

## Math 208, Class test, October 23, 2016

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Time: 1 Hour and 30 minutes

Answer all questions. Only approved calculators are allowed

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT \frac{(1+i)^n - 1}{i}, \quad PV = PMT \frac{1 - (1+i)^{-n}}{i}$$

1. (10 points) For a quadratic function

$$f(x) = 1.25x^2 - 3.75x + 2.2$$

Find

a)  $x$  and  $y$  intercepts algebraically.

b) The vertex form of  $f(x)$

c) The vertex and the minimum of  $f(x)$ .

2. (10 points) Solve the following equations for  $x$

$$(A) \quad 4^{\sqrt{x+1}} = 64 \cdot 2^{\sqrt{x+1}}$$

$$(B) \quad 2^{x^2-1} - 3^{x^2} = 3^{x^2-1} - 2^{x^2+2}$$

$$(C) \quad \log_{10}(5-x) + 2 \log_{10} \sqrt{3-x} = 1$$

$$(D) \quad \frac{\ln(x^2)}{\ln(6x-5)} = 1$$

3. (10 points)

(A) In the arithmetic sequence

$$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}$$

$a_3 = 7$  and  $a_6 = -2$ . Find

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}$$

(B) In the geometric sequence

$$b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}$$

$\frac{b_3}{b_1} = 4$  and  $b_{10} = 64$ . Find  $b_2$ .

4. (10 points)

(A) What is Annual Nominal Rate (compounded continuously) that gives Annual Percentage Yield 9% ?

(B) What is Annual Nominal Rate (compounded bi-monthly) if the Principle Amount doubles in 10 years ?

5. (10 points) Beginning in February 2017, you will be depositing 500\$ at the end of each two months period into an account earning 8% compounded bi-monthly. Find the interest earned during each year for the first four years.

6. (10 points) A family is thinking about buying a new house costing 380 000\$. They must pay 110 000\$ down and the rest is to be amortized over 25 years in equal monthly payments. If money costs 7% compounded monthly

(A) What will their monthly payment be?

(B) What will be unpaid balance after 20 years?

(C) How much total interest will be paid over the 25 years?

MATH 208 : MIDTERM TEST  
FALL 2016 : SOLUTION.

Q1.  $f(x) = 1.25x^2 - 3.75x + 2.2$   
 $a = 1.25, b = -3.75, c = 2.2$

a) x intercept set  $f(x) = 0$

$$1.25x^2 - 3.75x + 2.2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3.75 \pm \sqrt{(-3.75)^2 - 4(1.25)(2.2)}}{2(1.25)}$$

$$x = \frac{3.75 \pm 1.75}{2.5} = 2.2, 0.8$$

y intercept = 2.2

b) Vertex  $V(h, k), h = -\frac{b}{2a} = \frac{3}{2}$

$$k = \frac{4ac - b^2}{4a} = -0.6125$$

$$f(x) = 1.25(x - \frac{3}{2})^2 - 0.6125$$

c) Minimum at  $(\frac{3}{2}, -0.6125)$

$$\text{Min } f(\frac{3}{2}) = -0.6125$$

2A)  $(2^x)^{\sqrt{x+1}} = 2^6 \cdot 2^{\sqrt{x+1}} \Rightarrow 2^{2\sqrt{x+1}} = 2^6$

$$2\sqrt{x+1} = 6 \Rightarrow \sqrt{x+1} = 3$$

$$x+1 = 9 \Rightarrow x = 8$$

B)  $2^{x^2-1} + 2^{x^2-1} \cdot \frac{3}{2} = 3 + 3 \cdot \frac{3}{2}$

$$2^{x^2-1} (1 + \frac{3}{2}) = 3 \cdot \frac{5}{2}$$

$$\left(\frac{2}{3}\right)^{x^2-1} = \frac{4}{9} \Rightarrow \left(\frac{2}{3}\right)^{x^2-1} = \left(\frac{2}{3}\right)^2$$

$$x^2 - 1 = 2 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

C)  $\log_{10}(5-x)(3-x) = 1$

$$(5-x)(3-x) = 10$$

$$x^2 - 8x + 5 = 0 \Rightarrow x^2 - 8x + 5 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 20}}{2} = 4 \pm \sqrt{11}$$

D)  $\ln x^2 = \ln(6x-1)$

$$\ln\left(\frac{x^2}{6x-1}\right) = 0 \Rightarrow \frac{x^2}{6x-1} = 1$$

$$x^2 - (6x-1) = 0$$

$$(x-5)(x-1) = 0$$

$$x = 1, 5$$

3 A.  $a_n = a_1 + (n-1)d$

$$a_5 = a_1 + 4d \Rightarrow a_1 + 4d = -2$$

$$a_3 = a_1 + 2d \Rightarrow a_1 + 2d = 7$$

$$3d = -9$$

Thus:  $d = -3, a_1 = 13$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d] \Rightarrow S_{10} = 5[26 - 27] = -5$$

$$S_{10} = -5$$

3 B)  $b_n = b_1 r^{n-1} \Rightarrow \frac{b_3}{b_1} = 4 \Rightarrow r^2 = 4 = 4$

$$r = \pm 2 \text{ but } b_{10} = b_1 r^9 = 64$$

$$b_1 = \frac{64}{(\pm 2)^9} = \pm \frac{1}{8} \Rightarrow b_2 = (\pm \frac{1}{8})(\pm 2)$$

$$b_2 = (-\frac{1}{8})(-2) = \frac{1}{4}$$

4 A)  $APY = e^r - 1 \Rightarrow 0.09 = e^r - 1$

A)  $e^r = 1.09 \Rightarrow r = \ln(1.09) =$

B)  $m = 6, t = 10, r = p, A = 2P, r =$

Formula  $A = P(1 + \frac{r}{m})^{mt}$   
 $2P = P(1 + \frac{r}{6})^{60} \Rightarrow (1 + \frac{r}{6}) = 2^{\frac{1}{50}}$   
 $\frac{r}{6} = 2^{\frac{1}{50}} - 1 \Rightarrow r = 6[2^{\frac{1}{50}} - 1] = 6.97\%$

5:  $m = 6, r = 0.08, i = 0.01333, PMT = 450$

Yr 1:  $FV = \frac{PMT[(1+i)^n - 1]}{i} = \$3101.87$

Interest Yr 1 =  $3101.87 - 3000 = \$101.87$

Yr 2:  $FV = \frac{500[(1.0133)^{12} - 1]}{0.0133} = 6458.58$

Int Yr 2 =  $6458.58 - 3000 - 3101.87 = \$356.71$

Int Yr 3 =  $\$636.23, \text{ Int Yr 4} = \$936.89$

#6  $PV = \$170,000, i = 0.07, t = 12, r = 0.0553$   
 $PMT = \frac{PV \cdot i}{1 - (1+i)^{-n}} = \frac{170,000 \cdot 0.0553}{1 - (1.0553)^{-12}} = \$1908.30$   
 Unpaid balance 20yr =  $PMT[1 - (1+i)^{-60}] = 1908.30[1 - (1.0553)^{-60}] = \$96,268.92$   
 Total Int Paid =  $300 \times 1908.30 - 270,000 = \$302,490$

