Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT1320B | **Test 2** | **20 November 2017**

Calculus I | **Elizabeth Maltais**

LAST NAME: ______________________

FIRST NAME: ______________________

STUDENT NUMBER: ______________________

SOLUTIONS VERSION B

DGD (circle yours):	Oleksii	Reyhaneh	Oleksii	Dina	Hayley
	10:00–11:30	11:30–13:00	13:00–14:30	14:30–16:00	16:00–17:30
	FTX 135	SMD 422	FTX 359	SMD 227	UCU 205

- No calculators are permitted. No notes, books, papers or any other aids.
- Print your name and student number on this page.
- Verify that your copy of the test has all 7 pages (including this one).
- Write your solutions in the space provided (use the backs of the pages if necessary). You must show all of your work.
- It is forbidden to use or have in your possesion a cellular telephone or other electronic device. Turn off your devices and put them in your bag.
- Sign below to acknowledge that you have read these instructions.

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- Do not write below this line.

1	2	3	4	5	6	7	total
/3	/2	/2	/2	/9	/6	/6	/30

[3] **1.** Let $f(x) = \sqrt{x^2+8}$. Estimate $f(1.03)$, using the linearization $L(x)$ of the function $f(x)$ at the point $a = 1$.

$$f'(x) = \frac{1}{2}(x^2+8)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+8}}$$

at $a=1$

$$f(1) = \sqrt{1^2+8} = 3$$

$$f'(1) = \frac{1}{\sqrt{1^2+8}} = \frac{1}{3}$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$\Rightarrow L(x) = 3 + \frac{1}{3}(x-1)$$

$$f(1.03) \approx L(1.03) = 3 + \frac{1}{3}(1.03-1) = 3 + \frac{1}{3}(0.03) = 3.01$$

[2] **2.** Give the Riemann sum using $n = 4$ rectangles for $\int_0^2 e^{2x}\,dx$. Use right-hand heights for the rectangles. You should give an explicit formula, but you do not need to evaluate it numerically. You may use sigma-notation if you wish.

$\int_a^b f(x)dx$ $\quad \int_0^2 e^{2x}dx$ $\quad$ so $b=2$, $a=0$ $\quad f(x)=e^{2x}$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = 0.5 \qquad x_i^* = x_i = a + i\Delta x = 0 + 0.5i$$

$$R_4 = \sum_{i=1}^{4} f(x_i^*)\Delta x = \Delta x\left[f(0.5) + f(1) + f(1.5) + f(2)\right]$$

$$\therefore \quad R_4 = 0.5\left[e^{2(0.5)} + e^{2(1)} + e^{2(1.5)} + e^{2(2)}\right]$$

[2] **3.** A particle moves along a straight line, with velocity $v(t) = 3t - 4$. Find the net displacement between $t = 1$ and $t = 2$.

Net displacement from $t=1$ to $t=2 = \int_1^2 v(t)\,dt$

$= \int_1^2 (3t-4)\,dt$

$= \left.\frac{3t^2}{2} - 4t\right]_1^2$

$= \frac{3(2^2)}{2} - 4(2) - \left(\frac{3(1^2)}{2} - 4(1)\right)$

$= 6 - 8 - \frac{3}{2} + 4$

$= \frac{1}{2}$

[2] **4.** Evaluate $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_x^{x^2} \sin(t)\,\mathrm{d}t$.

