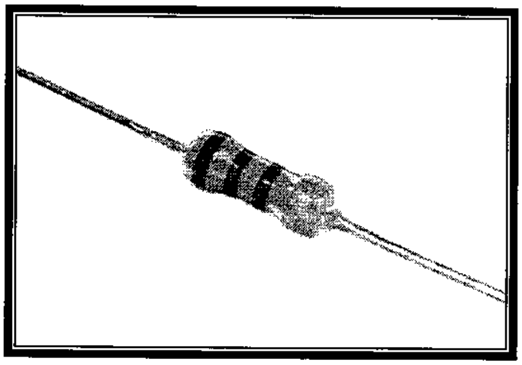


A 100 Ω resistor.
(commons.wikimedia.org).

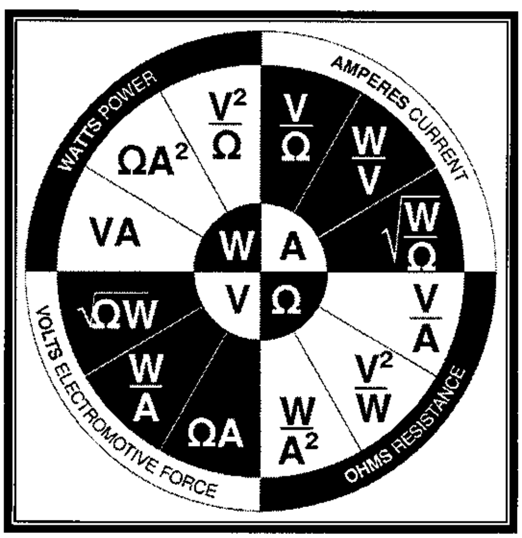
ENGG 2450: Electric Circuits

Professor: Dr. Christopher Collier

MODULE 2.0

UNIVERSITY of GUELPH

Basic Laws



The Ohm's Law Wheel
(wikipedia.org).

2.0 Fundamentals of Electric Circuits 6th Ed. corresponding textbook material for this module.

Chapter 1 Basic Concepts
1.1 Introduction
1.2 Systems of Units
1.3 Charge and Current
1.4 Voltage
1.5 Power and Energy
1.6 Circuit Elements
1.7 Applications
1.8 Problem Solving
1.9 Summary

Chapter 2 Basic Laws
2.1 Introduction
2.2 Ohm's Law
2.3 Nodes, Branches, and Loops
2.4 Kirchhoff's Laws
2.5 Series Resistors and Voltage Division
2.6 Parallel Resistors and Current Division
2.7 Wye-Delta Transformations
2.8 Applications
2.9 Summary

Chapter 3 Methods of Analysis
3.1 Introduction
3.2 Nodal Analysis
3.3 Nodal Analysis with Voltage Sources
3.4 Mesh Analysis
3.5 Mesh Analysis with Current Sources
3.6 Nodal and Mesh Analyses by Inspection
3.7 Nodal Versus Mesh Analysis
3.8 Circuit Analysis with PSpice
3.9 Applications: DC Transistor Circuits
3.10 Summary

Chapter 4 Circuit Theorems
4.1 Introduction
4.2 Linearity Property
4.3 Superposition
4.4 Source Transformation
4.5 Thevenin's Theorem
4.6 Norton's Theorem
4.7 Derivations of Thevenin's and Norton's Theorems
4.8 Maximum Power Transfer
4.9 Verifying Circuit Theorems with PSpice
4.10 Applications
4.11 Summary

Chapter 5 Operational Amplifiers
5.1 Introduction
5.2 Operational Amplifiers
5.3 Ideal Op Amp
5.4 Inverting Amplifier
5.5 Noninverting Amplifier
5.6 Summing Amplifier
5.7 Difference Amplifier
5.8 Cascaded Op Amp Circuits
5.9 Op Amp Circuit Analysis with PSpice
5.10 Applications
5.11 Summary

Chapter 6 Capacitors and Inductors
6.1 Introduction
6.2 Capacitors
6.3 Series and Parallel Capacitors
6.4 Inductors
6.5 Series and Parallel Inductors
6.6 Applications
6.7 Summary

