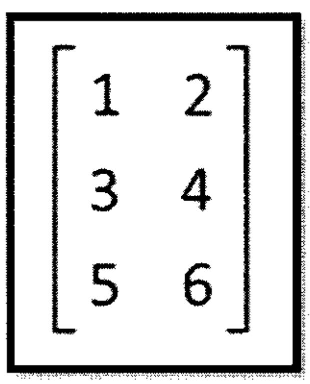
ENGG 2450: Electric Circuits

Professor: Dr. Christopher Collier

MODULE 3.0

UNIVERSITY of GUELPH

Methods of Analysis



Matrix algebra is often used to solve circuits with nodal and mesh analysis (freestockphotos.com).

3.0 Fundamentals of Electric Circuits 6th Ed. corresponding textbook material for this module.

Chapter 1 Basic Concepts
1.1 Introduction
1.2 Systems of Units
1.3 Charge and Current
1.4 Voltage
1.5 Power and Energy
1.6 Circuit Elements
1.7 Applications
1.8 Problem Solving
1.9 Summary

Chapter 2 Basic Laws
2.1 Introduction
2.2 Ohm's Law
2.3 Nodes, Branches, and Loops
2.4 Kirchhoff's Laws
2.5 Series Resistors and Voltage Division
2.6 Parallel Resistors and Current Division
2.7 Wye-Delta Transformations
2.8 Applications
2.9 Summary

Chapter 3 Methods of Analysis
3.1 Introduction
3.2 Nodal Analysis
3.3 Nodal Analysis with Voltage Sources
3.4 Mesh Analysis
3.5 Mesh Analysis with Current Sources
3.6 Nodal and Mesh Analyses by Inspection
3.7 Nodal Versus Mesh Analysis
3.8 Circuit Analysis with PSpice
3.9 Applications: DC Transistor Circuits
3.10 Summary

Chapter 4 Circuit Theorems
4.1 Introduction
4.2 Linearity Property
4.3 Superposition
4.4 Source Transformation
4.5 Thevenin's Theorem
4.6 Norton's Theorem
4.7 Derivations of Thevenin's and Norton's Theorems
4.8 Maximum Power Transfer
4.9 Verifying Circuit Theorems with PSpice
4.10 Applications
4.11 Summary

Chapter 5 Operational Amplifiers
5.1 Introduction
5.2 Operational Amplifiers
5.3 Ideal Op Amp
5.4 Inverting Amplifier
5.5 Noninverting Amplifier
5.6 Summing Amplifier
5.7 Difference Amplifier
5.8 Cascaded Op Amp Circuits
5.9 Op Amp Circuit Analysis with PSpice
5.10 Applications
5.11 Summary

Chapter 6 Capacitors and Inductors
6.1 Introduction
6.2 Capacitors
6.3 Series and Parallel Capacitors
6.4 Inductors
6.5 Series and Parallel Inductors
6.6 Applications
6.7 Summary

Chapter 7 First-Order Circuits
7.1 Introduction
7.2 The Source-Free RC Circuit
7.3 The Source-Free RL Circuit
7.4 Singularity Functions
7.5 Step Response of an RC Circuit
7.6 Step Response of an RL Circuit
7.7 First-Order Op Amp Circuits
7.8 Transient Analysis with PSpice
7.9 Applications
7.10 Summary

Chapter 8 Second-Order Circuits
8.1 Introduction
8.2 Finding Initial and Final Values
8.3 The Source-Free Series RLC Circuit
8.4 The Source-Free Parallel RLC Circuit
8.5 Step Response of a Series RLC Circuit
8.6 Step Response of a Parallel RLC Circuit
8.7 General Second-Order Circuits
8.8 Second-Order Op Amp Circuits
8.9 PSpice Analysis of RLC Circuits
8.10 Duality
8.11 Applications
8.12 Summary

Chapter 9 Sinusoids and Phasors
9.1 Introduction
9.2 Sinusoids
9.3 Phasors
9.4 Phasor Relationships for Circuit Elements
9.5 Impedance and Admittance
9.6 Kirchhoff's Laws in the Frequency Domain
9.7 Impedance Combinations
9.8 Applications
9.9 Summary

Chapter 10 Sinusoidal Steady-State
10.1 Introduction
10.2 Nodal Analysis
10.3 Mesh Analysis
10.4 Superposition Theorem
10.5 Source Transformation
10.6 Thevenin and Norton Equivalent Circuits
10.7 Op Amp AC Circuits
10.8 AC Analysis Using PSpice
10.9 Applications
10.10 Summary

Chapter 11 AC Power Analysis
11.1 Introduction
11.2 Instantaneous and Average Power
11.3 Maximum Average Power Transfer
11.4 Effective or RMS Value
11.5 Apparent Power and Power Factor
11.6 Complex Power
11.7 Conservation of AC Power
11.8 Power Factor Correction
11.9 Applications
11.10 Summary

