

Assignment 6:
KINEMATICS 1-D
Motion

Assigned: Oct20 Due: Oct 30 12:00 sharp!

- 1 A fast car, driving at 30.0 m/s, enters a one-lane tunnel. The driver observes a slow-moving truck 140 m ahead traveling at 6.00 m/s. She applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van. (4p)
Take the original moment in time to be when car driver notices the van. Choose the origin of the x-axis as the position of the car at that moment. We have $x_{is} = 0$, $v_{is} = 30.0 \text{ m/s}$, $a_s = -2.00 \text{ m/s}^2$ so car's position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)t^2.$$

For the van, $x_{iv} = 140 \text{ m}$, $v_{iv} = 6.00 \text{ m/s}$, $a_v = 0$ and $x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 140 + (6.00 \text{ m/s})t$.

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 140 + 6.00t_c \Rightarrow 0 = t_c^2 - 24.0t_c + 140.$$

From the quadratic formula

$$t_c = \frac{24.0 - \sqrt{(24.0)^2 - 4(140)}}{2} = \frac{24 - 4}{2} = 10.0 \text{ s} \quad \text{or} \quad t_c = \frac{24.0 + \sqrt{(24.0)^2 - 4(140)}}{2} = \frac{24 + 4}{2} = 14.0 \text{ s}$$

The smaller value is the collision time.

(The larger value tells when the van would rear end the car again if the vehicles could move through each other). The wreck happens at position

$$140 \text{ m} + (6.00 \text{ m/s})(10 \text{ s}) = \boxed{200 \text{ m}}.$$

Collision takes place at 200m
From the moment the car's driver applies the brakes

2. The height of a helicopter above the ground is given by $h = 2.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground? (4p)

SOLUTION: We need to find the position and velocity of the helicopter at $t=2\text{s}$ (moment when the bag is released)

$$y = 2.00t^3: \text{ At } t = 2.00 \text{ s}, y = 2.00(2.00)^3 = 16.0 \text{ m and}$$

$$v_y = \frac{dy}{dt} = 6.00t^2 = 24.0 \text{ m/s } \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_{i}t - \frac{1}{2}gt^2 = 16.0 + 24.0t - \frac{1}{2}(9.80)t^2.$$

$$\text{Setting } y_b = 0, \quad 0 = 16.0 + 24.0t - 4.90t^2.$$

Solving for t , (only positive values of t count), $t = \boxed{5.49 \text{ s}}$.

- 3 Livingston's Cheetah can reach 115km/h in 2 s and maintain this speed for 16 s. After this time, it must rest. An antelope can reach 90km/h in 2 s and sustain it for a long time. Suppose they are initially separated by 100m and the antelope reacts in 0.5s.

a) can cheetah get the antelope?

b) if not, how close does it get?

Assume both start from the rest.

/Provide full solution on opposite page/

Assignment 6: KINEMATICS

1-D Motion CONT

- 4 Two railroad tracks intersect at right angles at station O. At 10AM the train A, moving west with constant speed of 50 km/h, leaves the station O. One hour later train B, moving south with the constant speed of 60 km/h, passes through the station O. Find minimum distance between these trains. :

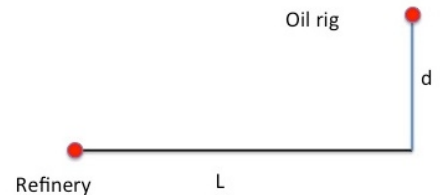
$$D(t) = \sqrt{(v_A t)^2 + (60 - v_B t)^2} \Rightarrow \frac{dD}{dt} = 0 \Rightarrow \frac{2v_A^2 t + 2(60 - v_B t)(-v_B)}{2\sqrt{(v_A t)^2 + (60 - v_B t)^2}} = 0$$

$$\frac{2v_A^2 t + 2(60 - v_B t)(-v_B)}{2\sqrt{(v_A t)^2 + (60 - v_B t)^2}} = 0 \Rightarrow v_A^2 t + (60 - v_B t)(-v_B) = 0 \Rightarrow 2500t - 3600 + 3600t = 0$$

$$t = \frac{3600}{6100} = \frac{36}{61} h = 35 \text{ min } 24.6s \Rightarrow D = \sqrt{(v_A t)^2 + (60 - v_B t)^2} = 38.41 \text{ km}$$

- 5 John who is member of NGO missed the meeting of his protest group at the refinery and now needs to get to the oil rig in the shortest time to join the demonstrators trying to disrupt the work of the petroleum company. John can run at 10km/h but can paddle only 3km/h.

