

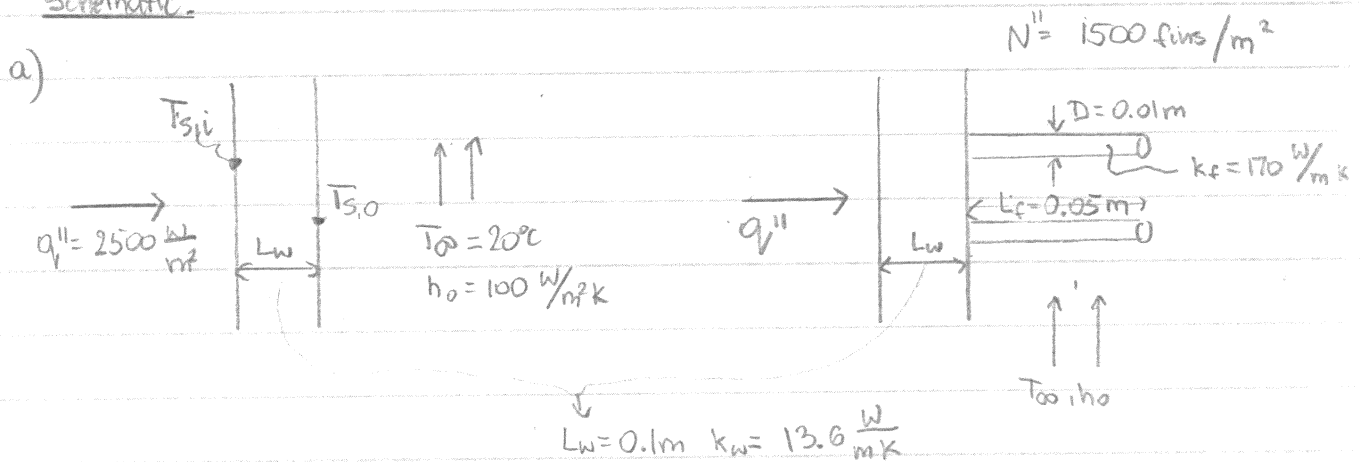
## CHG 2314 - Midterm Solutions

Winter 2016

Problem 1

Known: Wall inside surface receives constant heat flux ( $q''$ ). The outside surface is convectively cooled as a) flat surface, or b) with addition of fins

Find: Inner surface ( $T_{s,i}$ ) and outer surface ( $T_{s,o}$ ) temperatures of the wall for cases a) and b)

Schematic:

- Assumptions:
- 1) Steady-state 1-D conduction in wall & fins
  - 2)  $h_o$  not affected by the presence of fins
  - 3) Negligible radiation effects
  - 4) constant properties

Analysis: a) Outside surface is flat

• Thermal circuit  $q'' \rightarrow T_{s,i} \xrightarrow{R''_{\text{cond}}} T_{s,o} \xrightarrow{R''_{\text{conv}}} T_{\infty}$

$$\text{where: } R''_{\text{cond}} = \frac{L_w}{k} = \frac{0.1 \text{ m}}{13.6 \text{ W/m}\cdot\text{K}} = 0.00735 \frac{\text{K}\cdot\text{m}^2}{\text{W}}$$

$$R''_{\text{conv}} = \frac{1}{h_o} = \frac{1}{100 \text{ W/m}^2\cdot\text{K}} = 0.01 \frac{\text{K}\cdot\text{m}^2}{\text{W}}$$

$$q'' = \frac{T_{s,o} - T_{\infty}}{R''_{\text{conv}}} = h_o(T_{s,o} - T_{\infty}) \Rightarrow T_{s,o} = \frac{q''}{h_o} + T_{\infty} = \frac{2500 \frac{\text{W}}{\text{m}^2}}{100 \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + 20$$

$$\text{OneClass: } T_{s,o} = 25 + 20 = 45^\circ\text{C}$$

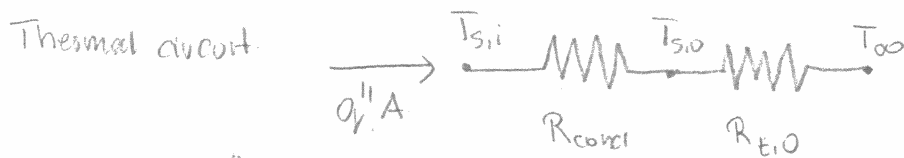
Problem 1 - continued

$$\text{similarly: } q'' = \frac{T_{s,i} - T_{s,o}}{R''_{\text{cond}}} \Rightarrow T_{s,i} = q'' R''_{\text{cond}} + T_{s,o} = 2500 \cdot \frac{\text{W}}{\text{m}^2} \cdot 0.00735 \frac{\text{K m}^2}{\text{W}} + 45^\circ\text{C}$$

