Assignment 5: ENTROPY and KINEMATICS 1-D Motion
Assigned: Oct 14 Due: Oct 20 6:00PM

UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/1331 FALL 2017
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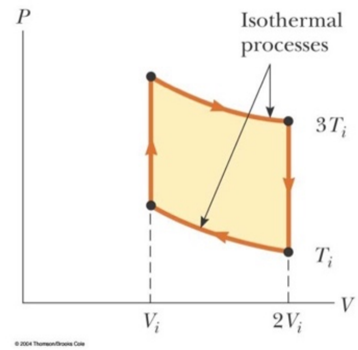
STUDENT #: ________________

NAME: ____________________

1 In 1816 Robert Stirling, a Scottish clergyman, patented the *Stirling engine*, which has found a wide variety of applications ever since. Fuel is burned externally to warm one of the engine's two cylinders. A fixed quantity of inert gas moves cyclically between the cylinders, expanding in the hot one and contracting in the cold one. Figure below represents a model for its thermodynamic cycle. Consider *n* mol of an ideal monatomic gas being taken once through the cycle, consisting of two isothermal processes at temperatures $3T_i$ and T_i and two constant-volume processes. Determine, in terms of *n*, *R*, and T_i, (a) the net energy transferred by heat to the gas. (b) its efficiency.



SOLUTION:
A Stirling engine is easier to manufacture than an internal combustion engine or a turbine. It can run on burning garbage. It can run on the energy of sunlight and produce no material exhaust.

(a) For an isothermal process, $Q = nRT\ln\left(\frac{V_2}{V_1}\right)$

Therefore, $Q_1 = nR(3T_i)\ln 2$

and $Q_3 = nR(T_i)\ln\left(\frac{1}{2}\right)$

For the constant volume processes, $Q_2 = \Delta E_{int,2} = \frac{3}{2}nR(T_i - 3T_i)$

and $Q_4 = \Delta E_{int,4} = \frac{3}{2}nR(3T_i - T_i)$

The net energy by heat transferred is then

$Q = Q_1 + Q_2 + Q_3 + Q_4$

or $Q = \boxed{2nRT_i\ln 2}$.

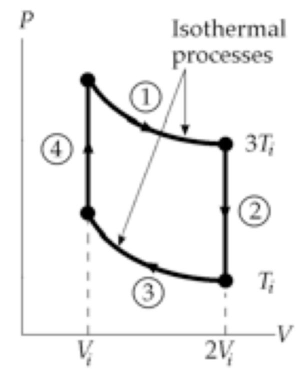


FIG. P22.57

(b) A positive value for heat represents energy transferred into the system.
Therefore, $|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$
Since the change in temperature for the complete cycle is zero,

$\Delta E_{int} = 0$ and $W_{eng} = Q$

Therefore, the efficiency is $e_c = \frac{W_{eng}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2\ln 2}{3(1+\ln 2)} = \boxed{0.273}$

2. An ice tray contains 500 g of liquid water at 0°C. Calculate the change in entropy of the water as it freezes slowly and completely at 0°C. (It is a one-line problem so please fit this into the space provided here)

$$\Delta S = \frac{mL}{T} = \frac{0.5 \cdot (3.3 \cdot 10^5)}{273}\frac{J}{K} = 604.40\frac{J}{K}$$

3 A 1.00-kg iron horseshoe is taken from a forge at 900°C and dropped into 4.00 kg of water at 10.0°C. Assuming that no energy is lost by heat to the surroundings, determine the total entropy change of the horseshoe-plus-water system.

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4 What change in entropy occurs when a 27.9-g ice cube at –12°C is transformed into steam at 115°C?

We assume a constant specific heat for each phase. As the ice is warmed from –12°C to 0°C, its entropy increases by

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\text{ K}}^{273\text{ K}} \frac{mc_{\text{ice}}dT}{T} = mc_{\text{ice}}\int_{261\text{ K}}^{273\text{ K}} T^{-1}dT = mc_{\text{ice}}\ln T\Big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S = 0.027\,0\text{ kg}\left(2\,090\text{ J/kg}\cdot{}^\circ\text{C}\right)\left(\ln 273\text{ K} - \ln 261\text{ K}\right) = 0.027\,0\text{ kg}\left(2\,090\text{ J/kg}\cdot{}^\circ\text{C}\right)\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.54\text{ J/K}$$