$= \frac{d}{dx}\left[\int_x^0 \sin(t)dt + \int_0^{x^2} \sin(t)dt\right]$

$= \frac{d}{dx}\left[-\int_0^x \sin(t)dt + \int_0^{x^2} \sin(t)dt\right]$

$= -\frac{d}{dx}\int \sin(t)dt + \frac{d}{dx}\int_0^{u(x)} \sin(t)dt$ where $u = x^2$, $\frac{du}{dx} = 2x$

$= -\sin(x) + \sin(u)\cdot\frac{du}{dx}$ (chain rule)

$= -\sin(x) + 2x\sin(x^2)$

[9] **5.** Evaluate each of the following definite integrals.

a) $\int_1^2 e^{3t+1}\,\mathrm{d}t$ $= \frac{1}{3}e^{3t+1}\Big]_1^2$

$= \frac{1}{3}e^{3(2)+1} - \frac{1}{3}e^{3(1)+1}$

$= \frac{1}{3}e^7 - \frac{1}{3}e^4$

$u = x^3 + 1$

$\frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$

b) $\int_0^1 \frac{x^5}{(x^3+1)^{11}}\,\mathrm{d}x$

$\int$ $x=1 \Rightarrow u = 1^3 + 1 = 2$, $x=0 \Rightarrow u = 0^3 + 1 = 1$

$= \int_1^2 \frac{x^5}{u^{11}} \cdot \frac{du}{3x^2}$

$= \int_1^2 \frac{1}{3} x^3 (u^{-11}) du$ Note since $u = x^3 + 1$ it means $x^3 = u - 1$

$= \frac{1}{3}\int_1^2 (u-1)u^{-11}\,du$

$= \frac{1}{3}\int_1^2 u^{-10} - u^{-11}\,du$

$= \frac{1}{3}\left[\frac{1}{-9}u^{-9} - \left(\frac{1}{-10}u^{-10}\right)\right]_1^2$

$= \frac{1}{3}\left[\left(-\frac{1}{9\cdot 2^9} + \frac{1}{10\cdot 2^{10}}\right) - \left(-\frac{1}{9\cdot 1^9} + \frac{1}{10\cdot 1^{10}}\right)\right]$

c) $\displaystyle\int_1^2 \frac{\cos\left(\sqrt{2x-1}\right)}{\sqrt{2x-1}}\,\mathrm{d}x$

$u = \sqrt{2x-1}$

$\frac{du}{dx} = \frac{1}{2}(2x-1)^{-1/2}(3) = \frac{2}{2\sqrt{2x-1}}$

$\Rightarrow dx = \sqrt{2x-1}\,du$

$x=2 \Rightarrow u = \sqrt{2(2)-1} = \sqrt{3}$

$x=1 \Rightarrow u = \sqrt{2(1)-1} = 1$

$$= \int_1^{\sqrt{3}} \frac{\cos(u)}{\sqrt{2x-1}}\left(\sqrt{2x-1}\right)du$$

$$= \int_1^{\sqrt{3}} \cos(u)\,du$$

$$= \Big[\sin(u)\Big]_1^{\sqrt{3}}$$

$$= \sin(\sqrt{3}) - \sin(1)$$

[6] **6.** Determine the indefinite integrals. You must show your work!

a) $\int x^3 \ln(x^2)\,\mathrm{d}x$

Parts: $u = \ln(x^2)$ $\quad v' = x^3$

$u' = \frac{2}{x}$ $\quad v = \frac{x^4}{4}$

$= \ln(x^2)\left(\frac{x^4}{4}\right) - \int \left(\frac{2}{x}\right)\left(\frac{x^4}{4}\right)dx$

$= \frac{x^4}{4}\ln(x^2) - \frac{1}{2}\int x^3 dx$

$= \frac{x^4}{4}\ln(x^2) - \frac{1}{2}\left(\frac{x^4}{4}\right) + C$

b) $\underbrace{\int e^{-x}\sin(3x)\,\mathrm{d}x}_{I}$

Parts #1

$u = e^{-x}$ $\quad v' = \sin(3x)$

$u' = -e^{-x}$ $\quad v = -\frac{1}{3}\cos(3x)$

$= e^{-x}\left(-\frac{1}{3}\cos(3x)\right) - \int(-e^{-x})\left(-\frac{1}{3}\cos(3x)\right)dx$