$$\therefore T_{s,i} = 63.4^\circ\text{C}$$

b) Outside surface is finned

we know number of fins per  $\text{m}^2$ ; thus we will use  $A = A_i = A_o = 1 \text{ m}^2$



$$R_{\text{cond}} = R''_{\text{cond}} / A = 0.00735 \text{ K/W}$$

$$R_{t,o} = \frac{1}{h A_t \eta_o} \quad \text{where } \eta_o = \text{efficiency of finned surface}$$

$$\eta_o = 1 - \frac{N A_f}{A_t} (1 - \eta_f)$$

$$\text{where: } A_t = N A_f + A_b = N A_f + (1 - N A_c)$$

• we will use eqn for adiabatic fins, thus to take into consideration conduction from the tips, we will use  $L_c = L_f + \frac{D}{4} = 0.05 + \frac{0.01}{4} = 0.0525 \text{ m}$

$$A_c = \pi \frac{D^2}{4} = \pi \cdot \frac{(0.01)^2}{4} = 7.85 \cdot 10^{-5} \text{ m}^2$$

$$A_f = \pi D L_c = \pi \cdot 0.01 \cdot 0.0525 = 1.65 \cdot 10^{-3} \text{ m}^2$$

$$\therefore A_b = 1 - 2500 \cdot 7.85 \cdot 10^{-5} = 0.80375 \text{ m}^2$$

$$\therefore A_t = 2500 \cdot 1.65 \cdot 10^{-3} + 0.80375 = 4.925 \text{ m}^2$$

$$\eta_f = \frac{\tanh(m L_c)}{m L_c} \quad \text{where } m = \left( \frac{h P}{k A_c} \right)^{1/2} = \left( \frac{4 h}{k D} \right)^{1/2} = \left( \frac{4 \cdot 2500}{170 \cdot 0.01} \right)^{1/2}$$

$$\Rightarrow m = 15.34 \text{ m}^{-1} \Rightarrow m L_c = 15.34 \cdot 0.0525 = 0.805$$

$$\therefore m L_c = 0.805$$

$$\text{sub-in numerical values: } \eta_f = \frac{\tanh(0.805)}{0.805} = 0.828$$

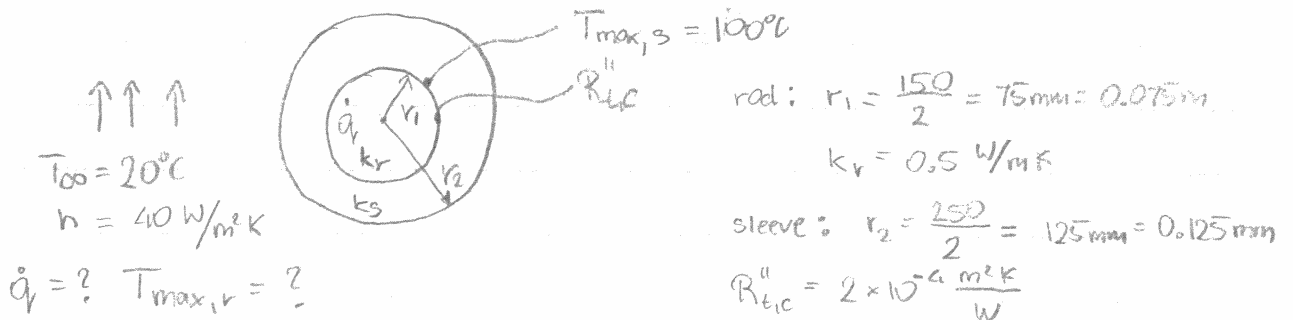
$$\eta_o = 1 - \frac{2500 \cdot 1.65 \cdot 10^{-3}}{4.925} (1 - 0.828) = 0.856 \Rightarrow R_{t,o} = \frac{1}{100 \cdot 4.925 \cdot 0.856} = 0.00237 \frac{\text{K}}{\text{W}}$$

Problem 2

Known: long cylindrical rod experiencing uniform volumetric heat generation, which is encapsulated by a circular sleeve exposed to convective cooling; contact resistance between the sleeve and the rod