Chapter 7 First-Order Circuits
7.1 Introduction
7.2 The Source-Free RC Circuit
7.3 The Source-Free RL Circuit
7.4 Singularity Functions
7.5 Step Response of an RC Circuit
7.6 Step Response of an RL Circuit
7.7 First-Order Op Amp Circuits
7.8 Transient Analysis with PSpice
7.9 Applications
7.10 Summary

Chapter 8 Second-Order Circuits
8.1 Introduction
8.2 Finding Initial and Final Values
8.3 The Source-Free Series RLC Circuit
8.4 The Source-Free Parallel RLC Circuit
8.5 Step Response of a Series RLC Circuit
8.6 Step Response of a Parallel RLC Circuit
8.7 General Second-Order Circuits
8.8 Second-Order Op Amp Circuits
8.9 PSpice Analysis of RLC Circuits
8.10 Duality
8.11 Applications
8.12 Summary

Chapter 9 Sinusoids and Phasors
9.1 Introduction
9.2 Sinusoids
9.3 Phasors
9.4 Phasor Relationships for Circuit Elements
9.5 Impedance and Admittance
9.6 Kirchhoff's Laws in the Frequency Domain
9.7 Impedance Combinations
9.8 Applications
9.9 Summary

Chapter 10 Sinusoidal Steady-State
10.1 Introduction
10.2 Nodal Analysis
10.3 Mesh Analysis
10.4 Superposition Theorem
10.5 Source Transformation
10.6 Thevenin and Norton Equivalent Circuits
10.7 Op Amp AC Circuits
10.8 AC Analysis Using PSpice
10.9 Applications
10.10 Summary

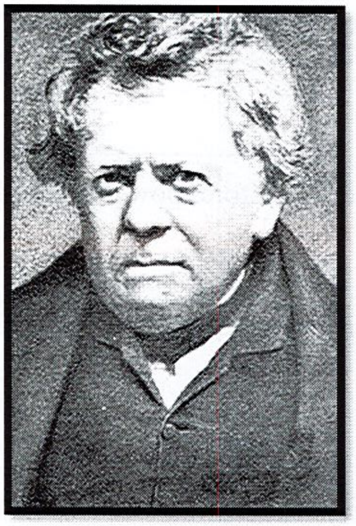
Chapter 11 AC Power Analysis
11.1 Introduction
11.2 Instantaneous and Average Power
11.3 Maximum Average Power Transfer
11.4 Effective or RMS Value
11.5 Apparent Power and Power Factor
11.6 Complex Power
11.7 Conservation of AC Power
11.8 Power Factor Correction
11.9 Applications
11.10 Summary

Chapter 12 Three-Phase Circuits
12.1 Introduction
12.2 Balanced Three-Phase Voltages
12.3 Balanced Wye-Wye Connection
12.4 Balanced Wye-Delta Connection
12.5 Balanced Delta-Delta Connection
12.6 Balanced Delta-Wye Connection
12.7 Power in a Balanced System
12.8 Unbalanced Three-Phase Systems
12.9 PSpice for Three-Phase Circuits
12.10 Applications
12.11 Summary