Chapter 12 Three-Phase Circuits
12.1 Introduction
12.2 Balanced Three-Phase Voltages
12.3 Balanced Wye-Wye Connection
12.4 Balanced Wye-Delta Connection
12.5 Balanced Delta-Delta Connection
12.6 Balanced Delta-Wye Connection
12.7 Power in a Balanced System
12.8 Unbalanced Three-Phase Systems
12.9 PSpice for Three-Phase Circuits
12.10 Applications
12.11 Summary

Chapter 13 Magnetically Coupled Circuits
13.1 Introduction
13.2 Mutual Inductance
13.3 Energy in a Coupled Circuit
13.4 Linear Transformers
13.5 Ideal Transformers
13.6 Ideal Autotransformers
13.7 Three-Phase Transformers
13.8 PSpice Analysis of Magnetically Coupled Circuits
13.9 Applications
13.10 Summary

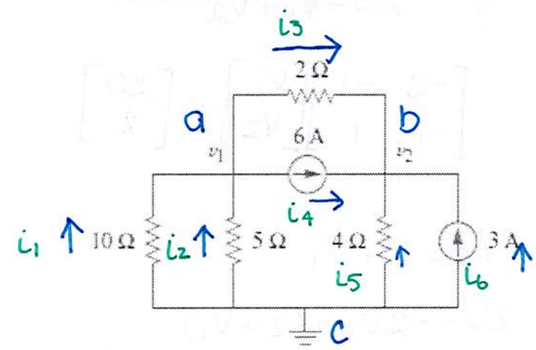
3.1 Nodal analysis.

In nodal analysis, circuits are analysed by investigating the current into/out of nodes. Here, a series of equations can be setup with fewer (or equal) unknowns than the number of equations. Problems of this nature are typically solved by solving this series of equations. This method can applied using three steps:

1. Select node as reference node (often ground) and assign voltages to other nodes.
   → Voltages are references with respect to the reference node
2. Apply KCL to each non-reference node
   → Use ohm's law to express currents as node voltages.
3. Solve equations to get unknown node voltages.

Tail minus head

Example. For the circuit, find v_1 and v_2.



(Fundamentals of electric circuits, 5th Ed.)

1. Set node c to be reference node.
2. KCL at node a:

$$\overset{+}{V_+}\ \overset{\xrightarrow{i_R}}{R}\ \overset{-}{V_-} \qquad i_R = \frac{V_+ - V_-}{R}$$

KCL: $i_1 + i_2 = i_3 + i_4$

$$\left(\frac{0-V_1}{10} + \frac{0-V_1}{5} = \frac{V_1 - V_2}{2} + 6\right)10$$

$$\Rightarrow -V_1 - 2V_1 = 5V_1 - 5V_2 + 60$$

$$-3V_1 - 5V_1 = -5V_2 + 60$$

(1) $$-8V_1 + 5V_2 = 60 \Rightarrow V_2 = \frac{60+8V_1}{5}$$

KCL at node b:

$$i_3 + i_4 + i_5 + i_6 = 0$$

$$\left(\frac{(V_1-V_2)}{2} + 6 + \frac{0-V_2}{4} + 3 = 0\right)4$$

$$2V_1 - 2V_2 + 24 - V_2 + 12 = 0$$

(2) $$-2V_1 + 3V_2 = 36$$

$$\begin{bmatrix} -8 & 5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 60 \\ 36 \end{bmatrix}$$

Substitution:

$$-2V_1 + 3\left(\frac{60+8V_1}{5}\right) = 36$$

$$\boxed{V_1 = 0V}$$

$$V_2 = \frac{60+8(0)}{5} = 12$$

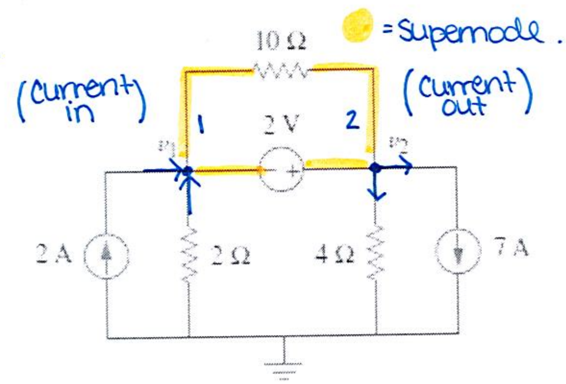
$$\boxed{V_2 = 12V}$$

3.2 Nodal analysis with voltage sources.

Nodal analysis gets slightly more complex when voltage sources are involved. Voltage sources can be thought of as a special type of node called a **supernode**. In this case, the nodal analysis can be supplemented with Kirchhoff's Voltage Law.

Supernode: a dependent or independent voltage source between two non reference nodes.