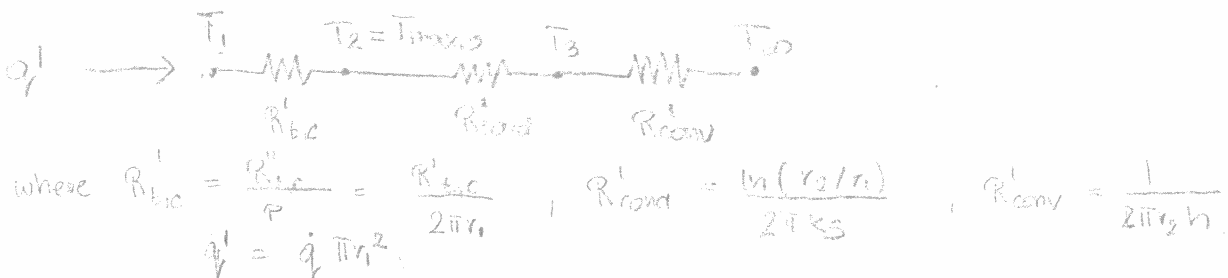
- Find:
- thermal circuit
  - volumetric heat generation so the sleeve does not exceed 100°C
  - corresponding maximum temperature of the rod

Schematic:



- Assumptions:
- 1-D conduction
  - Steady state
  - uniform heat generation
  - constant properties
  - negligible radiation

Analysis: a) Equivalent thermal circuit. Length of the rod is unknown, thus heat flow and resistances will be per unit length ( $q'$  and  $R'$ )



b) Determination of  $\dot{q}_v$  if  $T_2 = 100^{\circ}\text{C}$

From thermal circuit:

$$q' = \frac{T_2 - T_{\infty}}{R'_{cond} + R'_{conv}} = \dot{q}_v \pi r_1^2 \Rightarrow \dot{q}_v = \frac{T_2 - T_{\infty}}{(R'_{cond} + R'_{conv}) \pi r_1^2}$$

Problem 2 - continued

$$R'_{\text{cond},s} = \frac{\ln(0.125/0.075)}{2\pi \cdot 8} = 1.017 \cdot 10^{-2} \frac{\text{m} \cdot \text{K}}{\text{W}}$$

$$R'_{\text{conv}} = \frac{1}{2\pi \cdot 0.125 \cdot 40} = 3.184 \cdot 10^{-2} \frac{\text{m} \cdot \text{K}}{\text{W}}$$

Sub-in numerical values:

$$\dot{q}' = \frac{100 - 20}{(1.017 \cdot 10^{-2} + 3.184 \cdot 10^{-2}) \pi \cdot (0.075)^2} = 1.065 \cdot 10^5 \frac{\text{W}}{\text{m}^3}$$

c) Determination of  $T_{\text{max},r}$

$T_{\text{max},r}$  occurs at the centre of the rod

For the solid circular rod, the temperature distribution is given by:

$$T(r) = \frac{\dot{q}' r_0^2}{4k} \left(1 - \frac{r^2}{r_0^2}\right) + T_s \quad \text{where } r=0, \quad r_0=r_1, \quad k=k_r$$

$$T_s = T_1 = ?$$

From thermal circuit:

$$\dot{q}' = \frac{T_1 - T_2}{R'_{\text{tot}}} \Rightarrow T_1 = \dot{q}' R'_{\text{tot}} + T_2$$

$$\text{where: } \dot{q}' = 1.065 \cdot 10^5 \cdot \pi (0.075)^2 = 1882 \frac{\text{W}}{\text{m}}$$

$$R'_{\text{tot}} = \frac{R'_{\text{tot}}}{2\pi r_0} = \frac{2 \cdot 10^{-4}}{2\pi \cdot 0.075} = 4.25 \cdot 10^{-4} \frac{\text{m} \cdot \text{K}}{\text{W}}$$

$$\text{Sub-in numerical values: } T_1 = 1882 \cdot 4.25 \cdot 10^{-4} + 100 = 102.8^\circ\text{C}$$