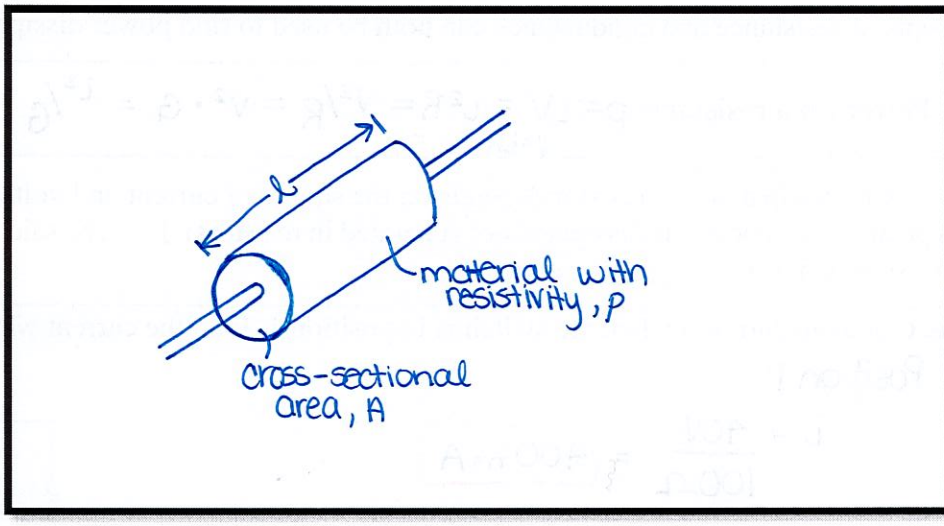
Chapter 13 Magnetically Coupled Circuits
13.1 Introduction
13.2 Mutual Inductance
13.3 Energy in a Coupled Circuit
13.4 Linear Transformers
13.5 Ideal Transformers
13.6 Ideal Autotransformers
13.7 Three-Phase Transformers
13.8 PSpice Analysis of Magnetically Coupled Circuits
13.9 Applications
13.10 Summary

2.1 Ohm's Law.

Georg Ohm noticed that materials tend to resist the flow of current. The tendency of a material to resist the flow of current is its resistivity, ρ, with units of Ω m. Ohm also noticed that devices made from resistive materials will have greater resistance if they have a large length, l, and small cross-sectional area, A.



Georg Ohm (commons.wikimedia.org).



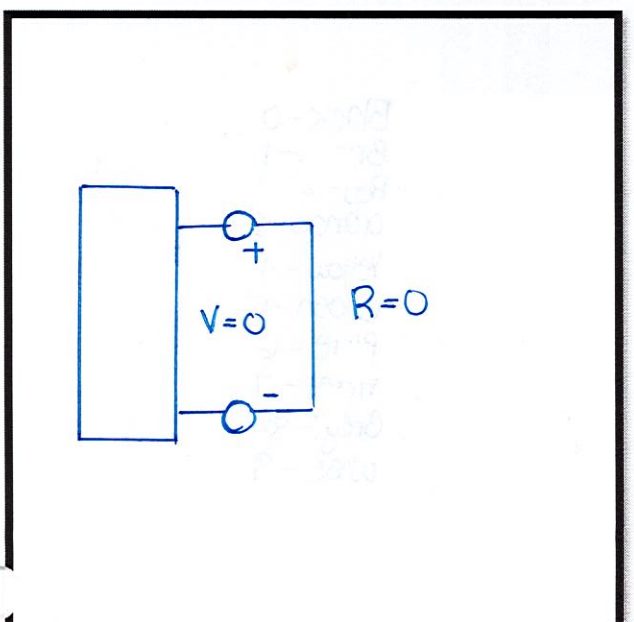
Ohm's law.

Resistance: $R=\rho\frac{l}{A}$ [units = Ω]

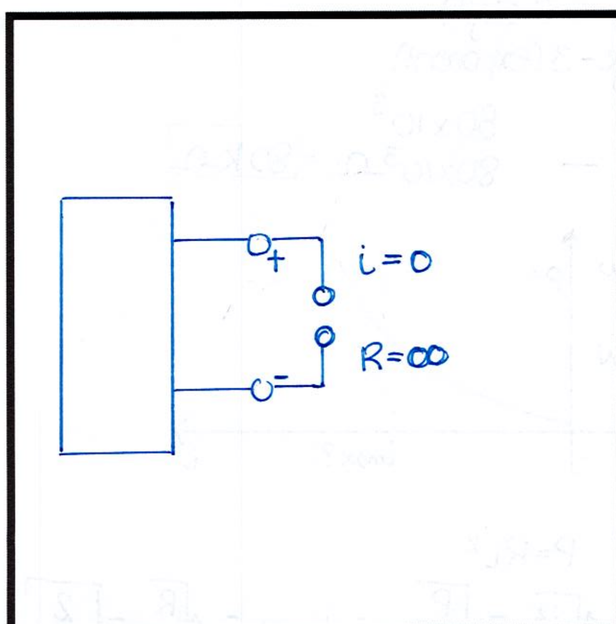
Ohm also noticed that the voltage over a resistor is linearly proportional to the current flowing through (i.e., higher voltage means higher current). This idea is now called **Ohm's Law**.