As the ice melts its entropy change is $\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{0.027\,0\text{ kg}\left(3.33\times10^5\text{ J/kg}\right)}{273\text{ K}} = 32.9\text{ J/K}$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{mc_{\text{liquid}}dT}{T} = mc_{\text{liquid}}\ln\left(\frac{T_f}{T_i}\right) = 0.027\,0\text{ kg}\left(4\,186\text{ J/kg}\cdot{}^\circ\text{C}\right)\ln\left(\frac{373}{273}\right) = 35.3\text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + mc_{\text{steam}}\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.027\,0\text{ kg}\left(2.26\times10^6\text{ J/kg}\right)}{373\text{ K}} + 0.027\,0\text{ kg}\left(2\,010\text{ J/kg}\cdot{}^\circ\text{C}\right)\ln\left(\frac{388}{373}\right) = 164\text{ J/K} + 2.14\text{ J/K}$$

The total entropy change is $(2.54 + 32.9 + 35.3 + 164 + 2.14)\text{ J/K} = \boxed{236\text{ J/K}}$.

5 A heat engine operates between two reservoirs at T_2 = 600 K and T_1 = 350 K. It takes in 1 000 J of energy from the higher-temperature reservoir and performs 250 J of work. Find (a) the entropy change of the Universe ΔS_U for this process and (b) the work W that could have been done by an ideal Carnot engine operating between these two reservoirs. (c) Show that the difference between the amounts of work done in parts (a) and (b) is $T_1 \Delta S_U$.

a) $\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{-1000}{600} + \frac{750}{350}\frac{J}{K} = 0.476\frac{J}{K}$
b) e $= \frac{|W|}{|Q_h|} = \frac{|Q_h|-|Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$ so for the Carnot engine $e = 1 - \frac{|T_c|}{|T_h|} = \frac{5}{12}$ and thus
$W = \frac{5}{12}1000J = 416.667J \Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{-1000}{600} + \frac{583.33}{350}\frac{J}{K} = 0\frac{J}{K}$
Work difference is 166.67J, while $T_c\Delta S = T_1\Delta S = 350(0.47618) = 166.7\,J$

6 By algebraic manipulation of the first two kinematic equations for one-dimensional motion:

$$1)\,v_f = v_i + at \qquad 2)\,x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

Obtain the other two kinematic equations: $3)\,v_f^{\,2} - v_i^{\,2} = 2a\Delta x \qquad 4)\,x_f = x_i + \tfrac{1}{2}(v_i + v_f)t$

Q3 SOLUTION:

$c_{\text{iron}} = 448\ \text{J/kg}\cdot°\text{C}$; $c_{\text{water}} = 4\,186\ \text{J/kg}\cdot°\text{C}$ since $Q_{\text{cold}} = -Q_{\text{hot}}$:

we have $4.00\ \text{kg}(4\,186\ \text{J/kg}\cdot°\text{C})(T_f - 10.0°\text{C}) = -(1.00\ \text{kg})(448\ \text{J/kg}\cdot°\text{C})(T_f - 900°\text{C})$

which yields $T_f = 33.2°\text{C} = 306.2\ \text{K}$

$$\Delta S = \int_{283\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1173\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1\,173}\right)$$

$$\Delta S = (4\,186\ \text{J/kg}\cdot\text{K})(4.00\ \text{kg})(0.078\,8) + (448\ \text{J/kg}\cdot\text{K})(1.00\ \text{kg})(-1.34)$$

$$\Delta S = \boxed{718\ \text{J/K}}$$

Q6 SOLUTION:

$$1)\ v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)\ x_f = x_i + v_i\frac{(v_f - v_i)}{a} + \tfrac{1}{2}a\frac{(v_f - v_i)^2}{a^2} \Rightarrow x_f - x_i = v_i\frac{(v_f - v_i)}{a} + \tfrac{1}{2a}\left(v_f^2 - 2v_iv_f + v_i^2\right) \Rightarrow$$

$$\Rightarrow 2a(x_f - x_i) = 2v_i(v_f - v_i) + \left(v_f^2 - 2v_iv_f + v_i^2\right) \Rightarrow 2a(x_f - x_i) = 2v_iv_f - 2v_iv_i + v_f^2 - 2v_iv_f + v_i^2 \Rightarrow 2a(x_f - x_i) = v_f^2 - v_i^2$$

$$1)\ v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)\ x_f = x_i + v_it + \tfrac{1}{2}a\frac{(v_f - v_i)}{a}t \Rightarrow x_f = x_i + v_it + \tfrac{1}{2}(v_f - v_i)t \Rightarrow x_f = x_i + v_it + \tfrac{1}{2}v_ft - \tfrac{1}{2}v_it \Rightarrow x_f = x_i + \tfrac{1}{2}(v_f + v_i)t$$