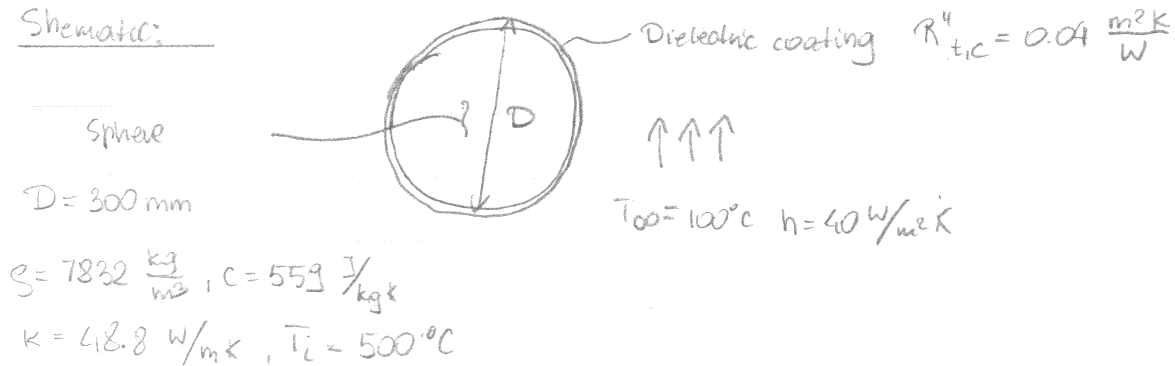
$$\text{Sub-in into temp distribution: } T_{\text{max},r} = T(0) = \frac{\dot{q}' r_0^2}{4k} + T_1$$

$$\therefore T_{\text{max},r} = \frac{1.065 \cdot 10^5 \cdot (0.075)^2}{4 \cdot 0.5} + 102.8 = 402.3^\circ\text{C}$$

Problem 3

Known: Solid steel sphere, coated with dielectric material of known thermal resistance.  
Coated sphere, initially at uniform temperature, is suddenly quenched in an oil bath

- Find:
- applicability of LTCM
  - time required for sphere centre to reach  $200^\circ\text{C}$
  - corresponding temperature of dielectric coating

Schematic:

- Assumptions:
- Dielectric coating has negligible thermal capacitance compared to sphere
  - Layer is very thin
  - Constant properties

Analysis

## a) Applicability of LTCM

Determination of Biot number  $Bi = \frac{R_{int}}{R_{ext}}$

Note that external resistance includes resistance to convection and thermal

resistance of coating, i.e.  $R_{ext} = R''_{t,c} + R''_{conv} \Rightarrow U = \frac{1}{R_{ext}}$

$$R_{ext} = 0.04 + \frac{1}{40} = 0.065 \frac{\text{K m}^2}{\text{W}} \Rightarrow U = 15.4 \frac{\text{W}}{\text{m}^2\text{K}}$$

$$\therefore Bi = \frac{U \cdot R}{k} = \frac{15.4 \cdot \frac{0.3}{2}}{48.8} = 0.047 < 0.1 \Rightarrow \text{yes LTCM is applicable}$$

b) Time for the center sphere temp to reach  $200^\circ\text{C}$ 

Since there is no heat generation

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = \exp\left(-\frac{UA}{\rho c V} t\right)$$

note: we use  $U$  instead of  $h$  because convection is not applicable

Problem 3- continued

$$\text{or: } \frac{T - T_{\infty}}{T_i - T_{\infty}} = \exp(-t/\tau) \quad \text{where } \tau = \frac{\rho C V}{AU} = \frac{\rho C \frac{\pi}{6} D^3}{\pi D^2 U} = \frac{\rho C D}{6U} = \frac{7832 \cdot 559 \cdot 0.3}{6 \cdot 15.4}$$

rearranging:

$$\therefore \tau = 14.215 \left[ \frac{\text{kg/m}^3 \cdot \text{J/kg} \cdot \text{m}}{\text{J/s m}^2 \text{K}} = \text{s} \right]$$

$$t = -\ln\left(\frac{T - T_{\infty}}{T_i - T_{\infty}}\right) \tau = -\ln\left(\frac{200 - 100}{500 - 100}\right) \cdot 14.215 = 19.706 \text{ s}$$

c) corresponding temp of coating

Coating provides a resistance (temperature drop) but no heat capacity; i.e. cannot store the energy; thus at any time:

$$\frac{T_s - T_c}{R_{t,c}''} = h(T_c - T_{\infty})$$

The only unknown is  $T_{s,0}$ . Rearranging:

$$(T_s - T_c) = h R_{t,c}'' (T_c - T_{\infty}) \Rightarrow T_c (1 + h R_{t,c}'') = T_s + h R_{t,c}'' T_{\infty}$$

$$\therefore T_{s,0} = \frac{T_{s,i} + h R_{t,c}'' T_{\infty}}{1 + h R_{t,c}''} = \frac{200 + 40 \cdot 0.04 \cdot 100}{1 + 40 \cdot 0.04} = 138.5^{\circ} \text{C}$$