Ohm's Law: $V=iR$ [units=V] $\rightarrow R=\frac{V}{i}$ [Ω]

If the resistance is zero, there is a **short circuit** (typically a metal connection). If the resistance is infinite, there is an **open circuit** (typically a disconnection).



Short circuit ($v=\lim\limits_{R\to 0} iR=0$).



Open circuit ($i=\lim\limits_{R\to\infty} v/R=0$).

Sometimes, engineers talk in terms of the **conductance** of a circuit element rather than the resistance. Conductance is the reciprocal of resistance.

Conductance: $G = \frac{1}{R} = \frac{i}{V} \quad [S = \frac{1}{\Omega} = \frac{A}{V}]$

The concepts of resistance and conductance can both be used to find power dissipated in a resistor.

Power (in a resistor): $p = iV = i^2 R = V^2/R = V^2 \cdot G = i^2/G$

$V = iR \quad i = \frac{V}{R}$

Note that power dissipated in a resistor depends on the square of current and voltage. Therefore, power in a resistor is always positive (i.e., power is dissipated not generated in resistors). It can be said that resistors are passive elements that do not supply power.

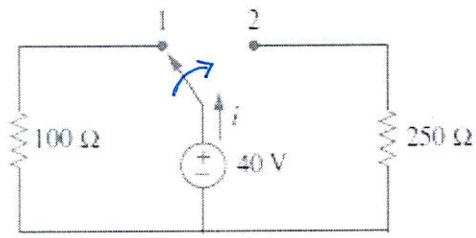
Example. Calculate current i when the switch is in position 1. Find the current when the switch is in position 2.

Position 1:

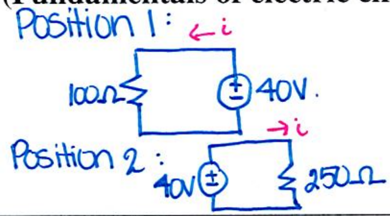
$$i = \frac{40V}{100\Omega} = \boxed{400mA}$$

Position 2:

$$i = \frac{40V}{250\Omega} = \boxed{160mA}$$



(Fundamentals of electric circuits, 5th Ed.)

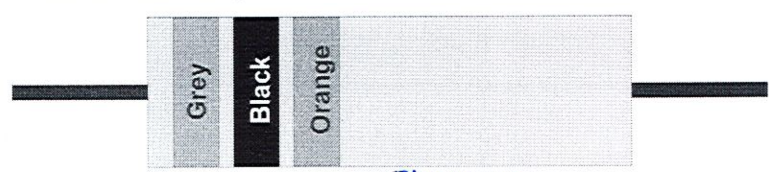


Example. Find the maximum current that a 2 W, 80 kΩ resistor can safely conduct.

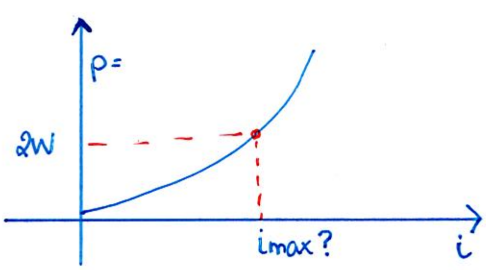
Grey - 8 (10 digit)
Black - 0 (1 digit)
Orange - 3 (exponent)

80×10^3

$80 \times 10^3 \Omega = \boxed{80k\Omega}$



Black - 0
Brown - 1
Red - 2
Orange - 3
Yellow - 4
Green - 5
Blue - 6
Violet - 7
Grey - 8
White - 9



$P = Ri^2$

$$\sqrt{i^2} = \sqrt{\frac{P}{R}} = i_{max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{2}{80K}} = \boxed{5mA}$$

