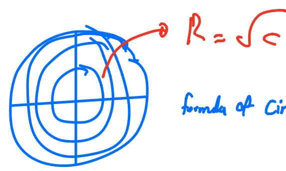
Exact equation

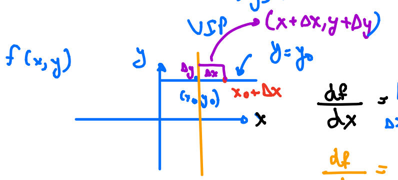
$f(x,y) = c$ (level line of $f(x,y)$)

Example: $f(x,y) = x^2 + y^2$

$f(x,y) = c$



formula of circle depending on circle.



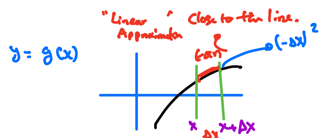
$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{df}{dy} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

$$\text{USP} = f(x,y) + \frac{df}{dx}\cdot \Delta x + \frac{df}{dy} + \Delta y + \text{Error}.$$

is small relative to Δx

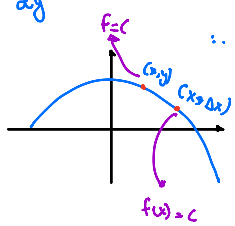
$$\approx f(x,y) + \frac{df}{dx}\cdot \Delta x + \frac{df}{dy}\cdot \Delta y$$



$$f(x+\Delta x) - f(x) \approx \frac{df}{dx}\,\Delta x$$

$$\frac{df}{dx}\Delta x + \frac{df}{dy}\Delta y = df(x,y)(\Delta x, \Delta y)$$ is called a first differential of $f(x,y)$.

$f(x,y) = c$

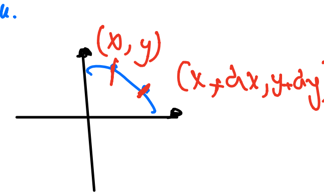


∴ The movement between the 2 points is 0.

$$f(x+\Delta x, y+\Delta y) - f(x,y)$$
$$\approx \frac{df}{dx}(x,y)\cdot \Delta x + \frac{df}{dy}(\ldots)\,\Delta y$$
$$\frac{df}{dx}\Delta x + \frac{df}{dy}\Delta y \approx 0$$

$dx \cdot dy$ uninfinitely small.

$$\frac{df}{dx}dx + \frac{df}{dy}dy = 0$$



$$\frac{dy}{dx} = \frac{\frac{df}{dx}}{\left(\frac{df}{dy}\right)}$$

function f is constant along the line.

Example: $f(x,y) = x^2 + y^2$

$$\frac{df}{dx} = 2x \quad \Rightarrow \quad 2x\,dx + 2y\,dy = 0$$

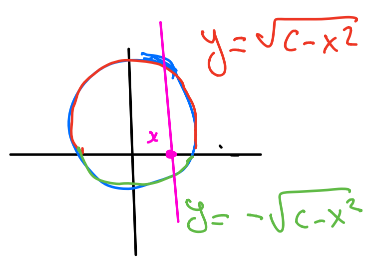
$$\frac{df}{dy} = 2y \qquad \frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y} \leftarrow \text{Seprapel equation}$$

$$\int x \cdot dx = -\int y \cdot dy + c$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + c$$

Circule equation $\boxed{x^2+y^2} = \cancel{2}c$ $\quad$ $2c = c$ because it a constant.

$R = \sqrt{c}$



$x \cdot dx + y\,dy = 0$ ∴ How to find equation because we not given the which is function of which

$x^2 + y^2 = c$

$\underline{\underline{M}}(x,y)\,dx + \underline{\underline{N}}(x,y)\,dy = 0$ ∴ If these exsit a function $f(x,y)$ Such that

Are giving functions.

$$M(x,y) = \frac{df}{dx}, \quad N(x,y) = \frac{df}{dy}$$

then the Solution of equation (x) has a form

$$f(x,y) = C$$

where C is arbiortary Constant.

Example 1 - given $M(x,y)$, $N(x,y)$, How to learn that these exsist $f(x,y)$

$$M = \frac{df}{Dt}, \quad N = \frac{df}{dy}$$

Suppose $M(x,y) = \frac{df}{dx}$

$$N(x,y) = \frac{df}{dy}$$

Find $\frac{dM}{dy}$ and $\frac{dN}{dx}$

$$\frac{dM}{dy} = \frac{d}{dy}\left(\frac{df}{dx}\right)$$

$$\frac{dN}{dx} = \frac{d}{dx}\left(\frac{df}{dy}\right)$$

The

If f, $\frac{df}{dx}$, $\frac{df}{dy}$

$\frac{d}{dx}\left(\frac{df}{Dx}\right)$, $\frac{d}{dy}\left(\frac{df}{dx}\right)$ are continous

then

$$\frac{d}{dx}\left(\frac{df}{dy}\right) = \frac{d}{dy}\left(\frac{df}{dx}\right)$$

If $f(x,y)$ exsist Such that

$$M(x,y) = \frac{df}{dx}, \quad N(x,y) = \frac{df}{dy}$$

Then

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$$x\frac{dx}{dy} + y\,dy = 0$$

$$M(x,y) = x$$

$$N(x,y) = y$$

$$\frac{dM}{dy} = 0, \quad \frac{dN}{dx} = 0$$

$$y(dx) - x\,dy = 0$$

$m = y$ $\qquad$ $N = -x$

$$\frac{dM}{dy} = 1$$

$$\frac{dN}{dx} = -1$$

Quiz Explination

$$\frac{dy}{dx} = 2y + e^{-x} + 1 \qquad y(1) = 0$$

$$\frac{dy}{dx} - 2y = e^{-x} + 1$$

$P(x) = -2$

$m(x) = e^{\int P(x)\cdot dx} = e^{\int -2\,dx} = e^{-2x}$

$e^{-2x}\frac{dy}{dx} - 2y = e^{-3x} + e^{-2x}$

$\frac{d}{dx}\left(e^{-2x}\cdot y\right) = e^{-3x} + e^{-2x}$

$e^{-2x} y = -\frac{1}{3}e^{-3x} - \frac{1}{2}e^{-2x} + C$

$y = -\frac{1}{3}e^{-x} - \frac{1}{2} + Ce^{2x}$

$y(1) = 0$

$-\frac{1}{3}e^{-1} - \frac{1}{2} + C = 0$

$C = \frac{1}{3}e^{-1} + \frac{1}{2}$

$y = -\frac{1}{3}e^{-x} - \frac{1}{2} + \left(\frac{1}{3}e^{-1} + \frac{1}{2}\right)e^{2x}$