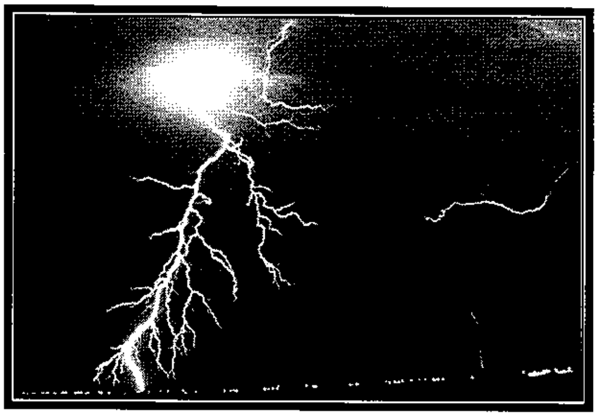
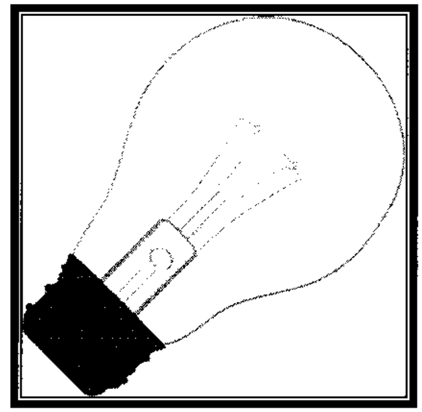


Lightning is a natural form of electricity (en.wikipedia.org).



A light bulb is a circuit element (publicdomainpictures.net).

ENGG 2450: Electric Circuits

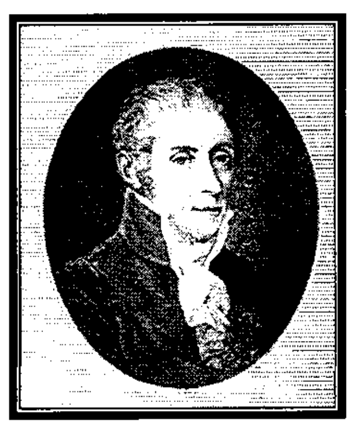
Professor: Dr. Christopher Collier

MODULE 1.0

UNIVERSITY of GUELPH

Basic Concepts

André-Marie Ampère (wikipedia.org)



Alessandro Volta (commons.wikimedia.org)

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1.1 System of units.

The international system of units in used in Canada and most other countries. The base units are the meter (m), second (s), Newton (N), etc. In contrast, the United States uses the Imperial system of units where the base units are the foot (ft), second (s), pound (lb), etc. In ENGG 2450 Electric Circuits, the international system will primarily be used.

In the international system of units, there are many **basic units**:

Quantity	Basic unit	Symbol
Length	metre	m
Mass	Kilogram	Kg
Time	seconds	S
Thermodynamic temperature	Kelvin	K
Amount of substance	mole	mol
Charge	Coulomb	C

Basic units.

There are also many **derived units**. These derived units are derived from combinations of the basic units. Here are some derived units:

Quantity	Unit Name (Symbol)	Formula
Frequency	hertz (Hz)	$1/s = s^{-1}$
Force	newton (N)	$Kg \cdot m/s^2$
Energy or work	joule (J)	Nm
Power	watt (W)	J/S
Electric charge	coulomb (C)	$A \cdot S$
Electric current	ampere (A)	C/S
Electric potential	volt (V)	J/C
Electric resistance	ohm (Ω)	V/A
Electric conductance	siemens (S)	A/V
Electric capacitance	farad (F)	C/V
Magnetic flux	weber (Wb)	$V \cdot S$
Inductance	henry (H)	Wb/A

Derived units.