- a) How far from the Refinery should John enter the water
b) What is the minimum time it will take to get to the oil rig?
L=12km, d=4km



While on land John's speed is $v_L=10\text{km/h}$. He will run at that speed the $L-x$ distance.

It will take him time t_L given by $t_L = \frac{L-x}{v_L} = \frac{12-x}{10}$ hr

The distance he will need to swim with speed v_W is $\sqrt{d^2 + x^2}$ it will take him $t_W = \frac{\sqrt{d^2 + x^2}}{v_W} = \frac{\sqrt{4^2 + x^2}}{3}$ hr

The total time to get from the R to OR is $t_L + t_W = \frac{L-x}{v_L} + \frac{\sqrt{d^2 + x^2}}{v_W} = \frac{12-x}{10} + \frac{\sqrt{4^2 + x^2}}{3}$

We need to find the entry point (coordinate x along the shore) such that this time is minimum

$$\frac{d(t_L + t_W)}{dx} = -\frac{1}{v_L} + \frac{1}{v_W} \frac{2x}{2\sqrt{d^2 + x^2}} = 0$$

$$-\frac{1}{10} + \frac{1}{3} \frac{2x}{2\sqrt{16+x^2}} = 0$$

$$\frac{1}{3} \frac{x}{\sqrt{16+x^2}} = \frac{1}{10} \Rightarrow 10x = 3\sqrt{16+x^2} \Rightarrow 100x^2 = 9(16+x^2) \Rightarrow 91x^2 = 144 \Rightarrow x = \sqrt{\frac{144}{91}} = 1.26(\text{km})$$

So the point at which John needs to enter the water is 10.74km away from refinery.

It will take

$$t_L + t_W = \frac{L-x}{v_L} + \frac{\sqrt{d^2 + x^2}}{v_W} = \frac{10.74}{10} + \frac{\sqrt{4^2 + 144/91}}{3} = 2.47 \text{ hr}$$

$$V(\text{cheetah}) = 31.94 \text{ m/s} \quad a(\text{cheetah}) = 15.98 \text{ m/s}^2$$

$$V(\text{antelope}) = 25 \text{ m/s} \quad A(\text{antelope}) = 12.5 \text{ m/s}^2$$

The easiest way to solve this problem is to find the moment of time when both animals move with constant speed.

This is 2.5 seconds from the moment cheetah started its chase.
 During this time cheetah accelerated for 2 seconds and then run with constant speed for 0.5 second.
 It covered the distance of $31.96 + 15.97 = 47.93$ meters

The antelope was initially not moving (0.5s) and then it accelerated for 2s. It run 25 meters away from the cheetah.
 After 2.5 seconds the animals are separated by $100 \text{ m} - 47.93 \text{ m} + 25 \text{ m} = 77.07 \text{ m}$

We are dealing with much simpler situation:
 Cheetah is separated from antelope by 77.07m it moves with constant speed of 31.94m/s vs antelope 25m/s. Question is will it get to the antelope within 15.5seconds?

$$X(\text{cheetah}) = 0 + 31.94(15.5) = 495.07 \text{ m}$$

$$X(\text{antelope}) = 77.07 + 25(15.5) = 464.57 \text{ m}$$

ANS: Since cheetah can cover longer distance while running for 15.5seconds it will get the antelope before the end of its run.
 What follows was asked in the problem but will satisfy our curiosity necessary
 To find the exact moment of time when this happens one needs to set up the equation for the $X(t)$ for cheetah and antelope and find t for which these two are equal

$$X_{\text{cheetah}}(t) = X_{\text{antelope}}(t)$$

$$31.95t = 77.07 + 25t$$

$$6.95t = 77.07$$

$$t = 11.09 \text{ (s)}$$

The cheetah will snatch the antelope at $t = 11.09 \text{ s}$ from the moment it started its chase.