

What follows is a list of types of problems you should be able to solve for the midterm. For derivatives, I have given a list of functions that you should be able to differentiate. On the final page, solutions are provided. It goes without saying that you should attempt to solve these problems in earnest before you look at the solutions. You should be able to:

1. compute the derivatives of the following functions:

i $f(x) = 3^{\ln(x)}$

ii $f(x) = x^x$

iii $f(x) = \sin(\ln(x^5 \arcsin(x)))$

iv $f(x) = \arctan(e^{4x} + xe^x)$

v $f(x) = \sqrt{x \ln(x)}$

vi $f(x) = \frac{\arcsin(x) + x^2}{(x-1)^3}$

2. find an equation of a line that's tangent to a given function at a given point.

3. use implicit differentiation to find an explicit expression for $\frac{dy}{dx}$ for equations which do not give y explicitly in terms of x

4. perform all of the steps necessary to graph a function (find intercepts, domain, asymptotes, critical points, intervals of increase/decrease, concavity, inflection points). You should have the mindset that even when you are not asked to graph functions, all of this analysis can extract useful information.

5. state the definition of the derivative and use it to find a derivative of a relatively simple function (polynomials, rational functions, and root functions are all fair game)

6. state the product rule, chain rule, and quotient rule (but you do NOT need to prove them!)

7. find the equilibrium points of a discrete-time dynamical system, which is not necessarily linear.

8. solve basic optimization problems (optimize fenced-in areas, optimize a given function, find the minimal distance between a point and a curve, etc...)

Computed derivatives:

$$\text{i } f'(x) = \frac{\ln(3)3^{\ln(x)}}{x}$$

$$\text{ii } f'(x) = x^x \ln(x) + x^x$$

$$\text{iii } f'(x) = \cos(\ln(x^5 \arcsin(x))) \frac{5x^4 \arcsin(x) + \frac{x^5}{\sqrt{1-x^2}}}{x^5 \arcsin(x)}$$

$$\text{iv } f'(x) = \frac{1}{1+(e^{4x} + xe^x)^2} (4e^{4x} + e^x + xe^x)$$

$$\text{v } f'(x) = \frac{\ln(x)+1}{2\sqrt{x\ln(x)}}$$

$$\text{vi } f'(x) = \frac{(\frac{1}{\sqrt{1-x^2}} + 2x)(x-1)^3 - 3(x-1)^2(\arcsin(x)+x^2)}{(x-1)^6}$$