$\therefore$ max current is 5mA

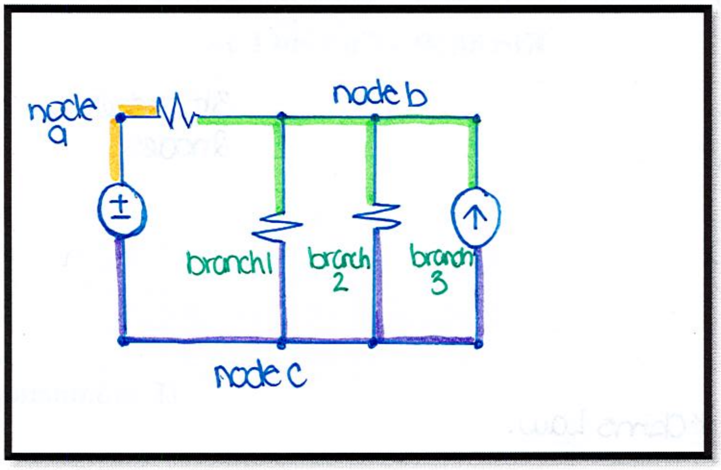
2.2 Nodes, branches, and loops.

In order to understand circuits, one has to be familiar with terms that are commonly used to describe circuits. Several of these terms will be discussed here. Specifically, **nodes**, **branches**, and **loops** will be discussed.

Branch: a single element such as a voltage source or a resistor

Node: point of connection between 2 or more branches

Loop: any closed path in a circuit



Nodes, branches, and loops.

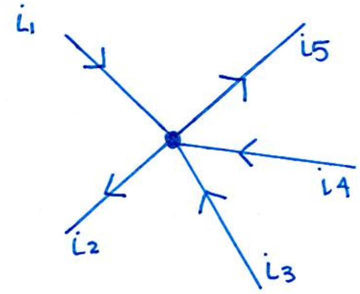
2.3 Kirchhoff's Laws.

The concepts of nodes, branch, and loop are used in explain Kirchhoff's Laws. Kirchhoff has two laws: **Kirchhoff's Current Law** and **Kirchhoff's Voltage Law**.

Gustav Kirchoff (commons.wikimedia.org).

Kirchhoff's Current Law relates to a conservation of current at a node.

Kirchhoff's Current Law (KCL): $\sum_{node} i = 0$ all current into and out of a node must sum to zero.

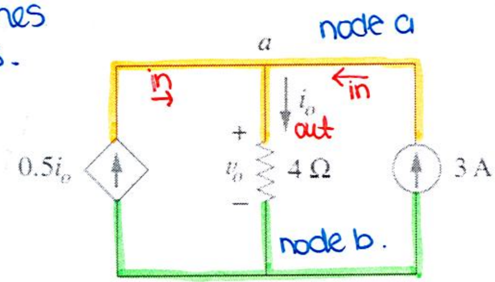


$\sum i = i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$

Kirchhoff's Current Law.

Example. Determine v_0 and i_0.

3 branches
2 nodes.



(Fundaments of electric circuits, 5th Ed.)

Apply KCL to node a:

$\sum i = 3 + 0.5i_0 - i_0 = 0$

$3 + (-0.5i_0) = 0$

$i_0 = \frac{3}{0.5} = \boxed{6A}$

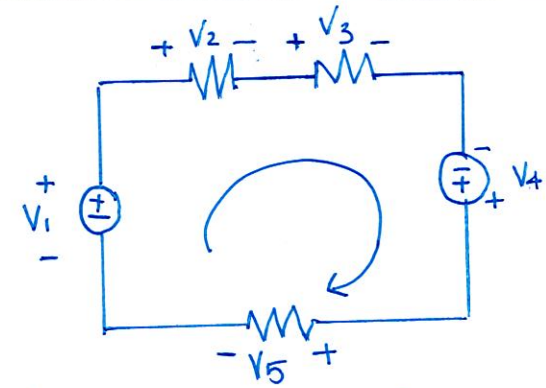
$V_0 = R i_0$ *Ohms Law.

$= (4\Omega)(6A)$

$= \boxed{24\ V}$

Kirchhoff's Voltage Law relates to a conservation of voltage over a loop.