Example. For the circuit, find v_1 and v_2.



(Fundamentals of electric circuits, 5th Ed.)

Supernode is 2V source, node 1 & 2 and 10Ω resistor

Apply KCL to supernode:

$$\left(2 + \frac{(0-V_1)}{2} = 7 + \frac{(V_2-0)}{4}\right)4$$

$$8 - 2V_1 = 28 + V_2$$

(1) $20 = -2V_1 - V_2$

KVL around centre loop:

$$V_1 + 2V - V_2 = 0$$

(2) $2 = -V_1 + V_2$

$$\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 2 \end{bmatrix}$$

$$V_2 = 2 + V_1$$

$$20 = -2V_1 - (2 + V_1)$$

$$20 = -2V_1 - 2 - V_1$$

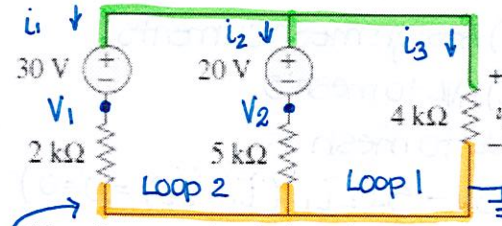
$$22 = -3V_1$$

$$\boxed{V_1 = -\frac{22}{3}V = -7.33V}$$

$$V_2 = 2 + V_1$$

$$\boxed{V_2 = -5.33V}$$

Example. For the circuit, find v_1 and v_2.



(Fundamentals of electric circuits, 5th Ed.)

reference node

KCL to top node:

$i_1 + i_2 + i_3 = 0$

$$20\left(\frac{V_1 - 0}{2K\Omega} + \frac{V_2 - 0}{5K\Omega} + \frac{V_o}{4K\Omega} = 0\right) \quad \text{Equation 1}$$

KVL to Loop 1:

$20V - V_o + V_2 = 0$ Equation 2

KVL to Loop 2:

$30V - V_o + V_1 = 0$ Equation 3

$$\begin{bmatrix} 10 & 4 & 5 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \\ -30 \end{bmatrix} \Rightarrow \boxed{\begin{aligned} V_o &= 20V \\ V_1 &= V_o - 30 = -10V \\ V_2 &= V_o - 20 = 0V \end{aligned}}$$

3.3 Mesh analysis.

Recall that nodal analysis used KCL and solved for unknown voltages. One can also, use KVL and solve for unknown currents. This is called **mesh analysis**. These meshes are the smallest contained loops in a circuit (i.e., they do not have other loops within them). Here, a series of equations can be setup with fewer (or equal) unknowns than the number of equations. Problems of this nature are typically solved by solving this series of equations.

Mesh: loop that is not containing another loop.

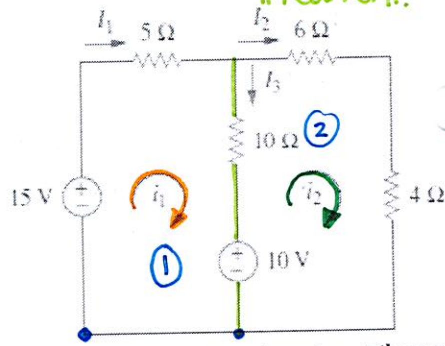
This method can applied using three steps:

1) Assign the mesh currents to the n meshes (typically clockwise)
2) Apply KVL to each n mesh → use Ohm's Law to express voltages in terms of mesh currents.
3) Solve resulting n equations, simultaneously to get mesh currents.

Example. For the circuit, find I_1, I_2, and I_3.

* shared difference in current!

$\boxed{* \ i_3 = i_1 - i_2}$



(Fundamentals of electric circuits, 5th Ed.)

1) Assign mesh currents
2) KVL to mesh's

KVL to mesh 1:

$$(15V - 5i_1 - 10(i_1 - i_2) - 10 = 0) \div 5$$

$$3V - i_1 - 2(i_1 - i_2) - 2 = 0$$

Eq. 1 $\quad 1 = 3i_1 - 2i_2$

KVL to mesh 2:

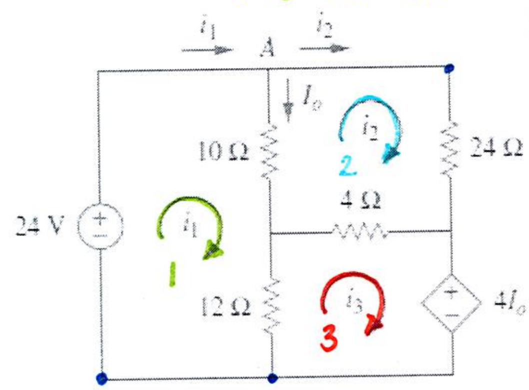
$$(10 - 10(i_2 - i_1) - 6(i_2) - 4(i_2) = 0) \div 10$$

Eq. 2 $\quad -1 = i_1 - 2i_2$

$$\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \boxed{\begin{array}{l} i_1 = 1A \\ i_2 = 1A \\ i_3 = 0 \end{array}}$$

Example. For the circuit, find i_1, i_2, and i_3.

