UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/31 Fall 2018

STUDENT #: ____________________

NAME: __________________________

ASSIGNMENT 4:
Maxwell Boltzmann Distribution
Heat Engines,

Released: Oct 6, Due: Oct 13 6PM Sharp!

1 Given is 1mole of oxygen molecules at atmospheric pressure and temperature of 20^{o}C.

a) Find the number of molecules having their speed between 300m/s and 302m/s

$P(v)dv = 4\pi\left[\frac{1}{2\pi}\frac{m}{kT}\right]^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}}dv$ so the $N_v = N_A P(v)dv = N_A 4\pi\left(\frac{1}{2\pi}\frac{M}{RT}\right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}}dv$

$N_v = N_A P(v)dv = (6.02\cdot 10^{23})4\pi\left(\frac{1}{2\pi}\frac{0.032}{8.31\cdot 293}\right)^{\frac{3}{2}}(301)^2 e^{-\frac{0.032\cdot 301^2}{2\cdot 8.31\cdot 293}}(2)$=2.2864x10^{21}=2.29 x10^{21}

b) Find the most probable speed: $\sqrt{\frac{2kT}{m}} = \sqrt{\frac{2N_A kT}{mN_A}} = \sqrt{\frac{2RT}{M}}$ ANS: 390m/s

c) Find the number of molecules with the most probable speed (within 1 m/s from it) ANS: 2.56x10^{21}

d) Find the number of molecules with the speed of 1000m/s (999;1001) ANS: 6.41x10^{19}

e) Find the number of molecules with the speed of 2000m/s (1999:2001) ANS: 7.03x10^{11}

NOTE: the same calculations (with interval of 2m/s) lead to the maximum speed of 2880m/s in this gas.
(the number of N_2 molecules with speed larger than 2880 is less than 1!)

2 A refrigerator has a coefficient of performance of 3.00. The ice tray compartment is at –20.0°C, and the room temperature is 22.0°C. The refrigerator can convert 30.0 g of water at 22.0°C to 30.0 g of ice at –20.0°C each minute. What input power is required? Give your answer in watts.

$\text{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_C}{t} = (0.030\,0\text{ kg})(4\,186\text{ J/kg}\,^\circ\text{C})(22.0^\circ\text{C}) + (0.030\,0\text{ kg})(3.33\times 10^5\text{ J/kg}) + (0.030\,0\text{ kg})(2\,090\text{ J/kg}\,^\circ\text{C})(20.0^\circ\text{C}) = 1.40\times 10^4\text{ J/min}$$

or, $\frac{Q_c}{t} = 233\text{ J/s}$.

Thus, the work done per sec $= P = \frac{233\text{ J/s}}{3.00} = \boxed{77.8\text{ W}}$.

3 A heat engine operating between 200°C and 80.0°C achieves 20.0% of the maximum possible efficiency. What energy input will enable the engine to perform 10.0 kJ of work?

The Carnot efficiency of the engine is $e_c = \frac{\Delta T}{T_h} = \frac{120\text{ K}}{473\text{ K}} = 0.253$

At 20.0% of this maximum efficiency, $e = 0.200(0.253) = 0.050\,6$

From the definition of efficiency $W_{eng} = |Q_h|e$ and $|Q_h| = \frac{W_{eng}}{e} = \frac{10.0\text{ kJ}}{0.050\,6} = \boxed{197\text{ kJ}}$

4 Using Maxwell=Boltzmann Distribution of speeds for Ideal Gas obtain the Boltzmann Distribution of Energies for Ideal Gas. <u>Follow Lecture Discussions (see the Video Link on the Brightspace.</u> Present your work on the opposite site of this page.

ASSIGNMENT 4: CONT

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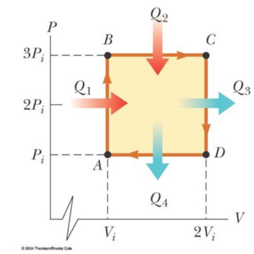
5 A 1.00-mol sample of a monatomic ideal gas is taken through the cycle shown. At point *A,* the pressure, volume, and temperature are P_i, V_i, and T_i, respectively. In terms of R and T_i, find (a) the total energy entering the system by heat per cycle, (b) the total energy leaving the system by heat per cycle, (c) the efficiency of an engine operating in this cycle,

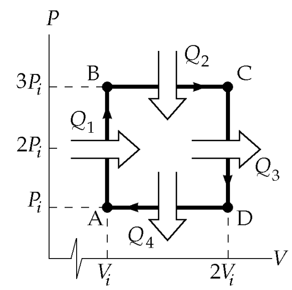


At point *A*, $P_iV_i = nRT_i$ and $n = 1.00$ mol

At point *B*, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point *C*, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point *D*, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$



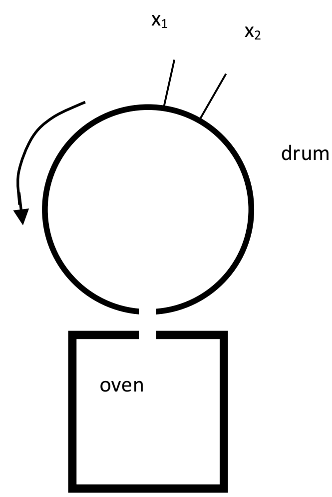
The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a)Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$ (b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c)Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

6 In the Zartman-Ko experiment Al atoms emerging from the oven (T= 400C) hit the plate on the opposite side of a drum. The drum radius is 5cm. We may assume that the speeds of atoms emerging from such oven are described by Maxwell-Boltzmann distribution. It was found that atoms with average speed strike the plate at x_1=0mm while the atoms with the most probable speed strike the plate at x_2 = 1.0mm. What is the angular velocity of the drum? Atomic mass of aluminum 27u.



Atoms travel along the straight line (diameter of the drum) before striking its wall. We can calculate the time necessare for each class of atoms to travel that distance

$$v_{avg} = \frac{2r}{t_1} \Longrightarrow t_1 = \frac{2r}{v_{avg}} = \frac{2r}{\sqrt{\frac{8kT}{\pi m}}} = \frac{0.1m}{726.3\frac{m}{s}} = 0.00013769s$$

$$v_{MP} = \frac{2r}{t_2} \Longrightarrow t_2 = \frac{2r}{v_{MP}} = \frac{2r}{\sqrt{\frac{2kT}{m}}} = \frac{0.1m}{643.6\frac{m}{s}} = 0.00015537s$$

$$\Delta t = t_2 - t_1 = 0.00015537 - 0.00013769 = 0.000017677s$$

During this time interval the drum will rotate by the angle (0.1cm/5cm) rad. Its angular velocity in rad/s is given by:

$$\omega = \frac{\theta}{\Delta t} = \frac{\frac{0.1cm}{5cm}\,rad}{0.000017677s} = 1131.4\frac{rad}{s}\left(= 180\frac{rev}{s}\right)$$