Kirchhoff's Voltage Law (KVL): $\sum_{loop} V = 0$ all voltages over a loop must sum to zero



$\sum V = -V_1 + V_2 + V_3 - V_4 + V_5 = 0$

Kirchhoff's Voltage Law.

Example. Determine v_0 and i.

*. gains are negative, losses are positive

KVL overloop:

$-12 + 4i + 2V_0 - 4 + 6i = 0$

Ohms Law:

$V_0 = (6\Omega)(-i)$

$= -6i$

$-16 + 10i + 2(-6i) = 0$

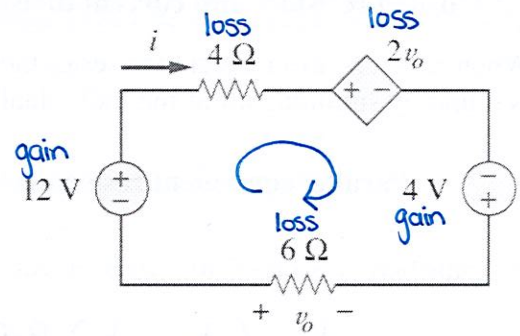
$-16 - 2i = 0$

$-2i = 16$

$\boxed{i = -8A}$

$V_0 = -6(-8A)$

$\boxed{= 48V}$



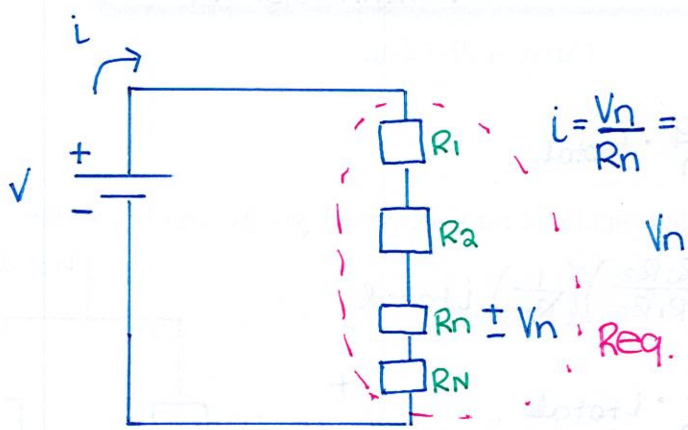
(Fundaments of electric circuits, 5th Ed.)

2.4 Series resistors and voltage division.

When resistors are connected in series they are connected one after another. The total resistance of all of the resistors is found by summing all of the individual resistances. This gives the **equivalent resistance**.

Series equivalent resistance: $R_{eq} = R_1 + R_2 + \ldots + R_n = \sum_{r=1}^{n} R_n$

The voltage drop over any one resistor is proportional to its resistance.



$i = \frac{V_n}{R_n} = \frac{V}{R_{eq}}$

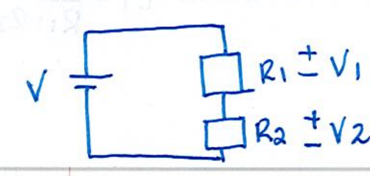
$V_n = \frac{R_n}{R_{eq}} \cdot V$

Voltage division.

Voltage division: $V_n = \frac{R_n}{R_{eq}} \cdot V = \frac{R_n}{R_1 + R_2 + \ldots + R_N} \cdot V$

For two resistors, this equation can be simplified.

Voltage division (two resistors): $V_1 = \frac{R_1}{R_1 + R_2} \cdot V \qquad V_2 = \frac{R_2}{R_1 + R_2} \cdot V$



2.5 Parallel resistors and current division.

When resistors are connected in series they are connected one after another. The total resistance of all of the resistors is found by summing all of the individual resistances. This gives the **equivalent resistance**.

