CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2010	2
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Special Instructions:	Only Sharp EL 531 or Casio FX 300 MS calculators are allowed	

MARKS

[8] **1.** **(a)** Sketch the graph of $f(x) = 2^{-|x|}$, and approximate the area between the graph $y = f(x)$ and the x-axis on the interval $[-2, 2]$ by the midpoint Riemann sum using partitioning of the interval into four subintervals of equal length.

(b) Calculate the derivative of the function $F(x) = \int_{\sin(x)}^{0} \sqrt{1+t^4}\, \mathrm{d}t$ and determine whether the slope of $F(x)$ is positive or negative at $x = 0$ (HINT: *use the Fundamental Theorem of Calculus to differentiate* F)

[10] **2.** Calculate the following indefinite integrals:

(**a**) $\displaystyle\int \frac{x^2}{x^2-4}\,\mathrm{d}x$ (**b**) $\displaystyle\int (x+1)\,\ln x\,\mathrm{d}x$

[12] **3.** Find the antiderivative $F(t)$ of the function $f(t)$ passing through the given point:

(**a**) $f(t) = \left(t + \dfrac{1}{t}\right)^2$, $F(1) = 0$. (**b**) $f(t) = \dfrac{e^t}{1+e^{2t}}$, $F(0) = \dfrac{\pi}{2}$.

[12] **4.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^1 (x^2-1)\,\sin(\pi x)\,\mathrm{d}x$ (**b**) $\displaystyle\int_0^{\frac{\pi}{2}} \sin(x)\,\sin(2x)\,\mathrm{d}x$

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[5] **5.** Evaluate the improper integral $\int_1^\infty \frac{\mathrm{d}x}{x \ln x}$ or show that it diverges.

[18] **6.** **(a)** Sketch the curves $y = \sin x$ and $y = \sin(2x)$ on the interval $[0, \pi]$ and find the area enclosed by the curves on this interval.
(HINT: find first the point of intersection of the curves between $x = 0$ and $x = \pi$.)

(b) Sketch the curves defined by $x = y^2$ and $y = \frac{x}{2}$ and find the volume of a solid of revolution of the region bounded by these curves about the y-axis.

(c) Find the average value of the function $f(x) = \frac{x}{\sqrt{1+x}}$ on the interval $[0, 3]$.

[10] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(**a**) $a_n = \dfrac{n^2 + \cos(\pi n)}{\sqrt{1 + 4n^4}}$ (**b**) $a_n = \ln(n) - \ln(n+2)$

[13] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n}{1 + n^2}$ (**b**) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+3}}{2^{2n}}$ (**c**) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$

[5] **9.** Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n+1}\,.$$

[7] **10.** Find **(a)** the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{4^n}$$

(b) within this radius, the sum of the series as a function of x:

[5] **Bonus Question.** Let $f(x) = \sqrt{4x - x^2}\,.$

(**a**) Determine the domain $[a, b]$ of f and graph this function.

(**b**) Calculate the definite integral of f over its domain: $\int_a^b f(x)\,\mathrm{d}x$.