$= -\frac{1}{3}e^{-x}\cos(3x) - \frac{1}{3}\int e^{-x}\cos(3x)\,dx$

Parts #2

$u = e^{-x}$ $\quad v' = \cos(3x)$

$u' = -e^{-x}$ $\quad v = \frac{1}{3}\sin(3x)$

$= -\frac{1}{3}e^{-x}\cos(3x) - \frac{1}{3}\left[e^{-x}\left(\frac{1}{3}\sin(3x)\right) - \int -e^{-x}\left(\frac{1}{3}\sin(3x)\right)dx\right]$

$\Rightarrow I = -\frac{1}{3}e^{-x}\cos(3x) - \frac{1}{9}e^{-x}\sin(3x) - \frac{1}{3}I$

$\Rightarrow \quad I = \frac{3}{4}\left[-\frac{1}{3}e^{-x}\cos(3x) - \frac{1}{9}e^{-x}\sin(3x)\right] + C$

[6] **7.** Determine the indefinite integrals. You must show your work!

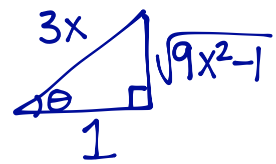
a) $\displaystyle\int \frac{\mathrm{d}x}{x^2\sqrt{9x^2-1}}$

$9x^2-1=(3x)^2-1$ like $\sec^2\theta-1$

$\hookrightarrow 3x=\sec\theta$

$\Rightarrow x=\frac{1}{3}\sec\theta$

$\Rightarrow dx=\frac{1}{3}\sec\theta\tan\theta\,d\theta$



$$=\int\frac{\frac{1}{3}\sec\theta\tan\theta\,d\theta}{(\frac{1}{3}\sec\theta)^2\sqrt{\sec^2\theta-1}}$$

$$=\int\frac{\frac{1}{3}\sec\theta\tan\theta\,d\theta}{\frac{1}{9}\sec^2\theta\sqrt{\tan^2\theta}}$$

$$=3\int\frac{\sec\theta\tan\theta}{\sec^2\theta\tan\theta}\,d\theta$$

$$=3\int\frac{1}{\sec\theta}\,d\theta$$

$$=3\int\cos\theta\,d\theta$$

$$=3\sin\theta+C$$

$$=3\,\frac{\sqrt{9x^2-1}}{3x}+C$$

b) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{2x^2+4x+20}}$

$2x^2+4x+20=2(x^2+2x+10)$

$=2(x^2+2x+1-1+10)$

$=2((x+1)^2+9)$

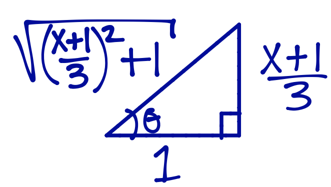
$=2\left(9\left(\frac{x+1}{3}\right)^2+9\right)$

$=18\left[\left(\frac{x+1}{3}\right)^2+1\right]$ like $18(\tan^2\theta+1)$

$\hookrightarrow \frac{x+1}{3}=\tan\theta$

$\Rightarrow x=3\tan\theta-1$

$\Rightarrow dx=3\sec^2\theta\,d\theta$



$$=\int\frac{3\sec^2\theta\,d\theta}{\sqrt{18(\tan^2\theta+1)}}$$

$$=\int\frac{3\sec^2\theta\,d\theta}{\sqrt{18}\sqrt{\tan^2\theta+1}}$$

$$=\frac{1}{\sqrt{2}}\int\frac{\sec^2\theta\,d\theta}{\sqrt{\sec^2\theta}}$$

$$=\frac{1}{\sqrt{2}}\int\sec\theta\,d\theta$$

$$=\frac{1}{\sqrt{2}}\ln|\sec\theta+\tan\theta|+C$$

$$=\frac{1}{\sqrt{2}}\ln\left|\sqrt{\left(\frac{x+1}{3}\right)^2+1}+\frac{x+1}{3}\right|+C$$