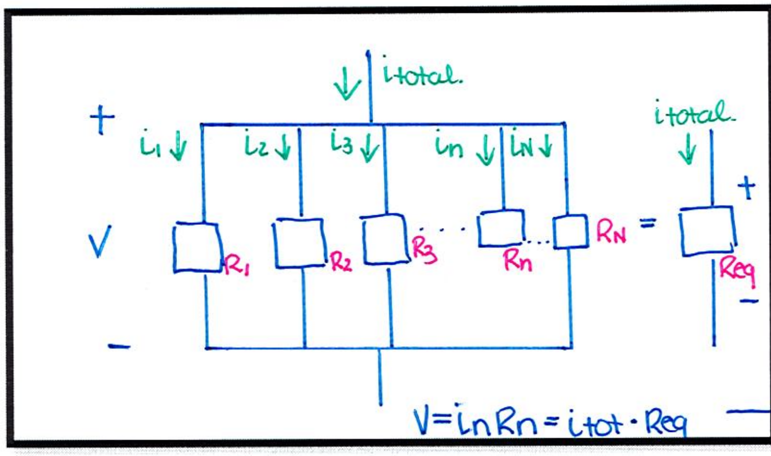
Parallel equivalent resistance: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$

A simplified version of this equation can be calculated for the case of two resistors in parallel.

$$\frac{1}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{R_1R_2}{R_1R_2} = \frac{R_2+R_1}{R_1R_2} \Rightarrow R_{eq} = \frac{R_1R_2}{R_1+R_2}$$

Parallel equivalent resistance (two resistors): $R_{eq} = \frac{R_1R_2}{R_1+R_2}$

When resistors are in parallel, the current is divided.



$\rightarrow i_n = \frac{R_{eq}}{R_n} \bullet i_{total}$

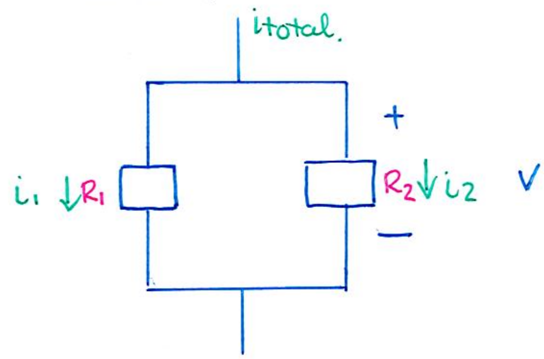
Current division.

Current division: $i_n = \frac{R_{eq}}{R_n} \cdot i_{total}$

A simplified version of this equation can be calculated for the case of two resistors in parallel.

$$i_1 = \frac{R_{eq}}{R_1} i_{total} = \left(\frac{R_1R_2}{R_1R_2}\right)\left(\frac{1}{R_1}\right) i_{total}$$

$$i_1 = \frac{R_2}{R_1+R_2} \cdot i_{total}$$



Current division (two resistors): $i_1 = \frac{R_2}{R_1+R_2} \cdot i_{total}$; $i_2 = \frac{R_1}{R_1+R_2} i_{total}$

In a similar way to equivalent resistances, equivalent series and parallel conductances can be found.

Series equivalent conductance: $\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \ldots + \frac{1}{G_N}$

Parallel equivalent conductance: $G_{eq} = G_1 + G_2 + \ldots + G_N$

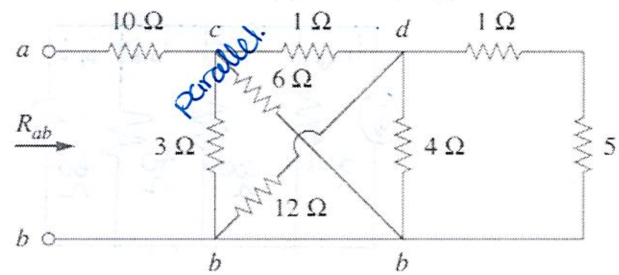
Example. Calculate the equivalent resistance R_{ab} in the circuit.