These units sometimes take a form that is very large (e.g., 1,000,000,000 Ω or 1/1,000,000 F). In these situations, it is helpful to use a **prefix**:

Prefix	Symbol	Power
Atto	a	10^{-18}
Femto	f	10^{-15}
Pico	p	10^{-12}
Nano	n	10^{-9}
Micro	µ	10^{-6}
Milli	m	10^{-3}
Centi	c	10^{-2}
Deci	d	10^{-1}
Deka	da	10^{1}
Hecto	h	10^{2}
Kilo	k	10^{3}
Mega	M	10^{6}
Giga	G	10^{9}
Tera	T	10^{12}

Example. If a signal can travel in a cable at 80% of the speed of light ($c = 3 \times 10^8$ m/s), what length of cable, in inches, represents 1 ns? (Note: 1" = 2.54 cm)

1" = 2.54cm

100cm = 1m

10^9 ns = 1s

Speed = $(0.80)c = 0.80(3\times10^8 m/s) = 2.4\times10^8 m/s$

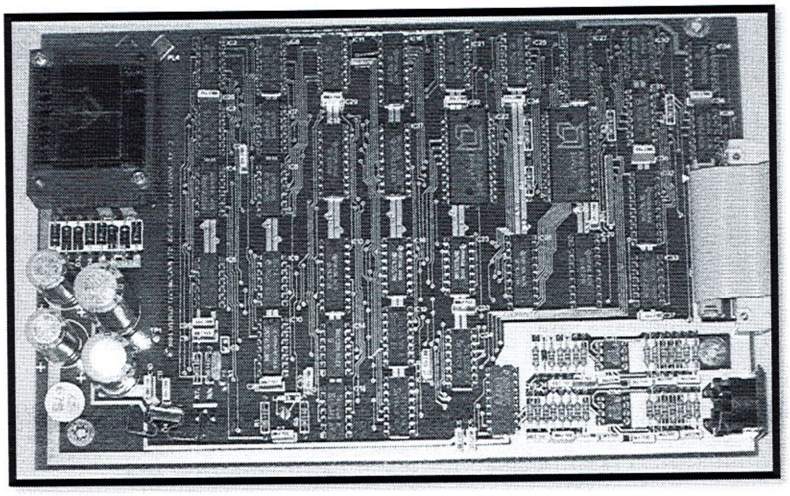
Length = speed · time

$= 2.4\times10^8 \cancel{m}/\cancel{s} \cdot 1\cancel{ns} \cdot \frac{1\cancel{s}}{10^9 \cancel{ns}} \cdot \frac{100\cancel{cm}}{1\cancel{m}} \cdot \frac{1"}{2.54\cancel{cm}}$.

$= \boxed{9.45 \text{ inches}}$

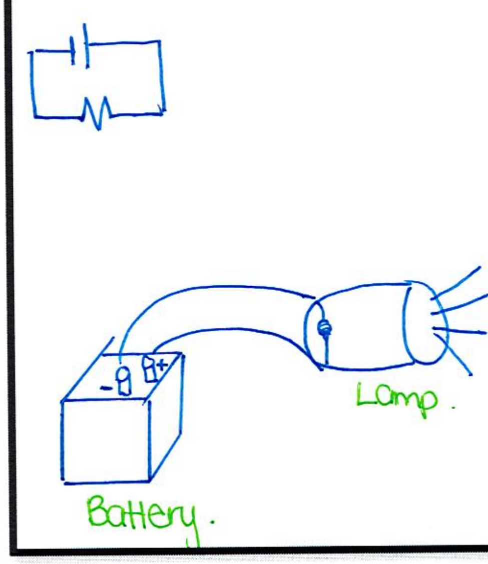
*units are important

Electric circuits are useful for all engineering disciplines. These electric circuits can seem very complex at times, but ultimately, electric circuits consist of many electrical elements (batteries, resistors, capacitors, etc.) which are interconnected together.



An Acorn Music 500 Synthesizer electric circuit (commons.wikimedia.org).

Electric circuit: an interconnection of electrical elements



A simple electric circuit.