$* \ I_o = i_1 - i_2$



(Fundamentals of electric circuits, 5th Ed.)

Mesh 1 KVL:

$$(24V - 10(i_1 - i_2) - 12(i_1 - i_3) = 0) \div 2$$

Eq. 1 $\quad 12 = 11i_1 - 5i_2 - 6i_3$

Mesh 2 KVL:

$$(-24i_2 - 4(i_2 - i_3) - 10(i_2 - i_1) = 0) \div 2$$

Eq. 2 $\quad 5i_1 - 19i_2 + 2i_3 = 0$

Mesh 3 KVL:

$$-12(i_3 - i_1) - 4(i_3 - i_2) - 4I_o = 0$$

Eq. 3 $\quad i_1 + i_2 - 2i_3 = 0$

$$\begin{bmatrix} 11 & -5 & -6 \\ 5 & -19 & 2 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Solve: $i_1 = 2.25A$

$i_2 = 0.75A$

$i_3 = 1.5A$

3.4 Mesh analysis with current sources.

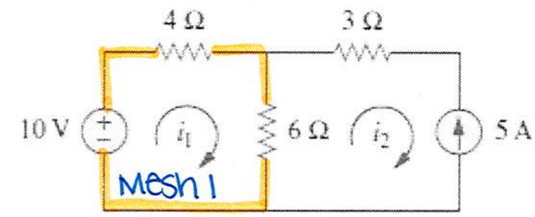
Mesh analysis gets slightly more complex when current sources are involved. If the current source is only in one mesh, the current is known in that wire. If two meshes share the same source, there is a **supermesh**. Here, the urce and any element in series with it can be excluded from the analysis.

Supermesh: two meshes that have a (dependent or independent) current source in common

Example. For the circuit, find i_1 and i_2.

*2 variables ... 2 equations

Note: No supermeshes in this circuit



(Fundamentals of electric circuits, 5th Ed.)

KVL of mesh 1:

$$10V - 4i_1 - 6(i_1 - i_2) = 0$$

$$(-10i_1 + 6i_2 = 10V) \div 2$$

$$5 - 5i_1 + 3i_2 = 0 \quad \text{Eq. 1}$$

$\boxed{i_2 = -5A}$ Eq. 2

Substitution:

$$5 - 5i_1 - 3(5) = 0$$

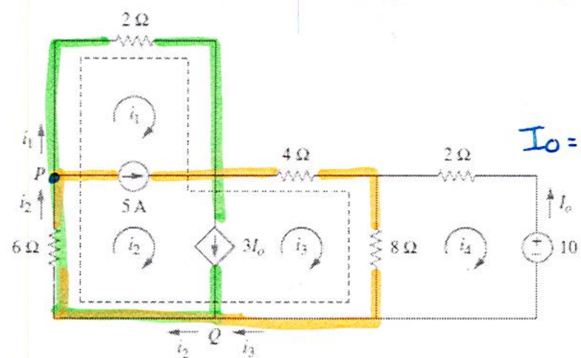
$$\boxed{i_1 = -2A}$$

* Note: If a current source is given, it is the same across the circuit.

Example. For the circuit, find I_1, I_2, and I_3.

*4 variables ... 4 equations

Note: There are 2 supermeshes.
↳ They intersect so they can combine



$I_o = -i_4$

(Fundamentals of electric circuits, 5th Ed.)

KVL of Large Supermesh:

$$-2i_1 - 4i_3 - 8(i_3 - i_4) - 6i_2 = 0$$

$$(-2i_1 - 4i_3 - 8i_3 + 8i_4 - 6i_2 = 0) \div 2$$

$$-i_1 - 3i_2 - 6i_3 + 4i_4 = 0 \quad \text{Eq. 1}$$

KCL node P:

$$5A = i_2 - i_1 \quad \text{Eq. 2}$$

KCL node Q:

$$3I_o + i_3 = i_2$$

↳ $I = -i_4$

$$i_2 - i_3 - 3(i_4) = 0$$

$$i_2 - i_3 + 3i_4 = 0 \quad \text{Eq. 3}$$

KVL i_4 loop:

$$-2i_4 - 10V - 8(i_4 - i_3) = 0$$

$$-4i_3 + 5i_4 = -5 \quad \text{Eq. 4.}$$

$$\begin{bmatrix} -1 & -3 & -6 & 4 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -4 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ -5 \end{bmatrix} \Rightarrow$$

$i_1 = -7.5A$

$i_2 = -2.5A$

$i_3 = 3.93A$

$i_4 = 2.143A$