Circuit1

$3\Omega \| 6\Omega = \frac{(3)(6)}{3+6} = \frac{3 \cdot 6}{3+6} = \frac{18}{9} = 2\Omega$

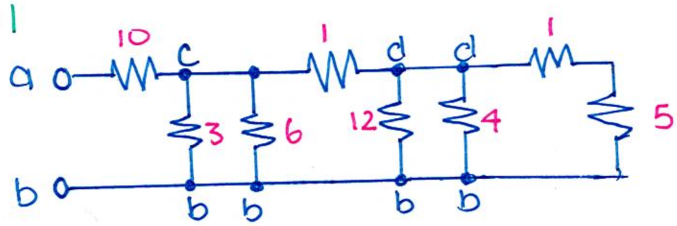
$12\Omega \| 4\Omega = \frac{(12)(4)}{12+4} = \frac{48}{16} = 3\Omega$

$1+5 = 6\Omega$



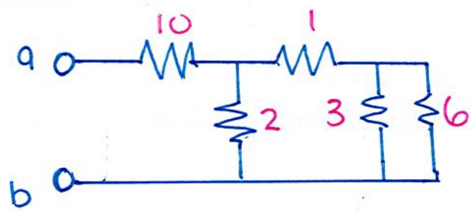
(Fundamentals of electric circuits, 5th Ed.)

Circuit 1



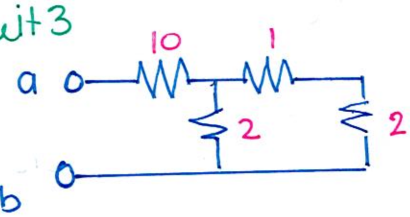
Circuit 2

$3 \| 6 = 2\Omega$



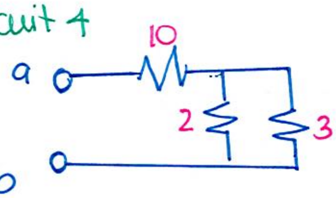
Circuit 3

$1 + 2 = 3\Omega$



Circuit 4

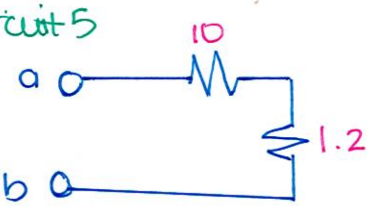
$2 \| 3 = \frac{2(3)}{2+3} = \frac{6}{5} = 1.2\Omega$



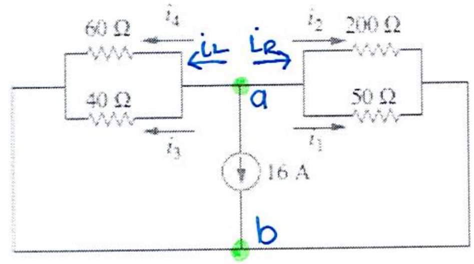
Circuit 5

$10 + 1.2 = 11.2\Omega$

$\boxed{R_{ab} = 11.2\Omega}$

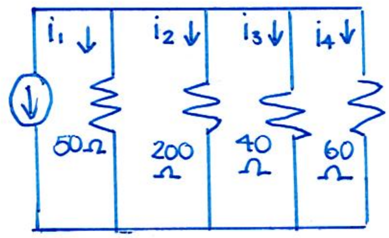


Example. Calculate i_1, i_2, i_3, i_4,



(Fundamentals of electric circuits, 5th Ed.)

$i_{total} = -16A$



* all in parallel

$$R_{eq} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right]^{-1} = 15$$

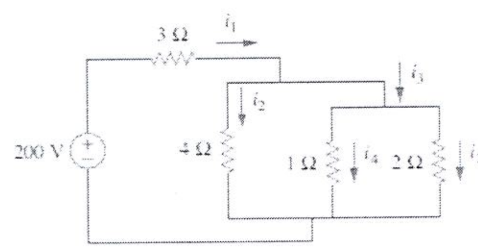
$$i_1 = \frac{15}{50} i_{total} = \boxed{-4.8\,A}$$

$$i_2 = \frac{15}{200} i_{total} = \boxed{-1.2A}$$

$$i_3 = \frac{15}{40} i_{total} = \boxed{-6A}$$

$$i_4 = \frac{15}{60} i_{total} = \boxed{-4A}$$

Practice problem (solve at home). Calculate i_1, i_2, i_3, i_4, and i_5.



(Fundamentals of electric circuits, 5th Ed.)

(Note: Answers are $i_1 = 11.2$ A, $i_2 = 1.6$ A, $i_3 = 9.6$ A, $i_4 = 6.4$ A, and $i_5 = 3.2$ A)