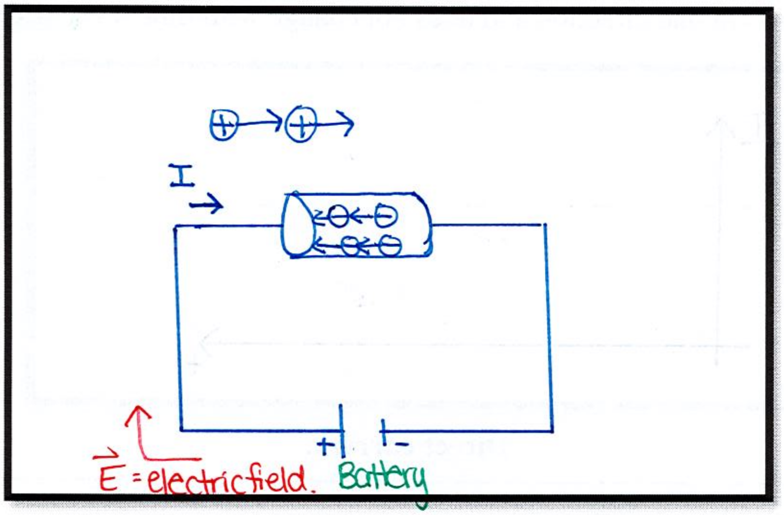
This interconnection of electrical elements is governed by *current* and *voltage*.

1.2 Charge and current.

To understand current, it is important to first understand **charge**. Charge is what a person feels when he/she walks across a carpet and receives a shock when touching a metal object. This shock comes from electrons being exchanged with the surroundings. This electric charge is measured in coulombs.

Coulomb: 1 electron = 1.602×10^{-19} C
(elementary charge).

In an electric circuit, this charge flows through electrical elements. This is an **electric current**.



An electric current in an electric circuit.

Mathematically, current is the first derivative of charge with respect to time.

Electric current:	time rate of change of charge (measured in Amperes). [A = C/S]

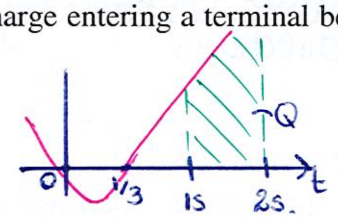
Electric current (mathematical definition):	$i = \frac{dq}{dt}$

Of course, if current is the *derivative* of charge, it makes sense that charge is the *integral* of current.

Charge (mathematical definition):	$Q = \int_{t_0}^{t} i\,dt$

Electric current can be thought of as a flow of electrons. With this in mind, it makes sense that electric current can take positive (i.e., flowing one direction) or negative values (flowing the opposite direction).

Example. Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing through the terminal is $i = (3t^2 - t)$ A.



Parabola $i = 3t^2 - t = t(3t-1)$

Zeroes at $3t - 1 = 0$

$t = 1/3$.

$t = 0, 1/3$.

$$Q = \int_{t=1}^{t=2} i\,dt = \int_1^2 3t^2 - t\,dt$$

$$= \left[t^3 - \frac{1}{2}t^2\right]\Big|_1^2$$

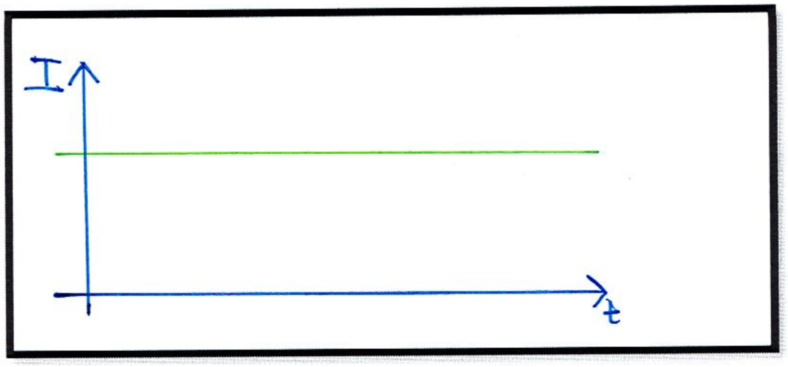
$$= (2^3 - \frac{2}{2}^2) - (1^3 - \frac{1}{2}^2)$$

$$= (8-2) - (1 - \frac{1}{2})$$

$$\boxed{= 5.5\,C}$$

Note: units.

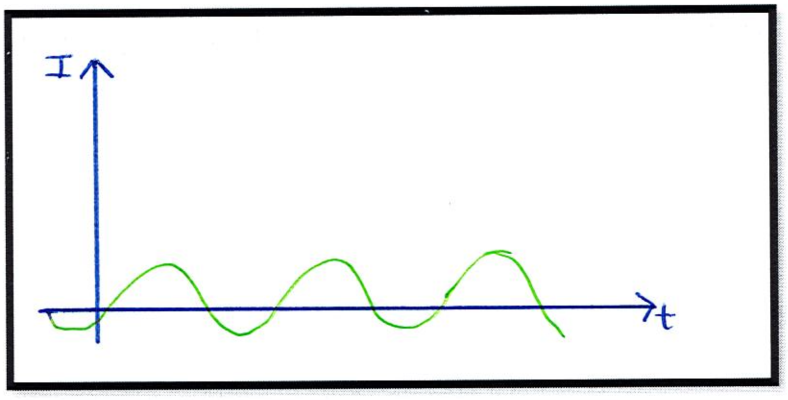
If an electric current only flows in one direction and does not change with time it is **a direct current.**



Direct current.

Direct current (DC): a current that remains constant in time. (current that does not change direction)

If an electric current alternates between flow in two directions (sinusoidally) it is an **alternating current**.

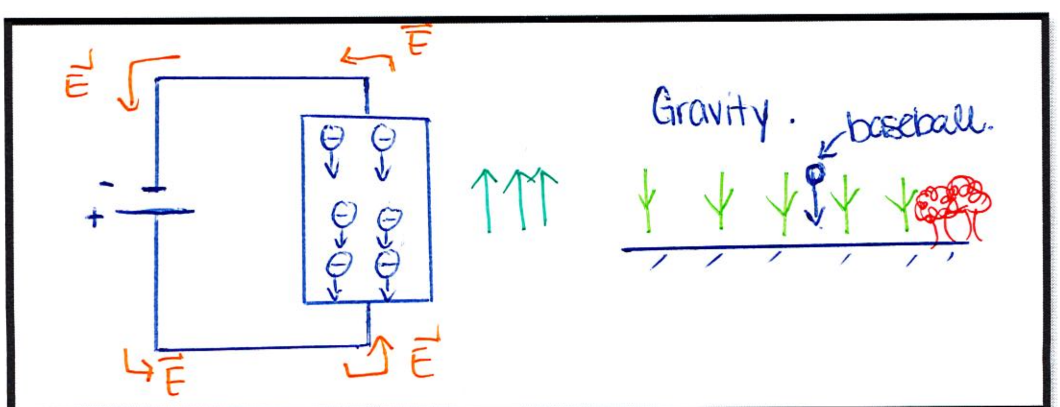


Alternating current.

Alternating current (AC): a current that varies sinusoidally in time. (does change direction)

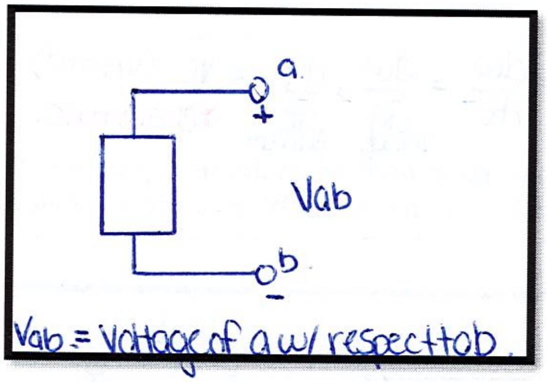
1.3 Voltage.

Current can be thought of as the flow of electrons (i.e., the time-rate-of-change of charge). Naturally, something must be causing the electrons to move. The movement of electrons is caused by an electromotive force (emf) which is often called a *voltage* or a *potential difference*. This electromotive force is causes by an electric field (typically from a battery) which puts a force on electrons. This force is analogous to forces from gravity.



Voltage and potential difference.

As voltage is a potential difference, the voltage between two points can be defined as being the energy (i.e., work) required to move a charge from one point to another (or through an element).



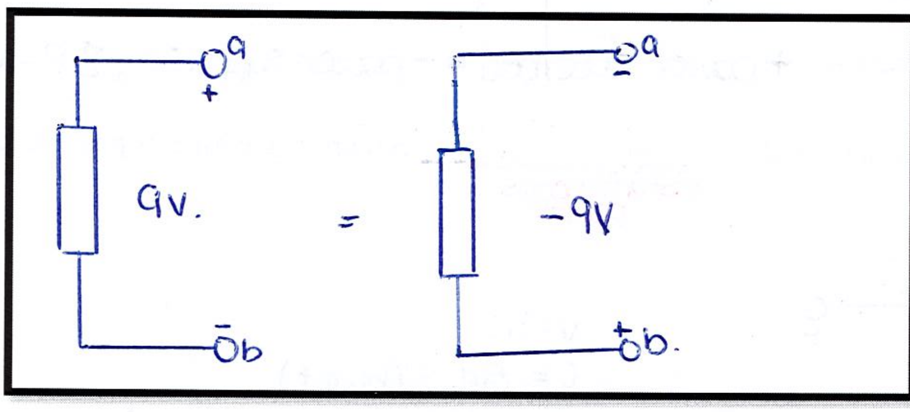
The voltage between point *a* and *b*.

Voltage (i.e., potential difference):	energy required to move a unit charge through an element.

Voltage (mathematical definition):	$V_{ab} = \frac{dw}{dq}$. $(V = J/C)$

It should be noted that the above mathematical definition of voltage is the voltage at point *a* **with respect to** point *b*. If point *a* is at a higher potential than point *b*, there would be $v_{ab} > 0$. It is possible to look at the opposite case, i.e., the voltage at point *b* with respect to point *a*. With the case of point *a* being at a higher potential than point *b*, there would be $v_{ba} < 0$. That is to say, the voltage would flip its sign.

Equivalent voltage:	$V_{ab} = -V_{ba}$.



Equivalent voltage.

1.4 Power and energy.

The concept of power can be used to find how much energy is being consumed over time. This is very useful. For example, home owners pay for the amount of energy consumed in a certain pay period. They would calculate this energy consumption by taking the average power consumption multiplied by the time of the pay period.

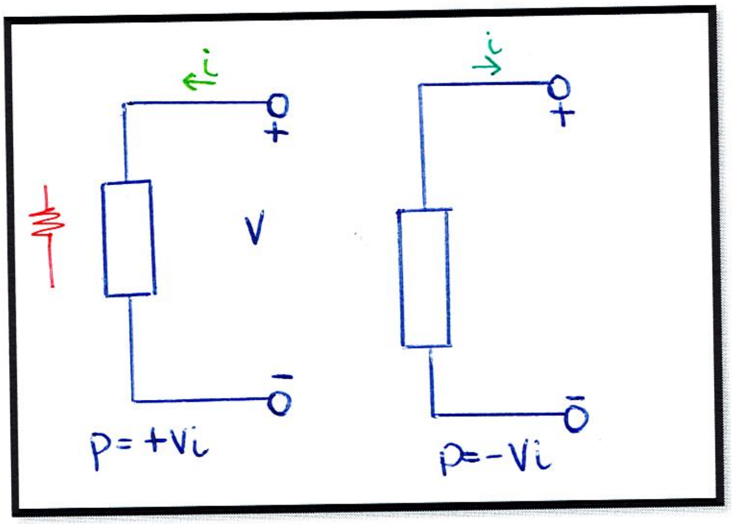
Power:	time rate of change. of expending/absorbing energy (w).

Power (mathematical definition):	$p = \frac{dw}{t}$ $(W = J/s)$

This mathematical definition of power can be expanded through the chain rule, $P = dw/dt = dw/dq \times dq/dt$. Remember that dw/dq is voltage and dq/dt is current. Therefore, the instantaneous power is the multiplication of instantaneous voltage and current.

Power (instantaneous): $P = \frac{dw}{dt} = \underbrace{\frac{dw}{dq}}_{\text{voltage}} \cdot \underbrace{\frac{dq}{dt}}_{\text{current}} = vi \quad (VA = W)$ *chain rule

It is also important to note the sign convention used when discussing power. When current is entering through the positive terminal of an element, the positive sign is used. When current is entering through the negative terminal of an element, the negative sign is used.



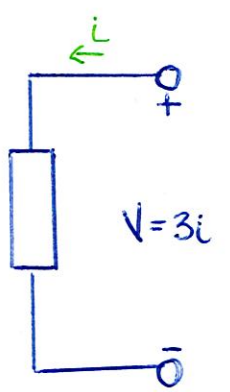
Sign convention for power.

The concept of conservation of power/energy applies to electric circuits. This is to say, all power that is absorbed in a circuit must also be supplied.

Power conservation: + power absorbed = − power supplied, $\sum P = 0$

Example. Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is $i = 5\cos(60\pi t)$ A and the voltage is $v = 3i$.

instantaneous power



$V = 3i$

$i = 5\cos(60\pi t)$

$P = +Vi = 3i(5\cos(60\pi t))$

$= 3(5\cos(60\pi t))(5\cos(60\pi t))$

$= 75\cos^2(60\pi t)$

$p(t = 3ms) = 75\cos^2(60\pi(3ms))$

$= 75\cos^2(60\pi(3\times10^{-3}))$

$= \boxed{53.5W}$

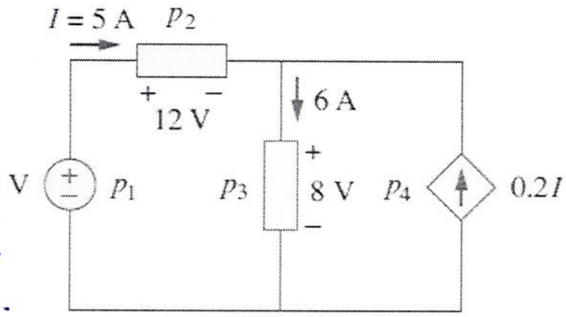
Example. Calculate the power supplied or absorbed by each element. State if absorbed or supplied. Check to see if power is conserved.

$P_1 = -(20)(5) = -100W$ (supplied power).

$P_2 = +(12)(5) = 60W$ (absorbed power).

$P_3 = +(8)(6) = 48W$ (absorbed power).

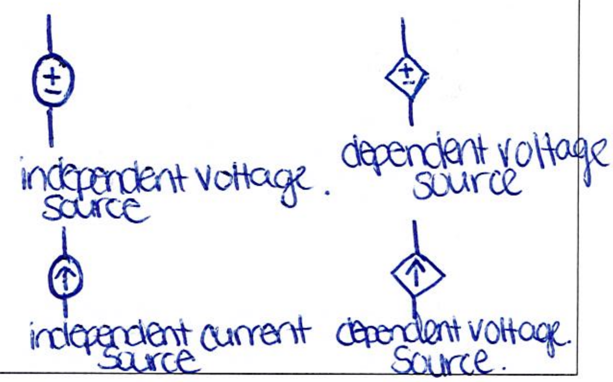
$P_4 = -(8)(0.2i) = -8(0.2\cdot5) = -8W$ (supplied power).



(Fundamentals of electric circuits, 5th Ed.)

$\Sigma p = 0 = -100 + 60 + 48 - 8$

$\boxed{= 0}$ ∴ Power is conserved.

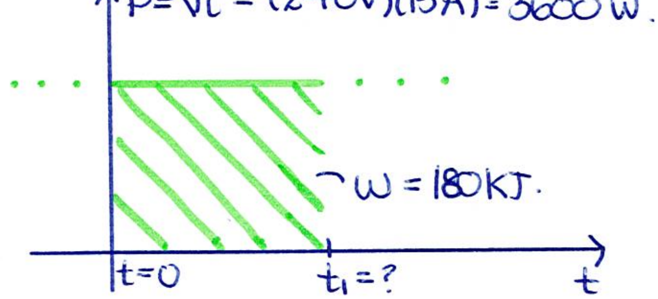


Recall that charge and current have a derivative/integral relationship. In the same way, energy and power have a derivative/integral relationship. That is to say, energy is the integral of power.

Energy (mathematical definition): $w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$.

Example. A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 180 kJ?

$P = Vi = (240V)(15A) = 3600\ W$.



(publicdomainpicture.net)

$$w = \int_{t_0}^{t_1} p\,dt = 180KJ = (3600)(t_1 - \overset{\nearrow 0}{t_0})$$
$$= 3600t_1.$$

$$t_1 = \frac{180KJ}{3600W} = \boxed{50s}$$